

Ratio, Proportion, and Percent

What You'll Learn

- **Lesson 6-1** Write ratios as fractions and find unit rates.
- **Lessons 6-2 and 6-3** Use ratios and proportions to solve problems, including scale drawings.
- **Lesson 6-4** Write decimals and fractions as percents and vice versa.
- **Lessons 6-5, 6-6, 6-7, and 6-8** Estimate and compute with percents.
- **Lesson 6-9** Find simple probability.

Key Vocabulary

- ratio (p. 264)
- rate (p. 265)
- proportion (p. 270)
- percent (p. 281)
- probability (p. 310)

Why It's Important

The concept of proportionality is the foundation of many branches of mathematics, including geometry, statistics, and business math. Proportions can be used to solve real-world problems dealing with scale drawings, indirect measurement, predictions, and money. *You will solve a problem about currency exchange rates in Lesson 6-2.*

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 6.

For Lesson 6-1

Convert Measurements

Complete each sentence. (For review, see pages 718–721.)

1. 2 ft = in.
2. 4 yd = ft
3. 2 mi = ft
4. 3 h = min
5. 8 min = s
6. 4 lb = oz
7. 2 T = lb
8. 5 gal = qt
9. 3 pt = c
10. 3 m = cm
11. 5.8 m = cm
12. 2 km = m
13. 5 cm = mm
14. 2.3 L = mL
15. 15 kg = g

For Lessons 6-2 and 6-3

Multiply Decimals

Find each product. (For review, see page 715.)

16. $7(3.4)$
17. $6.1(8)$
18. 2.8×5.9
19. 1.6×8.4
20. 0.8×9.3
21. $0.6(0.3)$
22. $12.4(3.8)$
23. 15.2×0.2

For Lesson 6-9

Write Fractions in Simplest Form

Simplify each fraction. If the fraction is already in simplest form, write *simplified*. (For review, see Lesson 4-5.)

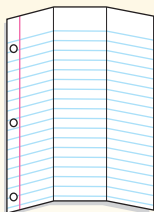
24. $\frac{4}{8}$
25. $\frac{5}{15}$
26. $\frac{6}{10}$
27. $\frac{12}{25}$
28. $\frac{22}{20}$
29. $\frac{15}{16}$
30. $\frac{36}{42}$
31. $\frac{36}{48}$

FOLDABLES[™] Study Organizer

Fractions, Decimals, and Percents Make this Foldable to help you organize your notes. Begin with a piece of notebook paper.

Step 1 Fold in Thirds

Fold in thirds lengthwise.



Step 2 Label

Draw lines along folds and label as shown.

Fraction	Decimal	Percent
○		
○		

Reading and Writing As you read and study the chapter, complete the table with the commonly-used fraction, decimal, and percent equivalents.

6-1 Ratios and Rates

What You'll Learn

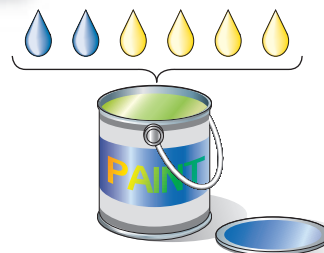
- Write ratios as fractions in simplest form.
- Determine unit rates.

Vocabulary

- ratio
- rate
- unit rate

How are ratios used in paint mixtures?

The diagram shows a gallon of paint that is made using 2 parts blue paint and 4 parts yellow paint.



- a. Which combination of paint would you use to make a smaller amount of the same shade of paint? Explain.



- b. Suppose you want to make the same shade of paint as the original mixture? How many parts of yellow paint should you use for each part of blue paint?

WRITE RATIOS AS FRACTIONS IN SIMPLEST FORM

A **ratio** is a comparison of two numbers by division. If a gallon of paint contains 2 parts blue paint and 4 parts yellow paint, then the ratio comparing the blue paint to the yellow paint can be written as follows.

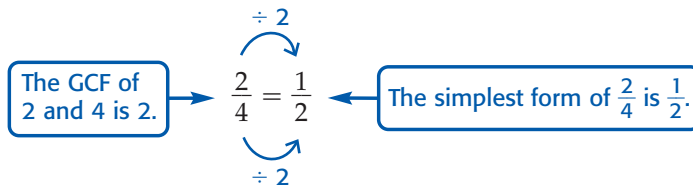
$$2 \text{ to } 4 \qquad 2:4 \qquad \frac{2}{4}$$

Recall that a fraction bar represents division. When the first number being compared is less than the second, the ratio is usually written as a fraction in simplest form.

Study Tip

Look Back

To review how to write a fraction in **simplest form**, see Lesson 4-5.



Example 1 Write Ratios as Fractions

Express the ratio 9 goldfish out of 15 fish as a fraction in simplest form.

$$\frac{9}{15} = \frac{3}{5}$$

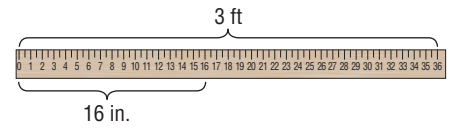
Divide the numerator and denominator by the GCF, 3.

The ratio of goldfish to fish is 3 to 5. This means that for every 5 fish, 3 of them are goldfish.

When writing a ratio involving measurements, both quantities should have the same unit of measure.

Example 2 Write Ratios as Fractions

Express the ratio 3 feet to 16 inches as a fraction in simplest form.



$$\frac{3 \text{ feet}}{16 \text{ inches}} = \frac{36 \text{ inches}}{16 \text{ inches}} \quad \text{Convert 3 feet to inches.}$$

$$= \frac{9 \text{ inches}}{4 \text{ inches}} \quad \text{Divide the numerator and denominator by the GCF, 4.}$$

Written in simplest form, the ratio is 9 to 4.

✓ Concept Check Give an example of a ratio in simplest form.

FIND UNIT RATES A **rate** is a ratio of two measurements having different kinds of units. Here are two examples of rates.

Miles and hours are different kinds of units.

65 miles in 3 hours

Dollars and pounds are different kinds of units.

\$16 for 2 pounds

When a rate is simplified so that it has a denominator of 1, it is called a **unit rate**. An example of a unit rate is \$5 per pound, which means \$5 per 1 pound.

Example 3 Find Unit Rate

SHOPPING A package of 20 recordable CDs costs \$18, and a package of 30 recordable CDs costs \$28. Which package has the lower cost per CD?

Find and compare the unit rates of the packages.

$$\frac{18 \text{ dollars}}{20 \text{ CDs}} = \frac{0.9 \text{ dollars}}{1 \text{ CD}} \quad \text{Divide the numerator and denominator by 20 to get a denominator of 1.}$$

For the 20-pack, the unit rate is \$0.90 per CD.

$$\frac{28 \text{ dollars}}{30 \text{ CDs}} = \frac{0.9\bar{3} \text{ dollars}}{1 \text{ CD}} \quad \text{Divide the numerator and denominator by 30 to get a denominator of 1.}$$

For the 30-pack, the unit rate is \$0.93 per CD.

So, the package that contains 20 CDs has the lower cost per CD.

Study Tip

Alternative Method

Another way to find the unit rate is to divide the cost of the package by the number of CDs in the package.

✓ Concept Check Is \$50 in 3 days a rate or a unit rate? Explain.

Study Tip

Look Back

To review **dimensional analysis**, see Lesson 5-3.

To convert a rate such as miles per hour to a rate such as feet per second, you can use dimensional analysis. Recall that this is the process of carrying units throughout a computation.

Example 4 Convert Rates

ANIMALS A grizzly bear can run 30 miles in 1 hour. How many feet is this per second?

You need to convert $\frac{30 \text{ mi}}{1 \text{ h}}$ to $\frac{\square \text{ ft}}{1 \text{ s}}$. There are 5280 feet in 1 mile and 3600 seconds in 1 hour. Write 30 miles per hour as $\frac{30 \text{ mi}}{1 \text{ h}}$.

$$\frac{30 \text{ mi}}{1 \text{ h}} = \frac{30 \text{ mi}}{1 \text{ h}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \div \frac{3600 \text{ s}}{1 \text{ h}}$$

Convert miles to feet and hours to seconds.

$$= \frac{30 \text{ mi}}{1 \text{ h}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ h}}{3600 \text{ s}}$$

The reciprocal of $\frac{3600 \text{ s}}{1 \text{ h}}$ is $\frac{1 \text{ h}}{3600 \text{ s}}$.

$$= \frac{\overset{1}{\cancel{30}} \text{ mi}}{\cancel{1} \text{ h}} \cdot \frac{\overset{44}{\cancel{5280}} \text{ ft}}{\cancel{1} \text{ mi}} \cdot \frac{\cancel{1} \text{ h}}{\underset{120}{\cancel{3600}} \text{ s}}$$

Divide the common factors and units.

$$= \frac{44 \text{ ft}}{\text{s}}$$

Simplify.

So, 30 miles per hour is equivalent to 44 feet per second.

Check for Understanding

Concept Check

1. Draw a diagram in which the ratio of circles to squares is 2:3.
2. Explain the difference between ratio and rate.
3. **OPEN ENDED** Give an example of a unit rate.

Guided Practice

Express each ratio as a fraction in simplest form.

4. 4 goals in 10 attempts
5. 15 dimes out of 24 coins
6. 10 inches to 3 feet
7. 5 feet to 5 yards

Express each ratio as a unit rate. Round to the nearest tenth, if necessary.

8. \$183 for 4 concert tickets
9. 9 inches of snow in 12 hours
10. 100 feet in 14.5 seconds
11. 254.1 miles on 10.5 gallons

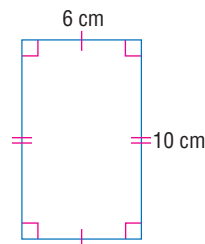
Convert each rate using dimensional analysis.

12. 20 mi/h = \square ft/min
13. 16 cm/s = \square m/h

Application

GEOMETRY For Exercises 14 and 15, refer to the figure below.

14. Express the ratio of width to length as a fraction in simplest form.
15. Suppose the width and length are each increased by 2 centimeters. Will the ratio of the width to length be the same as the ratio of the width to length of the original rectangle? Explain.



Practice and Apply

Homework Help

For Exercises	See Examples
16–27	1, 2
28–37	3
38–45	4
46, 47	3

Extra Practice
See page 736.

Express each ratio as a fraction in simplest form.

16. 6 ladybugs out of 27 insects
17. 14 girls to 35 boys
18. 18 cups to 45 cups
19. 12 roses out of 28 flowers
20. 7 cups to 9 pints
21. 9 pounds to 16 tons
22. 11 gallons to 11 quarts
23. 18 miles to 18 yards
24. 15 dollars out of 123 dollars
25. 17 rubies out of 118 gems
26. 155 apples to 75 oranges
27. 321 articles in 107 magazines

Express each ratio as a unit rate. Round to the nearest tenth, if necessary.

28. \$3 for 6 cans of tuna
 29. \$0.99 for 10 pencils
 30. 140 miles on 6 gallons
 31. 68 meters in 15 seconds
 32. 19 yards in 2.5 minutes
 33. 25 feet in 3.2 hours
 34. 236.7 miles in 4.5 days
 35. 331.5 pages in 8.5 weeks
36. **MAGAZINES** Which costs more per issue, an 18-issue subscription for \$40.50 or a 12-issue subscription for \$33.60? Explain.
37. **SHOPPING** Determine which is less expensive per can, a 6-pack of soda for \$2.20 or a 12-pack of soda for \$4.25. Explain.

Convert each rate using dimensional analysis.

38. 45 mi/h = ■ ft/s
39. 18 mi/h = ■ ft/s
40. 26 cm/s = ■ m/min
41. 32 cm/s = ■ m/min
42. 2.5 qt/min = ■ gal/h
43. 4.8 qt/min = ■ gal/h
44. 4 c/min = ■ qt/h
45. 7 c/min = ■ qt/h

46. **POPULATION** Population density is a unit rate that gives the number of people per square mile. Find the population density for each state listed in the table at the right. Round to the nearest whole number.

State	Population (2000)	Area (sq mi)
Alaska	626,932	570,374
New York	18,976,457	47,224
Rhode Island	1,048,319	1045
Texas	20,851,820	261,914
Wyoming	493,782	97,105

Source: U.S. Census Bureau



Online Research Data Update How has the population density of the states in the table changed since 2000? Visit www.pre-alg.com/data_update to learn more.

TRAVEL For Exercises 47 and 48, use the following information.

An airplane flew from Boston to Chicago to Denver. The distance from Boston to Chicago was 1015 miles and the distance from Chicago to Denver was 1011 miles. The plane traveled for 3.5 hours and carried 285 passengers.

47. About how fast did the airplane travel?
48. Suppose it costs \$5685 per hour to operate the airplane. Find the cost per person per hour for the flight.

More About . . .



Population

In 2000, the population density of the United States was about 79.6 people per square mile.

Source: *The World Almanac*



49. **CRITICAL THINKING** Marty and Spencer each saved money earned from shoveling snow. The ratio of Marty's money to Spencer's money is 3:1. If Marty gives Spencer \$3, their ratio will be 1:1. How much money did Marty earn?
50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are ratios used in paint mixtures?

Include the following in your answer:

- an example of a ratio of blue to yellow paint that would result in a darker shade of green, and
- an example of a ratio of blue to yellow paint that would result in a lighter shade of green.



51. Which ratio represents the same relationship as *for every 4 apples, 3 of them are green*?
- (A) 9:16 (B) 3:4 (C) 12:9 (D) 6:8
52. Joe paid \$2.79 for a gallon of milk. Find the cost per quart of milk.
- (A) \$0.70 (B) \$1.40 (C) \$0.93 (D) \$0.55

Extending the Lesson

53. Many objects such as credit cards or phone cards are shaped like golden rectangles.
- Find three different objects that are close to a golden rectangle. Make a table to display the dimensions and the ratio found in each object.
 - Describe how each ratio compares to the golden ratio.
 - RESEARCH** Use the Internet or another source to find three places where the golden rectangle is used in architecture.

A *golden rectangle* is a rectangle in which the ratio of the length to the width is approximately 1.618 to 1. This ratio is called the **golden ratio**.

Maintain Your Skills

Mixed Review

State whether each sequence is *arithmetic*, *geometric*, or *neither*. Then state the common difference or common ratio and write the next three terms of the sequence. (Lesson 5-10)

54. $-3, 6, -12, 24, \dots$ 55. $12.1, 12.4, 12.7, 13, \dots$

ALGEBRA Solve each equation. (Lesson 5-9)

56. $3.6 = x - 7.1$ 57. $y + \frac{3}{4} = \frac{2}{3}$ 58. $-4.8 = 6z$ 59. $\frac{3}{8}w = 5$
60. Find the quotient of $1\frac{1}{7}$ and $-\frac{4}{7}$. (Lesson 5-4)

Write each number in scientific notation. (Lesson 4-8)

61. 52,000,000 62. 42,240 63. 0.038
64. Write $8 \cdot (k + 3) \cdot (k + 3)$ using exponents. (Lesson 4-2)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation. (To review solving equations, see Lesson 3-4.)

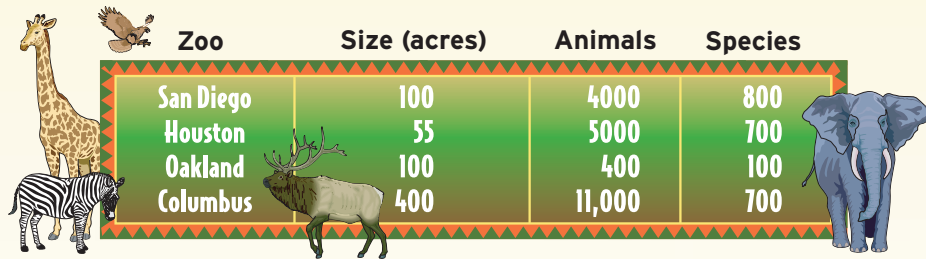
65. $10x = 300$ 66. $25m = 225$ 67. $8k = 320$
68. $192 = 4t$ 69. $195 = 15w$ 70. $231 = 33n$





Making Comparisons

In mathematics, there are many different ways to compare numbers. Consider the information in the table.



Zoo	Size (acres)	Animals	Species
San Diego	100	4000	800
Houston	55	5000	700
Oakland	100	400	100
Columbus	400	11,000	700

The following types of comparison statements can be used to describe this information.

Difference Comparisons

- The Houston Zoo has 1000 more animals than the San Diego Zoo.
- The Columbus Zoo is 345 acres larger than the Houston Zoo.
- The Oakland Zoo has 700 less species of animals than the San Diego Zoo.

Ratio Comparisons

- The ratio of the size of the San Diego Zoo to the size of the Columbus Zoo is 1:4. So, the San Diego Zoo is one-fourth the size of the Columbus Zoo.
- The ratio of the number of animals at the San Diego Zoo to the number of animals at the Oakland Zoo is 4000:400 or 10:1. So, San Diego Zoo has ten times as many animals as the Oakland Zoo.

Reading to Learn

1. Refer to the zoo information above. Write a difference comparison and a ratio comparison statement that describes the information.

Refer to the information below. Identify each statement as a difference comparison or a ratio comparison.

Florida <i>The Sunshine State</i>
Total area: 59,928 sq mi
Land area: 53,937 sq mi
Land forested: 26,478.4 sq mi

Ohio <i>The Buckeye State</i>
Total area: 44,828 sq mi
Land area: 40,953 sq mi
Land forested: 12,580.8 sq mi

Source: *The World Almanac*

2. The area of Florida is about 15,000 square miles greater than the area of Ohio.
3. The ratio of the amount of land forested in Ohio to the amount forested in Florida is about 1 to 2.
4. More than one-fourth of the land in Ohio is forested.

6-2 Using Proportions

What You'll Learn

- Solve proportions.
- Use proportions to solve real-world problems.

Vocabulary

- proportion
- cross products

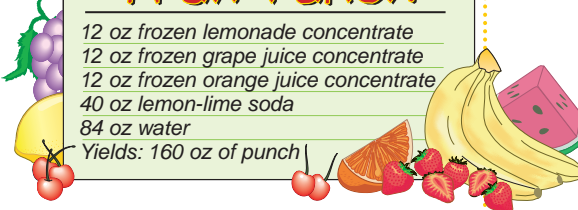
How are proportions used in recipes?

For many years, Phyllis Norman was famous in her neighborhood for making her flavorful fruit punch.

- For each of the first four ingredients, write a ratio that compares the number of ounces of each ingredient to the number of ounces of water.
- Double the recipe. Write a ratio for the ounces of each of the first four ingredients to the ounces of water as a fraction in simplest form.
- Are the ratios in parts a and b the same? Why or why not?

Fruit Punch

12 oz frozen lemonade concentrate
12 oz frozen grape juice concentrate
12 oz frozen orange juice concentrate
40 oz lemon-lime soda
84 oz water
Yields: 160 oz of punch



PROPORTIONS To solve problems that relate to ratios, you can use a proportion. A **proportion** is a statement of equality of two ratios.

Key Concept

Proportion

- **Words** A proportion is an equation stating that two ratios are equal.
- **Symbols** $\frac{a}{b} = \frac{c}{d}$
- **Example** $\frac{2}{3} = \frac{6}{9}$

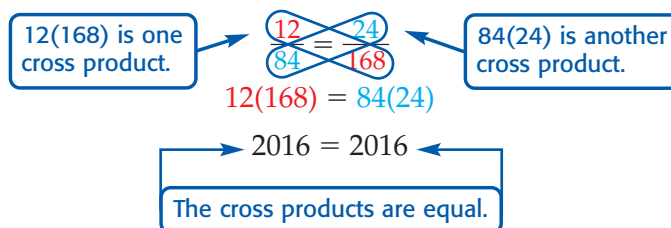
Consider the following proportion.

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} \cdot \frac{1}{1} \cdot \frac{1}{d} = \frac{c}{d} \cdot \frac{1}{1} \cdot \frac{1}{b} \quad \text{Multiply each side by } bd \text{ to eliminate the fractions.}$$

$$ad = cb \quad \text{Simplify.}$$

The products ad and cb are called the **cross products** of a proportion. Every proportion has two cross products.



Study Tip

Properties

When you multiply each side of an equation by bd , you are using the Multiplication Property of Equality.

Concept Check Write a proportion whose cross products are equal to 18.

Cross products can be used to determine whether two ratios form a proportion.

Key Concept

Property of Proportions

- **Words** The cross products of a proportion are equal.
- **Symbols** If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. If $ad = bc$, then $\frac{a}{b} = \frac{c}{d}$.

Example 1 Identify Proportions

Determine whether each pair of ratios forms a proportion.

a. $\frac{1}{3}, \frac{3}{9}$

$$\frac{1}{3} \stackrel{?}{=} \frac{3}{9} \quad \text{Write a proportion.}$$

$$1 \cdot 9 \stackrel{?}{=} 3 \cdot 3 \quad \text{Cross products}$$

$$9 = 9 \quad \text{Simplify.}$$

$$\text{So, } \frac{1}{3} = \frac{3}{9}.$$

b. $\frac{1.2}{4.0}, \frac{2}{5}$

$$\frac{1.2}{4.0} \stackrel{?}{=} \frac{2}{5} \quad \text{Write a proportion.}$$

$$1.2 \cdot 5 \stackrel{?}{=} 4.0 \cdot 2 \quad \text{Cross products}$$

$$6 \neq 8 \quad \text{Simplify.}$$

$$\text{So, } \frac{1.2}{4.0} \neq \frac{2}{5}.$$

Example 2 Solve Proportions

Solve each proportion.

a. $\frac{a}{25} = \frac{52}{100}$

$$\frac{a}{25} = \frac{52}{100}$$

$$a \cdot 100 = 25 \cdot 52 \quad \text{Cross products}$$

$$100a = 1300 \quad \text{Multiply.}$$

$$\frac{100a}{100} = \frac{1300}{100} \quad \text{Divide.}$$

$$a = 13$$

The solution is 13.

b. $\frac{12.5}{m} = \frac{15}{7.5}$

$$\frac{12.5}{m} = \frac{15}{7.5}$$

$$12.5 \cdot 7.5 = m \cdot 15 \quad \text{Cross products}$$

$$93.75 = 15m \quad \text{Multiply.}$$

$$\frac{93.75}{15} = \frac{15m}{15} \quad \text{Divide.}$$

$$6.25 = m$$

The solution is 6.25.

Study Tip

Cross Products

When you find cross products, you are *cross multiplying*.

USE PROPORTIONS TO SOLVE REAL-WORLD PROBLEMS When you solve a problem using a proportion, be sure to compare the quantities in the same order.

Example 3 Use a Proportion to Solve a Problem

FOOD Refer to the recipe at the beginning of the lesson. How much soda should be used if 16 ounces of each type of juice are used?

Explore You know how much soda to use for 12 ounces of each type of juice. You need to find how much soda to use for 16 ounces of each type of juice.

Plan Write and solve a proportion using ratios that compare juice to soda. Let s represent the amount of soda to use in the new recipe.

(continued on the next page)

More About . . .



Attractions

The world's largest baseball bat is located in Louisville, Kentucky. It is 120 feet long, has a diameter from 3.5 to 9 feet, and weighs 68,000 pounds.

Source: World's Largest Roadside Attractions

Solve

$$\frac{\text{juice in original recipe}}{\text{soda in original recipe}} = \frac{\text{juice in new recipe}}{\text{soda in new recipe}}$$

$$\frac{12}{40} = \frac{16}{s} \quad \text{Write a proportion.}$$

$$12 \cdot s = 40 \cdot 16 \quad \text{Cross products}$$

$$12s = 640 \quad \text{Multiply.}$$

$$\frac{12s}{12} = \frac{640}{12} \quad \text{Divide.}$$

$$s = 53\frac{1}{3} \quad \text{Simplify.}$$

$53\frac{1}{3}$ ounces of soda should be used.

Explore Check the cross products. Since $12 \cdot 53\frac{1}{3} = 640$ and $40 \cdot 16 = 640$, the answer is correct.

Proportions can also be used in measurement problems.

Example 4 Convert Measurements

ATTRACTIONS Louisville, Kentucky, is home to the world's largest baseball glove. The glove is 4 feet high, 10 feet long, 9 feet wide, and weighs 15 tons. Find the height of the glove in centimeters if $1 \text{ ft} = 30.48 \text{ cm}$.

Let x represent the height in centimeters.

$$\begin{array}{l} \text{customary measurement} \rightarrow \\ \text{metric measurement} \rightarrow \end{array} \frac{1 \text{ ft}}{30.48 \text{ cm}} = \frac{4 \text{ ft}}{x \text{ cm}} \quad \begin{array}{l} \leftarrow \text{customary measurement} \\ \leftarrow \text{metric measurement} \end{array}$$

$$1 \cdot x = 30.48 \cdot 4 \quad \text{Cross products}$$

$$x = 121.92 \quad \text{Simplify.}$$

The height of the glove is 121.92 centimeters.

Check for Understanding

Concept Check 1. Define *proportion*.

2. **OPEN ENDED** Find two counterexamples for the statement *Two ratios always form a proportion*.

Guided Practice Determine whether each pair of ratios forms a proportion.

3. $\frac{1}{4'} \frac{4}{16}$

4. $\frac{2.1}{3.5'} \frac{3}{7}$

ALGEBRA Solve each proportion.

5. $\frac{k}{35} = \frac{3}{7}$

6. $\frac{3}{t} = \frac{18}{24}$

7. $\frac{10}{8.4} = \frac{5}{m}$

Application 8. **PHOTOGRAPHY** A $3'' \times 5''$ photo is enlarged so that the length of the new photo is 7 inches. Find the width of the new photo.



Practice and Apply

Homework Help

For Exercises	See Examples
9–14	1
15–31	2
32–35, 36–42	3, 4

Extra Practice
See page 737.

Determine whether each pair of ratios forms a proportion.

9. $\frac{2}{3}$ ' $\frac{8}{12}$

10. $\frac{4}{2}$ ' $\frac{16}{5}$

11. $\frac{1.5}{5.0}$ ' $\frac{3}{9}$

12. $\frac{18}{2.4}$ ' $\frac{15}{2}$

13. $\frac{3.4}{1.6}$ ' $\frac{5.1}{2.4}$

14. $\frac{5.3}{15.9}$ ' $\frac{2.7}{8.1}$

ALGEBRA Solve each proportion.

15. $\frac{p}{6} = \frac{24}{36}$

16. $\frac{w}{11} = \frac{14}{22}$

17. $\frac{4}{10} = \frac{8}{a}$

18. $\frac{18}{12} = \frac{24}{q}$

19. $\frac{5}{h} = \frac{10}{30}$

20. $\frac{51}{z} = \frac{17}{7}$

21. $\frac{7}{45} = \frac{x}{9}$

22. $\frac{2}{15} = \frac{c}{72}$

23. $\frac{7}{5} = \frac{10.5}{b}$

24. $\frac{16}{7} = \frac{4.8}{h}$

25. $\frac{2}{9.4} = \frac{0.2}{v}$

26. $\frac{9}{7.2} = \frac{3.5}{k}$

27. $\frac{a}{0.28} = \frac{4}{1.4}$

28. $\frac{3}{14} = \frac{15}{m-3}$

29. $\frac{16}{x+5} = \frac{4}{5}$

30. Find the value of d that makes $\frac{5.1}{1.7} = \frac{7.5}{d}$ a proportion.

31. What value of m makes $\frac{6.5}{1.3} = \frac{m}{5.2}$ a proportion?

Write a proportion that could be used to solve for each variable. Then solve.

32. 8 pencils in 2 boxes
20 pencils in x boxes

33. 12 glasses in 3 crates
72 glasses in m crates

34. y dollars for 5.4 gallons
14 dollars for 3 gallons

35. 5 quarts for \$6.25
 d quarts for \$8.75

OLYMPICS For Exercises 36 and 37, use the following information.

There are approximately 3.28 feet in 1 meter.

36. Write a proportion that could be used to find the distance in feet of the 110-meter dash.

37. What is the distance in feet of the 110-meter dash?

38. **PHOTOGRAPHY** Suppose an 8" \times 10" photo is reduced so that the width of the new photo is 4.5 inches. What is the length of the new photo?

CURRENCY For Exercises 39 and 40, use the following information and the table shown.

The table shows the exchange rates for certain countries compared to the U.S. dollar on a given day.

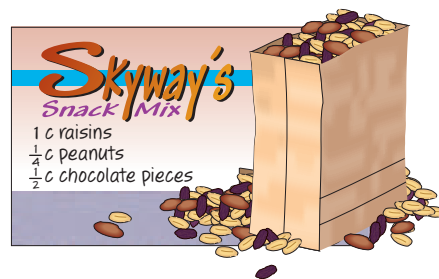
39. What is the cost of an item in U.S. dollars if it costs 14.99 in British pounds?

40. Find the cost of an item in U.S. dollars if it costs 12.50 in Egyptian pounds.

Country	Rate
United Kingdom	0.667
Egypt	3.481
Australia	1.712
China	8.280



41. **SNACKS** The Skyway Snack Company makes a snack mix that contains raisins, peanuts, and chocolate pieces. The ingredients are shown at the right. Suppose the company wants to sell a larger-sized bag that contains 6 cups of raisins. How many cups of chocolate pieces and peanuts should be added?



42. **PAINT** If 1 pint of paint is needed to paint a square that is 5 feet on each side, how many pints must be purchased in order to paint a square that is 9 feet 6 inches on each side?
43. **CRITICAL THINKING** The Property of Proportions states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. Write two proportions in which the cross products are ad and bc .
44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are proportions used in recipes?

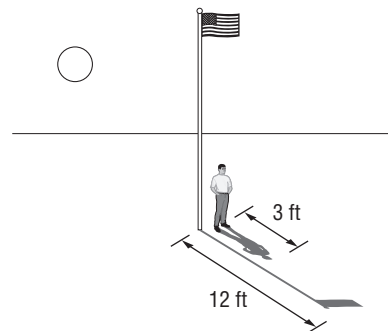
Include the following in your answer:

- an explanation telling how proportions can be used to increase or decrease the amount of ingredients needed, and
- an explanation of why adding 10 ounces to each ingredient in the punch recipe will not result in the same flavor of punch.



45. Jack is standing next to a flagpole as shown at the right. Jack is 6 feet tall. Which proportion could you use to find the height of the flagpole?

- (A) $\frac{3}{6} = \frac{x}{12}$ (B) $\frac{x}{6} = \frac{3}{12}$
 (C) $\frac{6}{3} = \frac{x}{12}$ (D) $\frac{3}{x} = \frac{12}{6}$



Maintain Your Skills

Mixed Review Express each ratio as a unit rate. Round to the nearest tenth, if necessary.
 (Lesson 6-1)

46. \$5 for 4 loaves of bread 47. 183.4 miles in 3.2 hours
48. Find the next three numbers in the sequence 2, 5, 8, 11, 14, . . .
 (Lesson 5-10)

ALGEBRA Find each quotient. (Lesson 5-4)

49. $\frac{x}{5} \div \frac{x}{20}$ 50. $\frac{3y}{4} \div \frac{5y}{8}$ 51. $\frac{4z}{w} \div \frac{7yz}{w}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Complete each sentence.
 (To review **converting measurements**, see pages 720 and 721.)

52. 5 feet = ■ inches 53. 8.5 feet = ■ inches
 54. 36 inches = ■ feet 55. 78 inches = ■ feet



Algebra Activity

A Follow-Up of Lesson 6-2

Capture-Recapture

Scientists often determine the number of fish in a pond, lake, or other body of water by using the *capture-recapture* method. A number of fish are captured, counted, carefully tagged, and returned to their habitat. The tagged fish are counted again and proportions are used to estimate the entire population. In this activity, you will model this estimation technique.

Collect the Data

Step 1 Copy the table below onto a sheet of paper.

Original Number Captured:		
Sample	Recaptured	Tagged
1		
2		
3		
4		
⋮		
10		
Total		



Step 2 Empty a bag of dried beans into a paper bag.

Step 3 Remove a handful of beans. Using a permanent marker, place an X on each side of each bean. These beans will represent the tagged fish. Record this number at the top of your table as the original number captured. Return the beans to the bag and mix.

Step 4 Remove a second handful of beans without looking. This represents the first sample of recaptured fish. Record the number of beans. Then count and record the number of beans that are tagged. Return the beans to the bag and mix.

Step 5 Repeat Step 4 for samples 2 through 10. Then use the results to find the total number of recaptured fish and the total number of tagged fish.

Analyze the Data

1. Use the following proportion to estimate the number of beans in the bag.

$$\frac{\text{original number captured}}{\text{total number in bag}} = \frac{\text{total number tagged}}{\text{total number recaptured}}$$

2. Count the number of beans in the bag. Compare the estimate to the actual number.

Make a Conjecture

- Why is it a good idea to base a prediction on several samples instead of one sample?
- Why does this method work?



What You'll Learn

- Use scale drawings.
- Construct scale drawings.

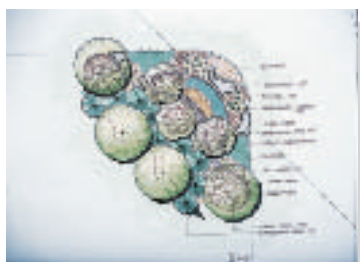
Vocabulary

- scale drawing
- scale model
- scale
- scale factor

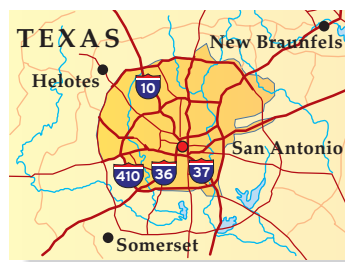
How are scale drawings used in everyday life?

A set of landscape plans and a map are shown.

Designers use blueprints when planning landscapes.



Maps are used to find actual distances between cities.



- Suppose the landscape plans are drawn on graph paper and the side of each square on the paper represents 2 feet. What is the actual width of a rose garden if its width on the drawing is 4 squares long?
- All maps have a scale. How can the scale help you estimate the distance between cities?

USE SCALE DRAWINGS AND MODELS A **scale drawing** or a **scale model** is used to represent an object that is too large or too small to be drawn or built at actual size. A few examples are maps, blueprints, model cars, and model airplanes.

Model cars are replicas of actual cars.

**✓ Concept Check** Why are scale drawings or scale models used?

The **scale** gives the relationship between the measurements on the drawing or model and the measurements of the real object. Consider the following scales.

$$1 \text{ inch} = 3 \text{ feet}$$

1 inch represents an actual distance of 3 feet.

$$1:24$$

1 unit represents an actual distance of 24 units.

Study Tip

Scale Factor

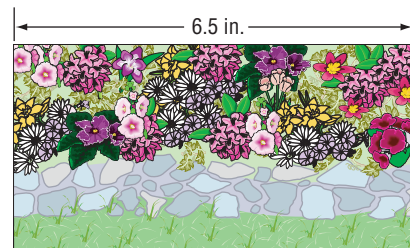
When finding the scale factor, be sure to use the same units of measure.

The ratio of a length on a scale drawing or model to the corresponding length on the real object is called the **scale factor**. Suppose a scale model has a scale of 2 inches = 16 inches. The scale factor is $\frac{2}{16}$ or $\frac{1}{8}$.

The lengths and widths of objects of a scale drawing or model are proportional to the lengths and widths of the actual object.

Example 1 Find Actual Measurements

DESIGN A set of landscape plans shows a flower bed that is 6.5 inches wide. The scale on the plans is 1 inch = 4 feet.



- a. What is the width of the actual flower bed?

Let x represent the actual width of the flower bed. Write and solve a proportion.

$$\begin{array}{l} \text{plan width} \rightarrow \frac{1 \text{ inch}}{4 \text{ feet}} = \frac{6.5 \text{ inches}}{x \text{ feet}} \quad \leftarrow \text{plan width} \\ \text{actual width} \rightarrow \end{array}$$

$$1 \cdot x = 4 \cdot 6.5 \quad \text{Find the cross products.}$$

$$x = 26 \quad \text{Simplify.}$$

The actual width of the flower bed is 26 feet.

- b. What is the scale factor?

To find the scale factor, write the ratio of 1 inch to 4 feet in simplest form.

$$\frac{1 \text{ inch}}{4 \text{ feet}} = \frac{1 \text{ inch}}{48 \text{ inches}} \quad \text{Convert 4 feet to inches.}$$

The scale factor is $\frac{1}{48}$. That is, each measurement on the plan is $\frac{1}{48}$ the actual measurement.

Example 2 Determine the Scale

ARCHITECTURE The inside of the Lincoln Memorial contains three chambers. The central chamber, which features a marble statue of Abraham Lincoln, has a height of 60 feet. Suppose a scale model of the chamber has a height of 4 inches. What is the scale of the model?

Write the ratio of the height of the model to the actual height of the statue. Then solve a proportion in which the height of the model is 1 inch and the actual height is x feet.

$$\begin{array}{l} \text{model height} \rightarrow \frac{4 \text{ inches}}{60 \text{ feet}} = \frac{1 \text{ inch}}{x \text{ feet}} \quad \leftarrow \text{model height} \\ \text{actual height} \rightarrow \end{array}$$

$$4 \cdot x = 60 \cdot 1 \quad \text{Find the cross products.}$$

$$4x = 60 \quad \text{Simplify.}$$

$$\frac{4x}{4} = \frac{60}{4} \quad \text{Divide each side by 4.}$$

$$x = 15 \quad \text{Simplify.}$$

So, the scale is 1 inch = 15 feet.

More About . . .



Architecture

The exterior of the Lincoln Memorial features 36 columns that represent the states in the Union when Lincoln died in 1865. Each column is 44 feet high.

Source: www.infoplease.com

CONSTRUCT SCALE DRAWINGS To construct a scale drawing of an object, use the actual measurements of the object and the scale to which the object is to be drawn.

Example 3 *Construct a Scale Drawing*

INTERIOR DESIGN Antonio is designing a room that is 20 feet long and 12 feet wide. Make a scale drawing of the room. Use a scale of 0.25 inch = 4 feet.

Step 1 Find the measure of the room's length on the drawing. Let x represent the length.

$$\begin{array}{l} \text{drawing length} \rightarrow \frac{0.25 \text{ inch}}{4 \text{ feet}} = \frac{x \text{ inches}}{20 \text{ feet}} \quad \leftarrow \text{drawing length} \\ \text{actual length} \rightarrow \quad \leftarrow \text{actual length} \end{array}$$

$$0.25 \cdot 20 = 4 \cdot x \quad \text{Find the cross products.}$$

$$5 = 4x \quad \text{Simplify.}$$

$$1.25 = x \quad \text{Divide each side by 4.}$$

On the drawing, the length is 1.25 or $1\frac{1}{4}$ inches.

Step 2 Find the measure of the room's width on the drawing. Let w represent the width.

$$\begin{array}{l} \text{drawing length} \rightarrow \frac{0.25 \text{ inch}}{4 \text{ feet}} = \frac{w \text{ inches}}{12 \text{ feet}} \quad \leftarrow \text{drawing length} \\ \text{actual length} \rightarrow \quad \leftarrow \text{actual length} \end{array}$$

$$0.25 \cdot 12 = 4 \cdot w \quad \text{Find the cross products.}$$

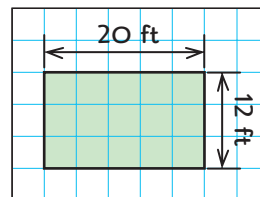
$$3 = 4w \quad \text{Simplify.}$$

$$\frac{3}{4} = \frac{4w}{4} \quad \text{Divide each side by 4.}$$

$$0.75 = w \quad \text{Simplify.}$$

On the drawing, the width is 0.75 or $\frac{3}{4}$ inch.

Step 3 Make the scale drawing. Use $\frac{1}{4}$ -inch grid paper. Since $1\frac{1}{4}$ inches = 5 squares and $\frac{3}{4}$ inch = 3 squares, draw a rectangle that is 5 squares by 3 squares.



Career Choices

Interior Designer

Interior designers plan the space and furnish the interiors of places such as homes, offices, restaurants, hotels, hospitals, and even theaters. Creativity and knowledge of computer-aided design software is essential.

Online Research
For information about a career as an interior designer, visit:
www.pre-alg.com/careers

Check for Understanding

- Concept Check**
- OPEN ENDED** Draw two squares in which the ratio of the sides of the first square to the sides of the second square is 1:3.
 - FIND THE ERROR** Montega and Luisa are rewriting the scale 1 inch = 2 feet in $a:b$ form.

Montega
1:36

Luisa
1:24

Who is correct? Explain your reasoning.



Guided Practice On a map of Pennsylvania, the scale is 1 inch = 20 miles. Find the actual distance for each map distance.

	From	To	Map Distance
3.	Pittsburgh	Perryopolis	2 inches
4.	Johnston	Homer City	$1\frac{3}{4}$ inches

- Applications** **STATUES** For Exercises 5 and 6, use the following information. The Statue of Zeus at Olympia is one of the Seven Wonders of the World. On a scale model of the statue, the height of Zeus is 8 inches.
- If the actual height of Zeus is 40 feet, what is the scale of the statue?
 - What is the scale factor?
7. **DESIGN** An architect is designing a room that is 15 feet long and 10 feet wide. Construct a scale drawing of the room. Use a scale of 0.5 in. = 10 ft.

Practice and Apply

Homework Help

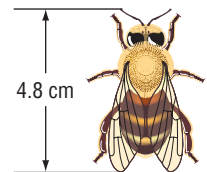
For Exercises	See Examples
8–17	1
18, 19	1, 2
20	3

Extra Practice
See page 737.

On a set of architectural drawings for an office building, the scale is $\frac{1}{2}$ inch = 3 feet. Find the actual length of each room.

	Room	Drawing Distance
8.	Conference Room	7 inches
9.	Lobby	2 inches
10.	Mail Room	2.3 inches
11.	Library	4.1 inches
12.	Copy Room	2.2 inches
13.	Storage	1.9 inches
14.	Exercise Room	$3\frac{3}{4}$ inches
15.	Cafeteria	$8\frac{1}{4}$ inches

- Refer to Exercises 8–15. What is the scale factor?
 - What is the scale factor if the scale is 8 inches = 1 foot?
18. **ROLLER COASTERS** In a scale model of a roller coaster, the highest hill has a height of 6 inches. If the actual height of the hill is 210 feet, what is the scale of the model?
19. **INSECTS** In an illustration of a honeybee, the length of the bee is 4.8 centimeters. The actual size of the honeybee is 1.2 centimeters. What is the scale of the drawing?



20. **GARDENS** A garden is 8 feet wide by 16 feet long. Make a scale drawing of the garden that has a scale of $\frac{1}{4}$ in. = 2 ft.

21. **CRITICAL THINKING** What does it mean if the scale factor of a scale drawing or model is less than 1? greater than 1? equal to 1?
22. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are scale drawings used in everyday life?

Include the following in your answer:

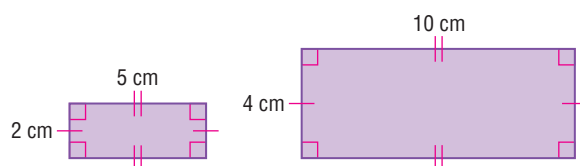
- an example of three kinds of scale drawings or models, and
- an explanation of how you use scale drawings in your life.



23. Which scale has a scale factor of $\frac{1}{18}$?
 (A) 3 in. = 6 ft (B) 6 in. = 9 ft (C) 3 in. = 54 ft (D) 6 in. = 6 ft
24. A model airplane is built using a 1:16 scale. On the model, the length of the wing span is 5.8 feet. What is the actual length of the wing?
 (A) 84.8 ft (B) 91.6 ft (C) 92.8 ft (D) 89.8 ft

Extending the Lesson

25. Two rectangles are shown. The ratio comparing their sides is 1:2.



- Write the ratio that compares their perimeters.
- Write the ratio that compares their areas.
- Find the perimeter and area of a 3-inch by 5-inch rectangle. Then make a conjecture about the perimeter and area of a 6-inch by 10-inch rectangle. Check by finding the actual perimeter and area.

Maintain Your Skills

Mixed Review

Solve each proportion. (Lesson 6-2)

26. $\frac{n}{20} = \frac{15}{50}$ 27. $\frac{14}{32} = \frac{x}{8}$ 28. $\frac{3}{2.2} = \frac{7.5}{y}$

Convert each rate using dimensional analysis. (Lesson 6-1)

29. 36 cm/s = ■ m/min 30. 66 gal/h = ■ qt/min

31. Find $1\frac{1}{4} + 4\frac{5}{6}$. Write the answer in simplest form. (Lesson 5-7)

ALGEBRA Find each product or quotient. Express in exponential form. (Lesson 4-6)

32. $4^3 \cdot 4^5$ 33. $3t^4 \cdot 6t$ 34. $7^{14} \div 7^8$ 35. $\frac{24m^5}{18m^2}$

36. **ALGEBRA** Find the greatest common factor of $14x^2y$ and $35xy^3$. (Lesson 4-4)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each fraction.

(To review **simplest form**, see Lesson 4-5.)

37. $\frac{5}{100}$ 38. $\frac{25}{100}$ 39. $\frac{40}{100}$ 40. $\frac{52}{100}$
 41. $\frac{78}{100}$ 42. $\frac{75}{100}$ 43. $\frac{82}{100}$ 44. $\frac{95}{100}$



Fractions, Decimals, and Percents

What You'll Learn

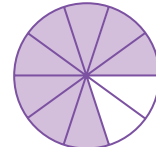
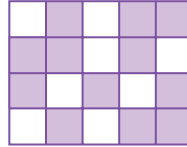
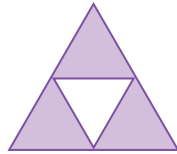
- Express percents as fractions and vice versa.
- Express percents as decimals and vice versa.

Vocabulary

- percent

How are percents related to fractions and decimals?

A portion of each figure is shaded.



- Write a ratio that compares the shaded region of each figure to its total region as a fraction in simplest form.
- Rewrite each fraction using a denominator of 100.
- Which figure has the greatest part of its area shaded?
- Was it easier to compare the fractions in part **a** or part **b**? Explain.

Reading Math

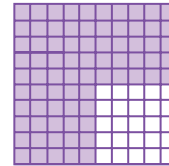
Percent

Root Word: Cent

There are 100 cents in one dollar.
Percent means *per hundred* or *hundredths*.

PERCENTS AND FRACTIONS

A **percent** is a ratio that compares a number to 100. The meaning of 75% is shown at the right. In the figure, 75 out of 100 squares are shaded.



To write a percent as a fraction, express the ratio as a fraction with a denominator of 100. Then simplify if possible. Notice that a percent can be greater than 100% or less than 1%.

Example 1 Percents as Fractions

Express each percent as a fraction in simplest form.

a. 45%

$$\begin{aligned} 45\% &= \frac{45}{100} \\ &= \frac{9}{20} \end{aligned}$$

b. 120%

$$\begin{aligned} 120\% &= \frac{120}{100} \\ &= \frac{6}{5} \text{ or } 1\frac{1}{5} \end{aligned}$$

c. 0.5%

$$\begin{aligned} 0.5\% &= \frac{0.5}{100} \\ &= \frac{0.5}{100} \cdot \frac{10}{10} \\ &= \frac{5}{1000} \text{ or } \frac{1}{200} \end{aligned}$$

Multiply by $\frac{10}{10}$ to eliminate the decimal in the numerator.

d. $83\frac{1}{3}\%$

$$\begin{aligned} 83\frac{1}{3}\% &= \frac{83\frac{1}{3}}{100} \\ &= 83\frac{1}{3} \div 100 \\ &= \frac{5}{3} \cdot \frac{1}{100} \text{ or } \frac{5}{6} \end{aligned}$$

The fraction bar indicates division.

To write a fraction as a percent, write an equivalent fraction with a denominator of 100.

Example 2 Fractions as Percents

Express each fraction as a percent.

a. $\frac{4}{5}$

$$\frac{4}{5} = \frac{80}{100} \text{ or } 80\%$$

b. $\frac{9}{4}$

$$\frac{9}{4} = \frac{225}{100} \text{ or } 225\%$$

PERCENTS AND DECIMALS Remember that *percent* means *per hundred*. In the previous examples, you wrote percents as fractions with 100 in the denominator. Similarly, you can write percents as decimals by dividing by 100.

Key Concept

Percents and Decimals

- To write a percent as a decimal, divide by 100 and remove the percent symbol.
- To write a decimal as a percent, multiply by 100 and add the percent symbol.

Example 3 Percents as Decimals

Express each percent as a decimal.

a. 28%

$$28\% = \frac{28}{100} = 0.28 \quad \text{Divide by 100 and remove the \%}$$

b. 8%

$$8\% = \frac{8}{100} = 0.08 \quad \text{Divide by 100 and remove the \%}$$

c. 375%

$$375\% = \frac{375}{100} = 3.75 \quad \text{Divide by 100 and remove the \%}$$

d. 0.5%

$$0.5\% = \frac{0.5}{100} = 0.005 \quad \text{Divide by 100 and remove the \%}$$

Study Tip

Mental Math

To divide a number by 100, move the decimal point two places to the left. To multiply a number by 100, move the decimal point two places to the right.

Example 4 Decimals as Percents

Express each decimal as a percent.

a. 0.35

$$0.35 = \frac{35}{100} = 35\% \quad \text{Multiply by 100 and add the \%}$$

b. 0.09

$$0.09 = \frac{9}{100} = 9\% \quad \text{Multiply by 100 and add the \%}$$

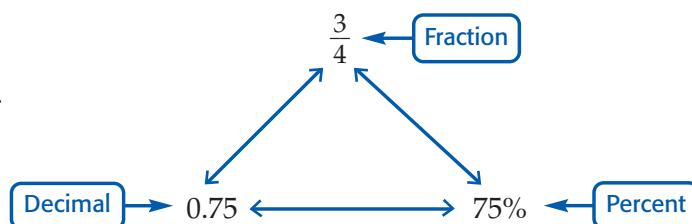
c. 0.007

$$0.007 = \frac{0.7}{100} = 0.7\% \quad \text{Multiply by 100 and add the \%}$$

d. 1.49

$$1.49 = \frac{149}{100} = 149\% \quad \text{Multiply by 100 and add the \%}$$

You have expressed fractions as decimals and decimals as percents. Fractions, decimals, and percents are all different names that represent the same number.



You can also express a fraction as a percent by first expressing the fraction as a decimal and then expressing the decimal as a percent.

Example 5 Fractions as Percents

Express each fraction as a percent. Round to the nearest tenth percent, if necessary.

a. $\frac{7}{8}$

$$\frac{7}{8} = 0.875$$

$$= 87.5\%$$

b. $\frac{2}{3}$

$$\frac{2}{3} = 0.6666666\dots$$

$$\approx 66.7\%$$

c. $\frac{3}{500}$

$$\frac{3}{500} = 0.006$$

$$= 0.6\%$$

d. $\frac{15}{7}$

$$\frac{15}{7} \approx 2.1428571$$

$$\approx 214.3\%$$

Study Tip

Fractions

When the numerator of a fraction is less than the denominator, the fraction is less than 100%. When the numerator of a fraction is greater than the denominator, the fraction is greater than 100%.

Example 6 Compare Numbers

SHOES In a survey, one-fifth of parents said that they buy shoes for their children every 4–5 months while 27% of parents said that they buy shoes twice a year. Which of these groups is larger?

Write one-fifth as a percent. Then compare.

$$\frac{1}{5} = 0.20 \text{ or } 20\%$$

Since 27% is greater than 20%, the group that said they buy shoes twice a year is larger.

Check for Understanding

Concept Check

- Describe two ways to express a fraction as a percent. Then tell how you know whether a fraction is greater than 100% or less than 1%.
- OPEN ENDED** Explain the method you would use to express $64\frac{1}{2}\%$ as a decimal.

Guided Practice

Express each percent as a fraction or mixed number in simplest form and as a decimal.

3. 30%

4. $12\frac{1}{2}\%$

5. 125%

6. 65%

7. 135%

8. 0.2%

Express each decimal or fraction as a percent. Round to the nearest tenth percent, if necessary.

9. 0.45

10. 1.3

11. 0.008

12. $\frac{1}{4}$

13. $\frac{12}{9}$

14. $\frac{3}{600}$

Application

- MEDIA** In a survey, 55% of those surveyed said that they get the news from their local television station while three-fifths said that they get the news from a daily newspaper. From which source do more people get their news?



Practice and Apply

Homework Help

For Exercises	See Examples
16–27	1, 3
28–39	2, 4, 5
40, 41	1
42	6

Extra Practice

See page 737.

Express each percent as a fraction or mixed number in simplest form and as a decimal.

- | | | | |
|----------|----------|-----------------------|-----------|
| 16. 42% | 17. 88% | 18. $16\frac{2}{3}\%$ | 19. 87.5% |
| 20. 150% | 21. 350% | 22. 18% | 23. 61% |
| 24. 117% | 25. 223% | 26. 0.8% | 27. 0.53% |

Express each decimal or fraction as a percent. Round to the nearest tenth percent, if necessary.

- | | | | |
|--------------------|--------------------|-----------------------|-----------------------|
| 28. 0.51 | 29. 0.09 | 30. 3.21 | 31. 2.7 |
| 32. 0.0042 | 33. 0.0006 | 34. $\frac{7}{25}$ | 35. $\frac{9}{40}$ |
| 36. $\frac{10}{3}$ | 37. $\frac{14}{8}$ | 38. $\frac{15}{2500}$ | 39. $\frac{20}{1200}$ |

40. **GEOGRAPHY** Forty-six percent of the world's water is in the Pacific Ocean. What fraction is this?

41. **GEOGRAPHY** The Arctic Ocean contains 3.7% of the world's water. What fraction is this?

- 42. **FOOD** According to a survey, 22% of people said that mustard is their favorite condiment while two-fifths of people said that they prefer ketchup. Which group is larger? Explain.

Choose the greatest number in each set.

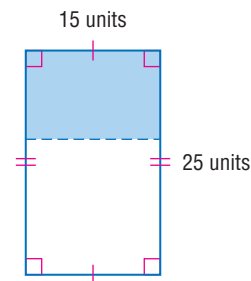
- | | |
|---|--|
| 43. $\left\{\frac{2}{5}, 0.45, 35\%, 3 \text{ out of } 8\right\}$ | 44. $\left\{\frac{3}{4}, 0.70, 78\%, 4 \text{ out of } 5\right\}$ |
| 45. $\left\{19\%, \frac{3}{16}, 0.155, 2 \text{ to } 15\right\}$ | 46. $\left\{89\%, \frac{10}{11}, 0.884, 12 \text{ to } 14\right\}$ |

Write each list of numbers in order from least to greatest.

- | | |
|-------------------------------|--------------------------------|
| 47. $\frac{2}{3}, 61\%, 0.69$ | 48. $\frac{2}{7}, 0.027, 27\%$ |
|-------------------------------|--------------------------------|

GEOMETRY For Exercises 49 and 50, use the information and the figure shown. Suppose that two fifths of the rectangle is shaded.

49. Write the decimal that represents the shaded region of the figure.
50. What is the area of the shaded region?



51. **CRITICAL THINKING** Find a fraction that satisfies the conditions below. Then write a sentence explaining why you think your fraction is or is not the only solution that satisfies the conditions.

- The fraction can be written as a percent greater than 1%.
- The fraction can be written as a percent less than 50%.
- The decimal equivalent of the fraction is a terminating decimal.
- The value of the denominator minus the value of the numerator is 3.

More About . . .



Food

The three types of mustard commonly grown are white or yellow mustard, brown mustard, and Oriental mustard.

Source: Morehouse Foods, Inc.

52. **CRITICAL THINKING** Explain why percents are rational numbers.
53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are percents related to fractions and decimals?

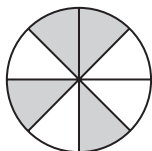
Include the following in your answer:

- examples of figures in which 25%, 30%, 40%, and 65% of the area is shaded, and
- an explanation of why each percent represents the shaded area.

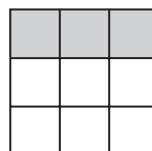


54. Assuming that the regions in each figure are equal, which figure has the greatest part of its area shaded?

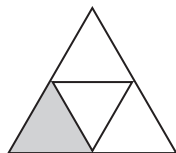
(A)



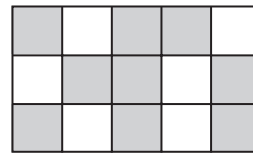
(B)



(C)



(D)



55. According to a survey, 85% of people eat a salad at least once a week. Which ratio represents this portion?
- (A) 17 to 20 (B) 13 to 20 (C) 9 to 10 (D) 4 to 5

Maintain Your Skills

Mixed Review Write the scale factor of each scale. (Lesson 6-3)

56. 3 inches = 18 inches

57. 2 inches = 2 feet

58. **ALGEBRA** Find the solution of $\frac{x}{54} = \frac{2}{3}$. (Lesson 6-2)

Find each product. Write in simplest form. (Lesson 5-3)

59. $\frac{4}{7} \cdot \frac{11}{12}$

60. $-\frac{3}{5} \cdot \frac{10}{18}$

61. $4 \cdot \frac{16}{52}$

62. Write 5.6×10^{-4} in standard form. (Lesson 4-8)

Determine whether each number is *prime* or *composite*. (Lesson 4-3)

63. 21

64. 47

65. 57

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each proportion.

(To review *proportions*, see Lesson 6-2.)

66. $\frac{25}{4} = \frac{x}{100}$

67. $\frac{56}{7} = \frac{y}{100}$

68. $\frac{75}{8} = \frac{n}{100}$

69. $\frac{m}{10} = \frac{9.4}{100}$

70. $\frac{h}{350} = \frac{46}{100}$

71. $\frac{86.4}{k} = \frac{27}{100}$



Algebra Activity

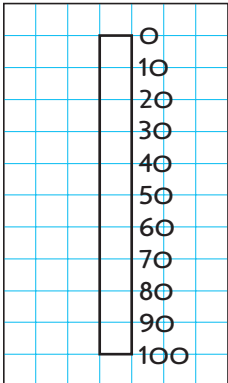
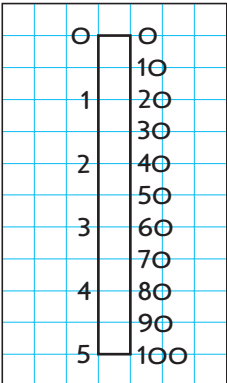
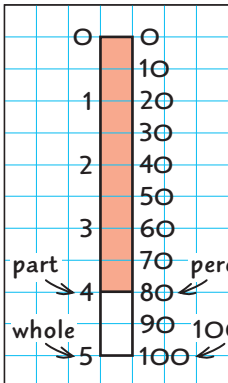
A Preview of Lesson 6-5

Using a Percent Model

Activity 1

When you see advertisements on television or in magazines, you are often bombarded with many claims. For example, you might hear that four out of five use a certain long-distance phone service. What percent does this represent?

You can find the percent by using a model.

Finding a Percent		
<p>Step 1</p> <p>Draw a 10-unit by 1-unit rectangle on grid paper. Label the units on the right from 0 to 100, because percent is a ratio that compares a number to 100.</p> 	<p>Step 2</p> <p>On the left side, mark equal units from 0 to 5, because 5 represents the whole quantity. Locate 4 on this scale.</p> 	<p>Step 3</p> <p>Draw a horizontal line from 4 on the left side to the right side of the model. The number on the right side is the percent. Label the model as shown.</p> 

Using the model, you can see that the ratio *4 out of 5* is the same as 80%. So, according to this claim, 80% of people prefer the certain long-distance phone service.

Model

Draw a model and find the percent that is represented by each ratio. If it is not possible to find the exact percent using the model, estimate.

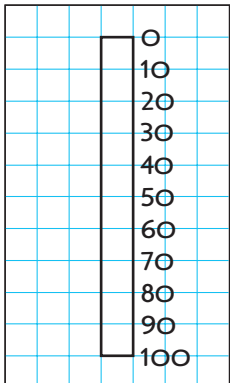
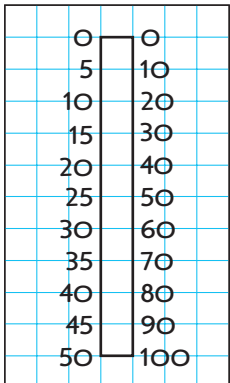
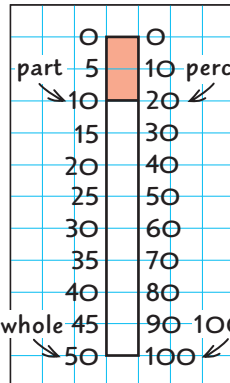
- 6 out of 10
- 9 out of 10
- 2 out of 5
- 3 out of 4
- 9 out of 20
- 8 out of 50
- 2 out of 8
- 3 out of 8
- 2 out of 3
- 5 out of 9

Activity 2

Suppose a store advertises a sale in which all merchandise is 20% off the original price. If the original price of a pair of shoes is \$50, how much will you save?

In this case, you know the percent. You need to find what part of the original price you'll save.

You can find the part by using a similar model.

Finding a Part		
<p>Step 1 Draw a 10-unit by 1-unit rectangle on grid paper. Label the units on the right from 0 to 100 because percent is a ratio that compares a number to 100.</p> 	<p>Step 2 On the left side, mark equal units from 0 to 50, because 50 represents the whole quantity.</p> 	<p>Step 3 Draw a horizontal line from 20% on the right side to the left side of the model. The number on the left side is the part. Label the model as shown.</p> 

Using the model, you can see that 20% of 50 is 10. So, you will save \$10 if you buy the shoes.

Model

Draw a model and find the part that is represented. If it is not possible to find an exact answer from the model, estimate.

- | | |
|-----------------------------|-----------------|
| 11. 10% of 50 | 12. 60% of 20 |
| 13. 90% of 40 | 14. 30% of 10 |
| 15. 25% of 20 | 16. 75% of 40 |
| 17. 5% of 200 | 18. 85% of 500 |
| 19. $33\frac{1}{3}\%$ of 12 | 20. 37.5% of 16 |

6-5

Using the Percent Proportion

What You'll Learn

- Use the percent proportion to solve problems.

Why are percents important in real-world situations?

Have you collected any of the new state quarters?



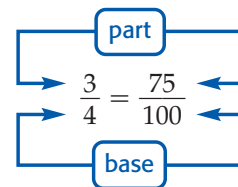
The quarters are made of a pure copper core and an outer layer that is an alloy of 3 parts copper and 1 part nickel.

- Write a ratio that compares the amount of copper to the total amount of metal in the outer layer.
- Write the ratio as a fraction and as a percent.

Vocabulary

- percent proportion
- part
- base

USE THE PERCENT PROPORTION In a **percent proportion**, one of the numbers, called the **part**, is being compared to the whole quantity, called the **base**. The other ratio is the percent, written as a fraction, whose base is 100.

**Key Concept***Percent Proportion*

- Words** $\frac{\text{part}}{\text{base}} = \frac{\text{percent}}{100}$
- Symbols** $\frac{a}{b} = \frac{p}{100}$, where a is the part, b is the base, and p is the percent.

Example 1 Find the Percent

Five is what percent of 8?

Five is being compared to 8. So, 5 is the part and 8 is the base. Let p represent the percent.

$$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{5}{8} = \frac{p}{100} \quad \text{Replace } a \text{ with 5 and } b \text{ with 8.}$$

$$5 \cdot 100 = 8 \cdot p \quad \text{Find the cross products.}$$

$$500 = 8p \quad \text{Simplify.}$$

$$\frac{500}{8} = \frac{8p}{8} \quad \text{Divide each side by 8.}$$

$$62.5 = p \quad \text{So, 5 is 62.5\% of 8.}$$

Study Tip**Estimation**

Five is a little more than one-half of eight. So, the answer should be a little more than 50%.

✓ Concept Check

In the percent proportion $\frac{15}{20} = \frac{75}{100}$, which number is the base?

Study Tip

Base

In percent problems, the base usually follows the word *of*.

Example 2 Find the Percent

What percent of 4 is 7?

Seven is being compared to 4. So, 7 is the part and 4 is the base.

Let p represent the percent.

$$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{7}{4} = \frac{p}{100} \quad \text{Replace } a \text{ with } 7 \text{ and } b \text{ with } 4.$$

$$7 \cdot 100 = 4 \cdot p \quad \text{Find the cross products.}$$

$$700 = 4p \quad \text{Simplify.}$$

$$\frac{700}{4} = \frac{4p}{4} \quad \text{Divide each side by } 4.$$

$$175 = p \quad \text{So, } 175\% \text{ of } 4 \text{ is } 7.$$



Log on for:

- Updated data
- More activities on finding a percent.

www.pre-alg.com/usa_today

Example 3 Apply the Percent Proportion

ENVIRONMENT The graphic shows the number of threatened species in the United States. What percent of the total number of threatened species are mammals?

Compare the number of species of mammals, 37, to the total number of threatened species, 443. Let a represent the part, 37, and let b represent the base, 443, in the percent proportion. Let p represent the percent.

$$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{37}{443} = \frac{p}{100}$$

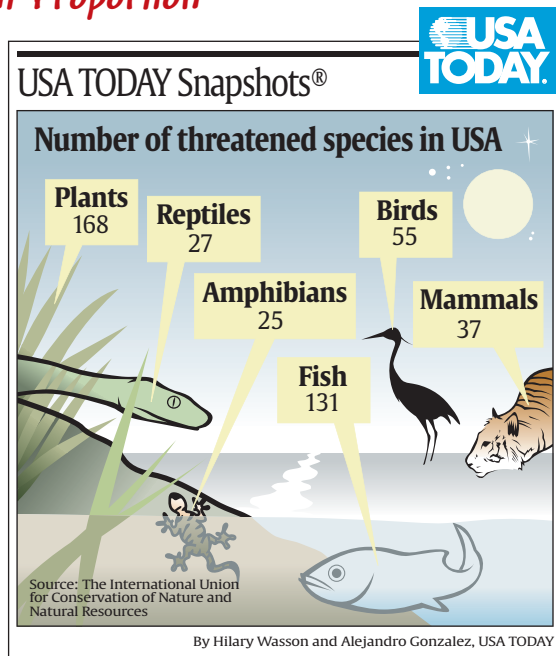
$$37 \cdot 100 = 443 \cdot p$$

$$3700 = 443p \quad \text{Simplify.}$$

$$\frac{3700}{443} = \frac{443p}{443} \quad \text{Divide each side by } 443.$$

$$8.4 \approx p \quad \text{Simplify.}$$

So, about 8.4% of the total number of threatened species are mammals.



Concept Summary

Types of Percent Problems

Type	Example	Proportion
Find the Percent	3 is <u>what percent</u> of 4?	$\frac{3}{4} = \frac{p}{100}$
Find the Part	<u>What number</u> is 75% of 4?	$\frac{a}{4} = \frac{75}{100}$
Find the Base	3 is 75% of <u>what number</u> ?	$\frac{3}{b} = \frac{75}{100}$



Example 4 Find the Part

What number is 5.5% of 650?

The percent is 5.5, and the base is 650. Let a represent the part.

$$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{a}{650} = \frac{5.5}{100} \quad \text{Replace } b \text{ with 650 and } p \text{ with 5.5.}$$

$$a \cdot 100 = 650 \cdot 5.5 \quad \text{Find the cross products.}$$

$$100a = 3575 \quad \text{Simplify.}$$

$$a = 35.75 \quad \text{Mentally divide each side by 100.}$$

So, 5.5% of 650 is 35.75.



Log on for:

- Updated data
 - More activities on using the percent proportion.
- www.pre-alg.com/usa_today

Example 5 Apply the Percent Proportion

CHORES Use the graphic to determine how many of the 1074 youths surveyed do not clean their room because there is not enough time.

The total number of youths is 1074. So, 1074 is the base. The percent is 29%.

To find 29% of 1074, let b represent the base, 1074, and let p represent the percent, 29%, in the percent proportion. Let a represent the part.

$$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{a}{1074} = \frac{29}{100}$$

$$a \cdot 100 = 1074 \cdot 29$$

$$100a = 31146 \quad \text{Simplify.}$$

$$a = 311.46 \quad \text{Mentally divide each side by 100.}$$

So, about 311 youths do not clean their room because there is not enough time.



USA TODAY Snapshots®

Kids don't enjoy cleaning their rooms

When 1,074 youths 19 and under were asked why they don't clean their room more often, these were their responses*:



Example 6 Find the Base

Fifty-two is 40% of what number?

The percent is 40% and the part is 52. Let b represent the base.

$$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{52}{b} = \frac{40}{100} \quad \text{Replace } a \text{ with 52 and } p \text{ with 40.}$$

$$52 \cdot 100 = b \cdot 40 \quad \text{Find the cross products.}$$

$$5200 = 40b \quad \text{Simplify.}$$

$$\frac{5200}{40} = \frac{40b}{40} \quad \text{Divide each side by 40.}$$

$$130 = b \quad \text{Simplify.}$$

So, 52 is 40% of 130.



Check for Understanding

Concept Check

- OPEN ENDED** Write a proportion that can be used to find the percent scored on an exam that has 50 questions.
- FIND THE ERROR** Judie and Pennie are using a proportion to find what number is 35% of 21. Who is correct? Explain your reasoning.

$$\text{Judie}$$

$$\frac{n}{21} = \frac{35}{100}$$

$$\text{Pennie}$$

$$\frac{21}{n} = \frac{35}{100}$$

Guided Practice

Use the percent proportion to solve each problem.

- 16 is what percent of 40?
- 21 is 30% of what number?
- What is 80% of 130?
- What percent of 5 is 14?

Applications

- BOOKS** Fifty-four of the 90 books on a shelf are history books. What percent of the books are history books?
- CHORES** Refer to Example 5 on page 290. How many of the 1074 youths surveyed do not clean their room because they do not like to clean?

Practice and Apply

Homework Help

For Exercises	See Examples
9–20	1, 2, 4, 6
21, 23, 24	3
22, 25	5

Extra Practice
See page 738.

Use the percent proportion to solve each problem. Round to the nearest tenth.

- 72 is what percent of 160?
- 17 is what percent of 85?
- 36 is 72% of what number?
- 27 is 90% of what number?
- What is 44% of 175?
- What is 84% of 150?
- 52.2 is what percent of 145?
- 19.8 is what percent of 36?
- 14 is $12\frac{1}{2}\%$ of what number?
- 36 is $8\frac{3}{4}\%$ of what number?
- 7 is what percent of 3500?
- What is 0.3% of 750?
- BIRDS** If 12 of the 75 animals in a pet store are parakeets, what percent are parakeets?
- FISH** Of the fish in an aquarium, 26% are angelfish. If the aquarium contains 50 fish, how many are angelfish?

SCIENCE For Exercises 23 and 24, use the information in the table.

- What percent of the world's fresh water does the Antarctic Icecap contain?
- RESEARCH** Use the Internet or another source to find the total volume of the world's fresh and salt water. What percent of the world's total water supply does the Antarctic Icecap contain?

World's Fresh Water Supply	
Source	Volume (mi ³)
Freshwater Lakes	30,000
All Rivers	300
Antarctic Icecap	6,300,000
Arctic Icecap and Glaciers	680,000
Water in the Atmosphere	3100
Ground Water	1,000,000
Deep-lying Ground Water	1,000,000
Total	9,013,400

Source: Time Almanac

25. **LIFE SCIENCE** Carbon constitutes 18.5% of the human body by weight. Determine the amount of carbon contained in a person who weighs 145 pounds.
26. **CRITICAL THINKING** A number n is 25% of some number a and 35% of a number b . Tell the relationship between a and b . Is $a < b$, $a > b$, or is it impossible to determine the relationship? Explain.
27. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are percents important in real-world situations?

Include the following in your answer:

- an example of a real-world situation where percents are used, and
- an explanation of the meaning of the percent in the situation.



28. The table shows the number of people in each section of the school chorale. Which section makes up exactly 25% of the chorale?

School Chorale	
Section	Number
Soprano	16
Alto	15
Tenor	12
Bass	17

- (A) Tenor (B) Alto
(C) Soprano (D) Bass

Maintain Your Skills

Mixed Review Write each percent as a fraction in simplest form. (Lesson 6-4)

29. 42% 30. 56% 31. 120%

32. **MAPS** On a map of a state park, the scale is 0.5 inch = 1.5 miles. Find the actual distance from the ranger's station to the beach if the distance on the map is 1.75 inches. (Lesson 6-3)

Find each sum or difference. Write in simplest form. (Lesson 5-5)

33. $\frac{2}{9} + \frac{5}{9}$ 34. $\frac{11}{12} - \frac{3}{12}$ 35. $2\frac{5}{8} + \frac{7}{8}$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Find each product. (To review multiplying fractions, see Lesson 5-3.)

36. $\frac{1}{2} \times 14$ 37. $\frac{1}{4} \times 32$ 38. $\frac{1}{5} \times 15$
39. $\frac{2}{3} \times 9$ 40. $\frac{3}{4} \times 16$ 41. $\frac{5}{6} \times 30$

Practice Quiz 1

Lessons 6-1 through 6-5

- Express \$3.29 for 24 cans of soda as a unit rate. (Lesson 6-1)
- What value of x makes $\frac{3}{4} = \frac{x}{68}$ a proportion? (Lesson 6-2)
- SCIENCE** A scale model of a volcano is 4 feet tall. If the actual height of the volcano is 12,276 feet, what is the scale of the model? (Lesson 6-3)
- Express 352% as a decimal. (Lesson 6-4)
- Use the percent proportion to find 32.5% of 60. (Lesson 6-5)



6-6

Finding Percents Mentally

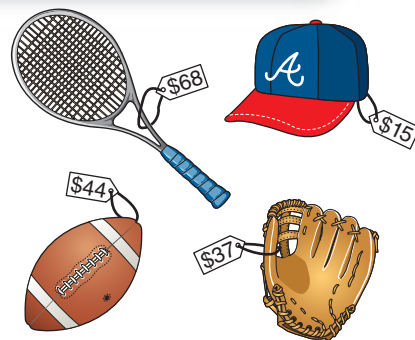
What You'll Learn

- Compute mentally with percents.
- Estimate with percents.

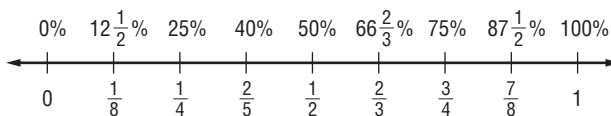
How is estimation used when determining sale prices?

A sporting goods store is having a sale in which all merchandise is on sale at half off. A few regularly priced items are shown at the right.

- What is the sale price of each item?
- What percent represents half off?
- Suppose the items are on sale for 25% off. Explain how you would determine the sale price.



FIND PERCENTS OF A NUMBER MENTALLY When working with common percents like 10%, 25%, 40%, and 50%, it may be helpful to use the fraction form of the percent. A few percent-fraction equivalents are shown.



Some percents are used more frequently than others. So, it is a good idea to be familiar with these percents and their equivalent fractions.

Concept Summary		Percent-Fraction Equivalents		
20% = 1/5	10% = 1/10	25% = 1/4	12½% = 1/8	16⅔% = 1/6
40% = 2/5	30% = 3/10	50% = 1/2	37½% = 3/8	33⅓% = 1/3
60% = 3/5	70% = 7/10	75% = 3/4	62½% = 5/8	66⅔% = 2/3
80% = 4/5	90% = 9/10		87½% = 7/8	83⅓% = 5/6

Example 1 Find Percent of a Number Mentally

Find the percent of each number mentally.

- 50% of 32

$$50\% \text{ of } 32 = \frac{1}{2} \text{ of } 32 \quad \text{Think: } 50\% = \frac{1}{2}.$$

$$= 16 \quad \text{Think: } \frac{1}{2} \text{ of } 32 \text{ is } 16.$$

So, 50% of 32 is 16.

Study Tip

Look Back

To review **multiplying fractions**, see Lesson 5-3.



Find the percent of each number mentally.

b. 25% of 48

$$25\% \text{ of } 48 = \frac{1}{4} \text{ of } 48 \quad \text{Think: } 25\% = \frac{1}{4}.$$

$$= 12 \quad \text{Think: } \frac{1}{4} \text{ of } 48 \text{ is } 12.$$

So, 25% of 48 is 12.

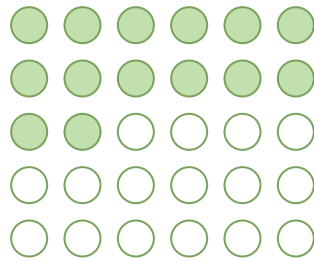
c. 40% of 45

$$40\% \text{ of } 45 = \frac{2}{5} \text{ of } 45 \quad \text{Think: } 40\% = \frac{2}{5}.$$

$$= 18 \quad \text{Think: } \frac{1}{5} \text{ of } 45 \text{ is } 9. \text{ So, } \frac{2}{5} \text{ of } 45 \text{ is } 18.$$

So, 40% of 45 is 18.

ESTIMATE WITH PERCENTS Sometimes, an exact answer is not needed. In these cases, you can estimate. Consider the following model.



- 14 of the 30 circles are shaded.
- $\frac{14}{30}$ is about $\frac{15}{30}$ or $\frac{1}{2}$.
- $\frac{1}{2} = 50\%$. So, about 50% of the model is shaded.

The table below shows three methods you can use to estimate with percents. For example, let's estimate 22% of 237.

Method	Estimate 22% of 237.
Fraction	22% is a bit more than 20% or $\frac{1}{5}$. 237 is a bit less than 240. So, 22% of 237 is about $\frac{1}{5}$ of 240 or 48. Estimate: 48
1%	$22\% = 22 \times 1\%$ $1\% \text{ of } 237 = 2.37$ or about 2. So, 22% of 237 is about 22×2 or 44. Estimate: 44
Meaning of Percent	22% means about 20 for every 100 or about 2 for every 10. 237 has 2 hundreds and about 4 tens. $(20 \times 2) + (2 \times 4) = 40 + 8$ or 48 Estimate: 48

Study Tip

Percents

To find 1% of any number, move the decimal point two places to the left.

You can use these methods to estimate the percent of a number.

Example 2 Estimate Percents

a. Estimate 13% of 120.

$$13\% \text{ is about } 12.5\% \text{ or } \frac{1}{8}.$$

$$\frac{1}{8} \text{ of } 120 \text{ is } 15.$$

So, 13% of 120 is about 15.

b. Estimate 80% of 296.

$$80\% \text{ is equal to } \frac{4}{5}.$$

296 is about 300.

$$\frac{4}{5} \text{ of } 300 \text{ is } 240.$$

So, 80% of 296 is about 240.

c. Estimate $\frac{1}{3}\%$ of 598.

$$\frac{1}{3}\% = \frac{1}{3} \times 1\%. \text{ 598 is almost 600.}$$

1% of 600 is 6.

So, $\frac{1}{3}\%$ of 598 is about $\frac{1}{3} \times 6$ or 2.

d. Estimate 118% of 56.

118% means about 120 for every 100 or about 12 for every 10.

56 has about 6 tens.

$$12 \times 6 = 72$$

So, 118% of 56 is about 72.

Estimating percents is a useful skill in real-life situations.

Example 3 Use Estimation to Solve a Problem

MONEY Amelia takes a taxi from the airport to a hotel. The fare is \$31.50. Suppose she wants to tip the driver 15%. What would be a reasonable amount of tip for the driver?

\$31.50 is about \$32.

$$15\% = 10\% + 5\%$$

10% of \$32 is \$3.20. **Move the decimal point 1 place to the left.**

5% of \$32 is \$1.60. **5% is one half of 10%.**

So, 15% is about $3.20 + 1.60$ or \$4.80.

A reasonable amount for the tip would be \$5.

Study Tip

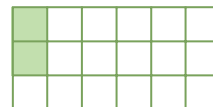
Finding Percents

To find 10% of a number, move the decimal point one place to the left.

Check for Understanding

Concept Check

1. Explain how to estimate 18% of 216 using the fraction method.
2. Estimate the percent of the figure that is shaded.
3. **OPEN ENDED** Tell which method of estimating a percent you prefer. Explain your decision.



Guided Practice

Find the percent of each number mentally.

4. 75% of 64
5. 25% of 52
6. $33\frac{1}{3}\%$ of 27
7. 90% of 80

Estimate. Explain which method you used to estimate.

8. 20% of 61
9. 34% of 24
10. $\frac{1}{2}\%$ of 396
11. 152% of 14

Application

12. **MONEY** Lu Chan wants to leave a tip of 20% on a dinner check of \$52.48. About how much should he leave?



Practice and Apply

Homework Help

For Exercises	See Examples
13–26	1
27–35	2
39, 40	3

Extra Practice
See page 738.

Find the percent of each number mentally.

- | | | |
|-----------------------------|-------------------------------|-----------------------------|
| 13. 50% of 28 | 14. 75% of 16 | 15. 60% of 55 |
| 16. 20% of 105 | 17. $87\frac{1}{2}\%$ of 56 | 18. $16\frac{2}{3}\%$ of 42 |
| 19. $12\frac{1}{2}\%$ of 32 | 20. $66\frac{2}{3}\%$ of \$24 | 21. 200% of 45 |
| 22. 150% of 54 | 23. 125% of 300 | 24. 175% of 200 |

MONEY For Exercises 25 and 26, use the following information.

In a recent year, the number of \$1 bills in circulation in the United States was about 7 billion.

- Suppose the number of \$5 bills in circulation was 25% of the number of \$1 bills. About how many \$5 bills were in circulation?
- If the number of \$10 bills was 20% of the number of \$1 bills, about how many \$10 bills were in circulation?

Estimate. Explain which method you used to estimate.

- | | | |
|----------------|----------------------------|---------------------------|
| 27. 30% of 89 | 28. 25% of 162 | 29. 38% of 88 |
| 30. 81% of 25 | 31. $\frac{1}{4}\%$ of 806 | 32. $\frac{1}{5}\%$ of 40 |
| 33. 127% of 64 | 34. 140% of 95 | 35. 295% of 145 |

SPACE For Exercises 36–38, refer to the information in the table.

- Which planet has a radius that measures about 50% of the radius of Mercury?
- Name two planets such that the radius of one planet is about one-third the radius of the other planet.
- Name two planets such that the mass of one planet is about 330% the mass of the other.

Radius and Mass of Each Planet

Planet	Radius (mi)	Mass
Mercury	1516	0.0553
Venus	3761	0.815
Earth	3960	1.000
Mars	2107	0.107
Jupiter	43,450	317.830
Saturn	36,191	95.160
Uranus	15,763	14.540
Neptune	15,304	17.150
Pluto	707	0.0021

Source: *The World Almanac*

- GEOGRAPHY** The United States has 88,633 miles of shoreline. Of the total amount, 35% is located in Alaska. About how many miles of shoreline are located in Alaska?
- GEOGRAPHY** About 8.5% of the total Pacific coastline is located in California. Use the information at the left to estimate the number of miles of coastline located in California.
- FOOD** A serving of shrimp contains 90 Calories and 7 of those Calories are from fat. About what percent of the Calories are from fat?
- FOOD** Fifty-six percent of the Calories in corn chips are from fat. Estimate the number of Calories from fat in a serving of corn chips if one serving contains 160 Calories.

More About . . .



Geography

There are four U.S. coastlines. They are the Atlantic, Gulf, Pacific, and Arctic coasts. Most of the coastline is located on the Pacific Ocean. It contains 40,298 miles.

Source: *The World Almanac*



43. **CRITICAL THINKING** In an election, 40% of the Democrats and 92.5% of the Republicans voted “yes”. Of all of the Democrats and Republicans, 68% voted “yes”. Find the ratio of Democrats to Republicans.
44. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is estimation used when determining sale prices?

Include the following in your answer:

- an example of a situation in which you used estimation to determine the sale price of an item, and
- an example of a real-life situation other than shopping in which you would use estimation with percents.



45. Which percent is greater than $\frac{3}{5}$ but less than $\frac{2}{3}$?
 (A) 68% (B) 54% (C) 64% (D) 38%
46. Choose the best estimate for 26% of 362.
 (A) 91 (B) 72 (C) 108 (D) 85

Maintain Your Skills

Mixed Review Use the percent proportion to solve each problem. (Lesson 6-5)

47. What is 28% of 75? 48. 37.8 is what percent of 84?

49. **FORESTRY** The five states with the largest portion of land covered by forests are shown in the graphic. For each state, how many square miles of land are covered by forests?

State	Percent of land covered by forests	Area of state (square miles)
Maine	89.9%	35,387
New Hampshire	88.1%	9351
West Virginia	77.5%	24,231
Vermont	75.7%	9615
Alabama	66.9%	52,423

Source: The Learning Kingdom, Inc.

Express each decimal as a percent. (Lesson 6-4)

50. 0.27 51. 1.6 52. 0.008

Express each percent as a decimal. (Lesson 6-4)

53. 77% 54. 8% 55. 421% 56. 3.56%

ALGEBRA Solve each equation. Check your solution. (Lesson 5-9)

57. $n + 4.7 = 13.6$ 58. $x + \frac{5}{6} = 2\frac{3}{8}$ 59. $\frac{3}{7}r = -9$

60. **GEOMETRY** The perimeter of a rectangle is 22 feet. Its length is 7 feet. Find its width. (Lesson 3-7)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Solve each equation. Check your solution.

(To review *solving equations*, see Lesson 3-4.)

61. $10a = 5$ 62. $20m = 4$
 63. $60h = 15$ 64. $28g = 1.4$
 65. $80w = 5.6$ 66. $125n = 15$

Using Percent Equations

What You'll Learn

- Solve percent problems using percent equations.
- Solve real-life problems involving discount and interest.

Vocabulary

- percent equation
- discount
- simple interest

How is the percent proportion related to an equation?

As of July 1, 1999, 45 of the 50 U.S. states had a sales tax. The table shows the tax rate for four U.S. states.

State	Tax Rate (percent)
Alabama	4%
Connecticut	6%
New Mexico	5%
Texas	6.25%

Source: www.taxadmin.org

- Use the percent proportion to find the amount of tax on a \$35 purchase for each state.
- Express each tax rate as a decimal.
- Multiply the decimal form of the tax rate by \$35 to find the amount of tax on the \$35 purchase for each state.
- How are the amounts of tax in parts a and c related?

PERCENT EQUATIONS The **percent equation** is an equivalent form of the percent proportion in which the percent is written as a decimal.

$$\frac{\text{Part}}{\text{Base}} = \text{Percent} \quad \leftarrow \text{The percent is written as a decimal.}$$

$$\frac{\text{Part}}{\text{Base}} \cdot \text{Base} = \text{Percent} \cdot \text{Base} \quad \text{Multiply each side by the base.}$$

$$\text{Part} = \text{Percent} \cdot \text{Base} \quad \leftarrow \text{This form is called the percent equation.}$$

Concept Summary

The Percent Equation

Type	Example	Equation
Missing Part	What number is 75% of 4?	$n = 0.75(4)$
Missing Percent	3 is what percent of 4?	$3 = n(4)$
Missing Base	3 is 75% of what number?	$3 = 0.75n$

Example 1 Find the Part

Find 52% of 85. **Estimate:** $\frac{1}{2}$ of 90 is 45.

You know that the base is 85 and the percent is 52%.
Let n represent the part.

$$n = 0.52(85) \quad \text{Write 52\% as the decimal 0.52.}$$

$$n = 44.2 \quad \text{Simplify.}$$

So, 52% of 85 is 44.2.

Study Tip

Estimation

To determine whether your answer is reasonable, estimate before finding the exact answer.

Example 2 Find the Percent

28 is what percent of 70? **Estimate:** $\frac{28}{70} \approx \frac{25}{75}$ or $\frac{1}{3}$, which is $33\frac{1}{3}\%$.

You know that the base is 70 and the part is 28.

Let n represent the percent.

$$28 = n(70)$$

$$\frac{28}{70} = n$$

Divide each side by 70.

$$0.4 = n$$

Simplify.

So, 28 is 40% of 70. The answer makes sense compared to the estimate.

Example 3 Find the Base

18 is 45% of what number? **Estimate:** 18 is 50% of 36.

You know that the part is 18 and the percent is 45.

Let n represent the base.

$$18 = 0.45n$$

Write 45% as the decimal 0.45.

$$\frac{18}{0.45} = \frac{0.45n}{0.45}$$

Divide each side by 0.45.

$$40 = n$$

Simplify.

So, 18 is 45% of 40. The answer is reasonable since it is close to the estimate.

DISCOUNT AND INTEREST The percent equation can also be used to solve problems involving discount and interest. **Discount** is the amount by which the regular price of an item is reduced.

Example 4 Find Discount

SKATEBOARDS Mateo wants to buy a skateboard. The regular price of the skateboard is \$135. Suppose it is on sale at a 25% discount. Find the sale price of the skateboard.

Method 1

First, use the percent equation to find 25% of 135. **Estimate:** $\frac{1}{4}$ of 140 = 35

Let d represent the discount.

$$d = 0.25(135)$$

The base is 135 and the percent is 25%.

$$d = 33.75$$

Simplify.

Then, find the sale price.

$$135 - 33.75 = 101.25 \quad \text{Subtract the discount from the original price.}$$

Method 2

A discount of 25% means the item will cost $100\% - 25\%$ or 75% of the original price. Use the percent equation to find 75% of 135.

Let s represent the sale price.

$$s = 0.75(135)$$

The base is 135 and the percent is 75%.

$$s = 101.25$$

Simplify.

The sale price of the skateboard will be \$101.25.

More About . . .



Skateboards

The popularity of the sport of skateboarding is increasing. An estimated 10,000,000 people worldwide participate in the sport.

Source: International Association of Skateboard Companies

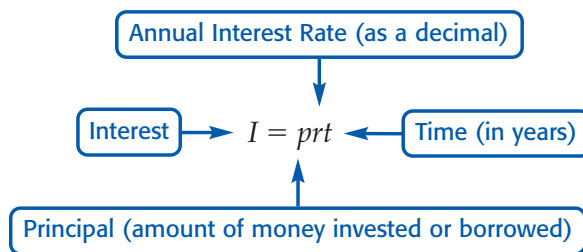


Simple interest is the amount of money paid or earned for the use of money. For a savings account, interest is earned. For a credit card, interest is paid. To solve problems involving interest, use the following formula.

Reading Math

Formulas

The formula $I = prt$ is read *Interest is equal to principal times rate times time.*



Concept Check Name a situation where interest is earned and a situation where interest is paid.

Example 5 Apply Simple Interest Formula

BANKING Suppose Miguel invests \$1200 at an annual rate of 6.5%. How long will it take until Miguel earns \$195?

$I = prt$	Write the simple interest formula.
$195 = 1200(0.065)t$	Replace I with 195, p with 1200, and r with 0.065.
$195 = 78t$	Simplify.
$\frac{195}{78} = \frac{78t}{78}$	Divide each side by 78.
$2.5 = t$	Simplify.

Miguel will earn \$195 in interest in 2.5 years.

Check for Understanding

- Concept Check**
- OPEN ENDED** Give an example of a situation in which using the percent equation would be easier than using the percent proportion.
 - Define *discount*.
 - Explain what I , p , r , and t represent in the simple interest formula.

- Guided Practice** Solve each problem using the percent equation.
- 15 is what percent of 60?
 - 30 is 60% of what number?
 - What is 20% of 110?
 - 12 is what percent of 400?
 - Find the discount for a \$268 DVD player that is on sale at 20% off.
 - What is the interest on \$8000 that is invested at 6% for $3\frac{1}{2}$ years? Round to the nearest cent.

- Applications**
- SHOPPING** A jacket that normally sells for \$180 is on sale at a 35% discount. What is the sale price of the jacket?
 - BANKING** How long will it take to earn \$252 in interest if \$2400 is invested at a 7% annual interest rate?



Practice and Apply

Homework Help


For Exercises	See Examples
12–27, 39	1–3
28–33	4
34–38	5

Extra Practice
See page 738.


Solve each problem using the percent equation.

12. 9 is what percent of 25?
13. 38 is what percent of 40?
14. 48 is 64% of what number?
15. 27 is 54% of what number?
16. Find 12% of 72.
17. Find 42% of 150.
18. 39.2 is what percent of 112?
19. 49.5 is what percent of 132?
20. What is 37.5% of 89?
21. What is 24.2% of 60?
22. 37.5 is what percent of 30?
23. 43.6 is what percent of 20?
24. 1.6 is what percent of 400?
25. 1.35 is what percent of 150?
26. 83.5 is 125% of what number?
27. 17.6 is $133\frac{1}{3}\%$ of what number?
28. **FOOD** A frozen pizza is on sale at a 25% discount. Find the sale price of the pizza if it normally sells for \$4.85.
29. **CALCULATORS** Suppose a calculator is on sale at a 15% discount. If it normally sells for \$29.99, what is the sale price?

Find the discount to the nearest cent.

30.  **SALE! 25% off**
Original Price: \$65
31. \$85 cordless phone, 20% off
32. \$489 stereo, 15% off
33. 25% off a \$74 baseball glove

Find the interest to the nearest cent.

34.  **First City Bank**
Amount: \$5432
Annual Interest Rate: 6.2%
Time: 3 years
35. \$4500 at 5.5% for $4\frac{1}{2}$ years
36. \$3680 at 6.75% for $2\frac{1}{4}$ years
37. 5.5% for $1\frac{3}{4}$ years on \$2543

38. **BANKING** What is the annual interest rate if \$1600 is invested for 6 years and \$456 in interest is earned?
39. **SPORTS** One season, a football team had 7 losses. This was 43.75% of the total games they played. How many games did they play?
40. **REAL ESTATE** A **commission** is a fee paid to a salesperson based on a percent of sales. Suppose a real estate agent earns a 3% commission. What commission would be earned for selling the house shown?
41. **BUSINESS** To make a profit, stores try to sell an item for more than it paid for the item. The increase in price is called the **markup**. Suppose a store purchases paint brushes for \$8 each. Find the markup if the brushes are sold for 15% over the price paid for them.



WebQuest

The percent equation can help you analyze the nutritional value of food. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

42. **CRITICAL THINKING** Determine whether $n\%$ of m is always equal to $m\%$ of n . Give examples to support your answer.
43. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is the percent proportion related to an equation?

Include the following in your answer:

- an explanation describing two methods for finding the amount of tax on an item, and
- an example of using both methods to find the amount of sales tax on an item.



44. What percent of 320 is 19.2?
 (A) 0.6% (B) 60% (C) 6% (D) 0.06%
45. Ryan wants to buy a tent that costs \$150 for his camping trip. The tent is on sale at a 30% discount. What will be the sale price of the tent?
 (A) 95 (B) 105 (C) 45 (D) 110

Maintain Your Skills

Mixed Review

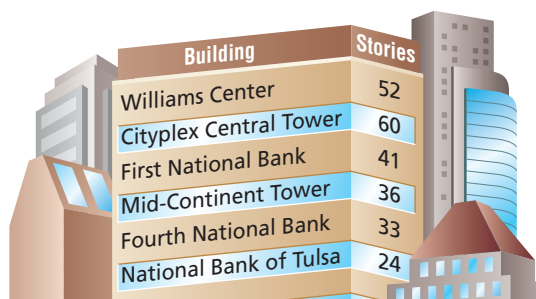
Estimate. Explain which method you used to estimate. (Lesson 6-6)

46. 47% of 84 47. 126% of 198 48. 9% of 514

Use the percent proportion to solve each problem. (Lesson 6-5)

49. What is 55% of 220? 50. 50.88 is what percent of 96?

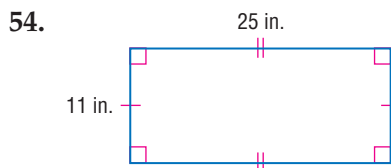
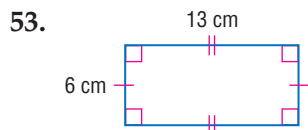
51. **POPULATION** The graphic shows the number of stories of certain buildings in Tulsa, Oklahoma. What is the mean of the data? (Lesson 5-8)



Source: The World Almanac

52. List all the factors of 30. (Lesson 4-1)

GEOMETRY Find the perimeter of each rectangle. (Lesson 3-7)



55. **ALGEBRA** Use the Distributive Property to rewrite $(w - 3)8$. (Lesson 3-1)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write each decimal as a percent. (To review writing decimals as percents, see Lesson 6-4.)

56. 0.58 57. 0.89 58. 0.125
 59. 1.56 60. 2.04 61. 0.224





Spreadsheet Investigation

A Follow-Up of Lesson 6-7

Compound Interest

Simple interest, which you studied in the previous lesson, is paid only on the initial principal of a savings account or a loan. **Compound interest** is paid on the initial principal and on interest earned in the past. You can use a spreadsheet to investigate the impact of compound interest.

SAVINGS Find the value of a \$1000 savings account after five years if the account pays 6% interest compounded semiannually.

6% interest compounded semiannually means that the interest is paid twice a year, or every 6 months. The interest rate is $6\% \div 2$ or 3%.

	A	B	C	D
1	RATE	0.03		
2				
3	PRINCIPAL	INTEREST	NEW PRINCIPAL	TIME (YR)
4	1000.00	30.00	1030.00	0.5
5	1030.00	30.90	1060.90	1.0
6	1060.90	31.83	1092.73	1.5
7	1092.73	32.78	1125.51	2.0
8	1125.51	33.77	1159.27	2.5
9	1159.27	34.78	1194.05	3.0
10	1194.05	35.82	1229.87	3.5
11	1229.87	36.90	1266.77	4.0
12	1266.77	38.00	1304.77	4.5
13	1304.77	39.14	1343.92	5.0

The value of the savings account after five years is \$1343.92.

Model and Analyze

1. Suppose you invest \$1000 for five years at 6% simple interest. How does the simple interest compare to the compound interest shown above?
2. Use a spreadsheet to find the amount of money in a savings account if \$1000 is invested for five years at 6% interest compounded quarterly.
3. Suppose you leave \$100 in each of three bank accounts paying 5% interest per year. One account pays simple interest, one pays interest compounded semiannually, and one pays interest compounded quarterly. Use a spreadsheet to find the amount of money in each account after three years.

Make a Conjecture

4. How does the amount of interest change if the compounding occurs more frequently?

6-8 Percent of Change

What You'll Learn

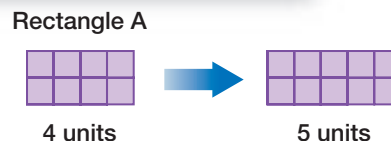
- Find percent of increase.
- Find percent of decrease.

Vocabulary

- percent of change
- percent of increase
- percent of decrease

How can percents help to describe a change in area?

Suppose the length of rectangle A is increased from 4 units to 5 units.



Rectangle A had an initial area of 8 square units. It increased to 10 square units. This is a change in area of 2 square units. The following ratio shows this relationship.

$$\frac{\text{change in area}}{\text{original area}} = \frac{2}{8} = \frac{1}{4} \text{ or } 25\%$$

This means that, compared to the original area, the new area increased by 25%.

Draw each pair of rectangles. Then compare the rectangles. Express the increase as a fraction and as a percent.

- X: 2 units by 3 units
Y: 2 units by 4 units
- G: 2 units by 5 units
H: 2 units by 6 units
- J: 2 units by 4 units
K: 2 units by 5 units
- P: 2 units by 6 units
Q: 2 units by 7 units
- e. For each pair of rectangles, the change in area is 2 square units. Explain why the percent of change is different.

FIND PERCENT OF INCREASE A **percent of change** tells the percent an amount has increased or decreased in relation to the original amount.

Example 1 Find Percent of Change

Find the percent of change from 56 inches to 63 inches.

Step 1 Subtract to find the amount of change.

$$63 - 56 = 7 \quad \text{new measurement} - \text{original measurement}$$

Step 2 Write a ratio that compares the amount of change to the original measurement. Express the ratio as a percent.

$$\text{percent of change} = \frac{\text{amount of change}}{\text{original measurement}}$$

$$= \frac{7}{56} \quad \text{Substitution.}$$

$$= 0.125 \text{ or } 12.5\% \quad \text{Write the decimal as a percent.}$$

The percent of change from 56 inches to 63 inches is 12.5%.



When an amount increases, as in Example 1, the percent of change is a **percent of increase**.

Example 2 Find Percent of Increase

FUEL In 1975, the average price per gallon of gasoline was \$0.57. In 2000, the average price per gallon was \$1.47. Find the percent of change.

Source: *The World Almanac*

Step 1 Subtract to find the amount of change.

$$1.47 - 0.57 = 0.9 \quad \text{new price} - \text{original price}$$

Step 2 Write a ratio that compares the amount of change to the original price. Express the ratio as a percent.

$$\begin{aligned} \text{percent of change} &= \frac{\text{amount of change}}{\text{original price}} \\ &= \frac{0.9}{0.57} && \text{Substitution.} \\ &\approx 1.58 \text{ or } 158\% && \text{Write the decimal as a percent.} \end{aligned}$$

The percent of change is about 158%. In this case, the percent of change is a percent of increase.



Example 3 Find Percent of Increase

Multiple-Choice Test Item

Refer to the table shown. Which county had the greatest percent of increase in population from 1990 to 2000?

- (A) Breckinridge (B) Bracken
(C) Calloway (D) Fulton

County	1990	2000
Breckinridge	16,312	18,648
Bracken	7766	8279
Calloway	30,735	34,177
Fulton	8271	7752

Read the Test Item

Percent of increase tells how much the population has increased in relation to 1990.

Solve the Test Item

Use a ratio to find each percent of increase. Then compare the percents.

• **Breckinridge**

$$\begin{aligned} \frac{18,648 - 16,312}{16,312} &= \frac{2336}{16,312} \\ &\approx 0.1432 \text{ or } 14.3\% \end{aligned}$$

• **Bracken**

$$\begin{aligned} \frac{8279 - 7766}{7766} &= \frac{513}{7766} \\ &\approx 0.0661 \text{ or } 6.6\% \end{aligned}$$

• **Calloway**

$$\begin{aligned} \frac{34,177 - 30,735}{30,735} &= \frac{3442}{30,735} \\ &\approx 0.112 \text{ or } 11.2\% \end{aligned}$$

• **Fulton**

Eliminate this choice because the population decreased.

Test-Taking Tip

If you are unsure of the correct answer, eliminate the choices you know are incorrect. Then consider the remaining choices.

Breckinridge County had the greatest percent of increase in population from 1990 to 2000. The answer is A.

PERCENT OF DECREASE When the amount decreases, the percent of change is negative. You can state a negative percent of change as a **percent of decrease**.

More About . . .



Stock Market

About 20 years ago, only 12.2% of Americans had money invested in the stock market. Today, more than 44% of Americans invest in the stock market.

Source: www.infoplease.com

Example 4 Find Percent of Decrease

STOCK MARKET One of the largest stock market drops on Wall Street occurred on October 19, 1987. On this day, the stock market opened at 2246.74 points and closed at 1738.42 points. What was the percent of change?

Step 1 Subtract to find the amount of change.

$$1738.42 - 2246.74 = -508.32 \quad \text{closing points} - \text{opening points}$$

Step 2 Compare the amount of change to the opening points.

$$\begin{aligned} \text{percent of change} &= \frac{\text{amount of change}}{\text{opening points}} \\ &= \frac{-508.32}{2246.74} && \text{Substitution.} \\ &\approx -0.226 \text{ or } -22.6\% && \text{Write the decimal as a percent.} \end{aligned}$$

The percent of change is -22.6% . In this case, the percent of change is a percent of decrease.

Check for Understanding

Concept Check

1. **Explain** how you know whether a percent of change is a percent of increase or a percent of decrease.
2. **OPEN ENDED** Give an example of a percent of decrease.
3. **FIND THE ERROR** Scott and Mark are finding the percent of change when a shirt that costs \$15 is on sale for \$10.

Scott

$$\frac{10 - 15}{10} = \frac{-5}{10} \text{ or } -50\%$$

Mark

$$\frac{10 - 15}{15} = \frac{-5}{15} \text{ or } -33\frac{1}{3}\%$$

Who is correct? Explain your reasoning.

Guided Practice

Find the percent of change. Round to the nearest tenth, if necessary. Then state whether the percent of change is a *percent of increase* or a *percent of decrease*.

4. from \$50 to \$67
5. from 45 in. to 18 in.
6. from 80 cm to 55 cm
7. from \$228 to \$251
8. **ANIMALS** In 2000, there were 356 endangered species in the U.S. One year later, 367 species were considered endangered. What was the percent of change?

Standardized Test Practice

(A) (B) (C) (D)

9. Refer to Example 3 on page 305. Suppose in 10 years, the population of Calloway is 36,851. What will be the percent of change from 1990?
 (A) 19.9% (B) 9.8% (C) 10.7% (D) 15.3%



Practice and Apply

Homework Help

For Exercises

10–18, 20, 21
19

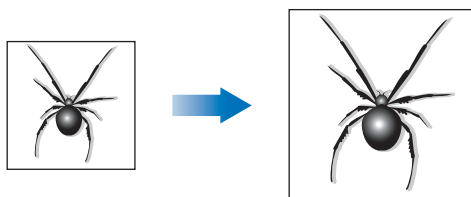
See Examples

1, 2, 4
3

Extra Practice
See page 739.

Find the percent of change. Round to the nearest tenth, if necessary. Then state whether the percent of change is a *percent of increase* or a *percent of decrease*.

10. from 25 cm to 36 cm
11. from \$10 to \$27
12. from 68 min to 51 min
13. from 50 lb to 44 lb
14. from \$135 to \$120
15. from 257 m to 243 m
16. from 365 ft to 421 ft
17. from \$289 to \$762
18. **WEATHER** Seattle, Washington, receives an average of 6.0 inches of precipitation in December. In March, the average precipitation is 3.8 inches. What is the percent of change in precipitation from December to March?
19. **POPULATION** In 1990, the population of Alabama was 4,040,587. In 2000, the population was 4,447,100. Find the percent of change from 1990 to 2000.
20. Suppose 36 videos are added to a video collection that has 24 videos. What is the percent of change?
21. A biology class has 28 students. Four of the students transferred out of the class to take chemistry. Find the percent of change in the number of students in the biology class.
22. **BUSINESS** A restaurant manager wants to reduce spending on supplies 10% in January and an additional 15% in February. In January, the expenses were \$2875. How much should the expenses be at the end of February?
23. **SCHOOL** Jiliana is using a copy machine to increase the size of a 2-inch by 3-inch picture of a spider. The enlarged picture needs to measure 3 inches by 4.5 inches.



What enlargement setting on the copy machine should she use?

24. **CRITICAL THINKING** Explain why a 10% increase followed by a 10% decrease is less than the original amount if the original amount was positive.
25. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can percents help to describe a change in area?

Include the following in your answer:

- an explanation describing how you can tell whether the percent of increase will be greater than 100%, and
- an example of a model that shows an increase less than 100% and one that shows an increase greater than 100%.

26. **RESEARCH** Use the Internet or another source to find the population of your town now and ten years ago. What is the percent of change?



For Exercises 27 and 28, refer to the information in the table.

27. What percent represents the percent of change in the number of beagles from 1998 to 1999?

- (A) -8.1% (B) -7.5%
(C) -9.7% (D) -8.0%

Kennel Club Registrations		
Breed	1998	1999
Labrador Retriever	157,936	157,897
Beagle	53,322	49,080
Maltese	18,013	16,358
Golden Retriever	65,681	62,652
Shih Tzu	38,468	34,576
Cocker Spaniel	34,632	29,958
Siberian Husky	21,078	18,106

28. Which breed had the largest percent of decrease?

- (A) Siberian Husky (B) Cocker Spaniel
(C) Golden Retriever (D) Labrador Retriever

Maintain Your Skills

Mixed Review

29. Find the discount to the nearest cent for a television that costs \$999 and is on sale at 15% off. *(Lesson 6-7)*
30. Find the interest on \$1590 that is invested at 8% for 3 years. Round to the nearest cent. *(Lesson 6-7)*
31. A calendar is on sale at a 10% discount. What is the sale price if it normally sells for \$14.95? *(Lesson 6-7)*

Estimate. Explain which method you used to estimate. *(Lesson 6-6)*

32. 60% of 134 33. 88% of 72 34. 123% of 32

Identify all of the sets to which each number belongs. *(Lesson 5-2)*

35. -8 36. $1\frac{1}{4}$ 37. -5.63

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write each fraction as a percent.

(To review writing fractions as percents, see Lesson 6-4.)

38. $\frac{3}{4}$ 39. $\frac{1}{5}$ 40. $\frac{2}{3}$ 41. $\frac{5}{6}$ 42. $\frac{3}{8}$

Practice Quiz 2

Lessons 6-6 through 6-8

Estimate. Explain which method you used to estimate. *(Lesson 6-6)*

1. 42% of 68 2. $66\frac{2}{3}\%$ of 34
3. Find the discount to the nearest cent on a backpack that costs \$58 and is on sale at 25% off. *(Lesson 6-7)*
4. Find the interest to the nearest cent on \$2500 that is invested at 4% for 2.5 years. *(Lesson 6-7)*
5. Find the percent of change from \$0.95 to \$2.45. *(Lesson 6-8)*



Algebra Activity

A Preview of Lesson 6-9

Taking a Survey

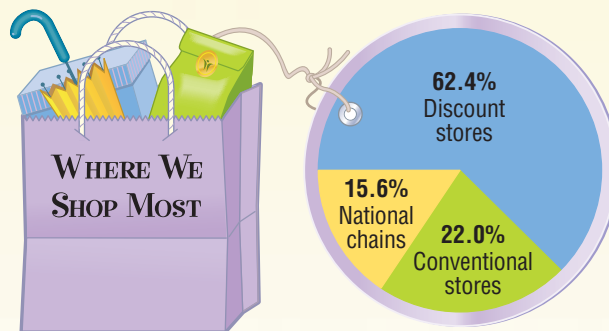
The graph shows the results of a survey about what types of stores people in the United States shop at the most. Since it would be impossible to survey everyone in the country, a sample was used. A **sample** is a subgroup or subset of the population.

It is important to obtain a sample that is unbiased. An **unbiased** sample is a sample that is:

- representative of the larger population,
- selected at **random** or without preference, and
- large enough to provide accurate data.

To insure an unbiased sample, the following sampling methods may be used.

- **Random** The sample is selected at random.
- **Systematic** The sample is selected by using every n th member of the population.
- **Stratified** The sample is selected by dividing the population into groups.



Source: International Mass Retail Association

Model and Analyze

Tell whether or not each of the following is a random sample. Then provide an explanation describing the strengths and weaknesses of each sample.

- | Type of Survey | Location of Survey |
|--|------------------------------|
| 1. travel preference | mall |
| 2. time spent reading | library |
| 3. favorite football player | Miami Dolphins football game |
| 4. Brad conducted a survey to find out which food people in his community prefer. He surveyed every second person that walked into a certain fast-food restaurant. Identify this type of sampling. Explain how the survey may be biased. | |
| 5. Suppose a study shows that teenagers who eat breakfast each day earn higher grades than teenagers who skip breakfast. Tell how you can use the stratified sampling technique to test this claim in your school. | |
| 6. Suppose you want to determine where students in your school shop the most. | |
| a. Formulate a hypothesis about where students shop the most. | |
| b. Design and conduct a survey using one of the sampling techniques described above. | |
| c. Organize and display the results of your survey in a chart or graph. | |
| d. Evaluate your hypothesis by drawing a conclusion based on the survey. | |



What You'll Learn

- Find the probability of simple events.
- Use a sample to predict the actions of a larger group.

Vocabulary

- outcomes
- simple event
- probability
- sample space
- theoretical probability
- experimental probability

How can probability help you make predictions?

A popular word game is played using 100 letter tiles. The object of the game is to use the tiles to spell words scoring as many points as possible. The table shows the distribution of the tiles.

Letter	Number of Tiles
E	12
A, I	9
O	8
N, R, T	6
D, L, S, U	4
G	3
B, C, F, H, M, P, V, W, Y, blank	2
J, K, Q, X, Z	1

- Write the ratio that compares the number of tiles labeled E to the total number of tiles.
- What percent of the tiles are labeled E?
- What fraction of tiles is this?
- Suppose a player chooses a tile. Is there a better chance of choosing a D or an N? Explain.

PROBABILITY OF SIMPLE EVENTS In the activity above, there are 27 possible tiles. These results are called **outcomes**. A **simple event** is one outcome or a collection of outcomes. For example, choosing a tile labeled E is a simple event.

You can measure the chances of an event happening with **probability**.

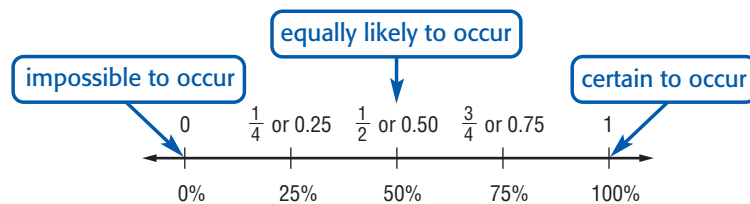
Study Tip**Probability**

Each of the outcomes must be equally likely to happen.

Key Concept**Probability**

- **Words** The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.
- **Symbols** $P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}}$

The probability of an event is always between 0 and 1, inclusive. The closer a probability is to 1, the more likely it is to occur.

**Concept Check**

Suppose there is a 45% chance that an event occurs. How likely is it that the event will occur?

Reading Math

$P(\text{prime})$

$P(\text{prime})$ is read as the probability of rolling a prime number.

Example 1 Find Probability

Suppose a number cube is rolled. What is the probability of rolling a prime number?



There are 3 prime numbers on a number cube: 2, 3, and 5.

There are 6 possible outcomes: 1, 2, 3, 4, 5, and 6.

$$P(\text{prime}) = \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} \\ = \frac{3}{6} \text{ or } \frac{1}{2}$$

So, the probability of rolling a prime number is $\frac{1}{2}$ or 50%.

The set of all possible outcomes is called the **sample space**. For Example 1, the sample space was {1, 2, 3, 4, 5, 6}. When you toss a coin, the sample space is {heads, tails}.

Example 2 Find Probability

Suppose two number cubes are rolled. Find the probability of rolling an even sum.

Make a table showing the sample space when rolling two number cubes.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

There are 18 outcomes in which the sum is even.

$$\text{So, } P(\text{even sum}) = \frac{18}{36} \text{ or } \frac{1}{2}.$$

This means there is a 50% chance of rolling an even sum.

The probabilities in Examples 1 and 2 are called theoretical probabilities. **Theoretical probability** is what *should* occur. **Experimental probability** is what *actually* occurs when conducting a probability experiment.

Example 3 Find Experimental Probability

The table shows the results of an experiment in which a coin was tossed. Find the experimental probability of tossing a coin and getting tails for this experiment.

Outcome	Tally	Frequency
Heads		14
Tails		11

$$\frac{\text{number of times tails occur}}{\text{number of possible outcomes}} = \frac{11}{14 + 11} \text{ or } \frac{11}{25}$$

The experimental probability of getting tails in this case is $\frac{11}{25}$ or 44%.



USE A SAMPLE TO MAKE PREDICTIONS You can use an athlete's past performance to predict whether she will get a hit or make a basket. You can also use the results of a survey to predict the actions of a larger group.



Log on for:

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- www.pre-alg.com/usa_today

Example 4 *Make a Prediction*

FOOD The graph shows the results of a survey. Out of a group of 450 people, how many would you expect to say that they prefer thin mint cookies?

The total number of people is 450. So, 450 is the base. The percent is 26%.

To find 26% of 450, let b represent the base, 450, and let p represent the percent, 26%, in the percent proportion. Let a represent the part.

$$\begin{array}{l} \text{part} \rightarrow \frac{a}{450} = \frac{26}{100} \leftarrow \text{percent} \\ \text{base} \rightarrow \end{array}$$

$$100 \cdot a = 26 \cdot 450$$

$$100a = 11700 \quad \text{Simplify.}$$

$$a = 117 \quad \text{Mentally divide each side by 100.}$$

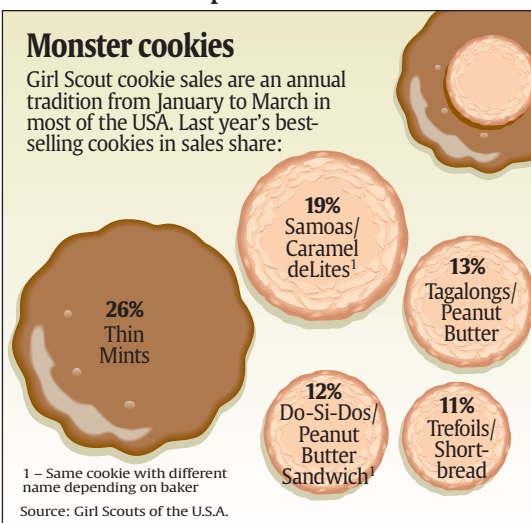
You can expect 117 people to say that they prefer thin mint cookies.



USA TODAY Snapshots®

Monster cookies

Girl Scout cookie sales are an annual tradition from January to March in most of the USA. Last year's best-selling cookies in sales share:



By Anne R. Carey and Quin Tian, USA TODAY

Check for Understanding

Concept Check

1. Tell what a probability of 0 means.
2. Compare and contrast theoretical and experimental probability.
3. **OPEN ENDED** Give an example of a situation in which the probability of the event is 25%.

Guided Practice

Ten cards are numbered 1 through 10, and one card is chosen at random. Determine the probability of each outcome. Express each probability as a fraction and as a percent.

4. $P(5)$
5. $P(\text{odd})$
6. $P(\text{less than } 3)$
7. $P(\text{greater than } 6)$

For Exercises 8 and 9, refer to the table in Example 2 on page 311. Determine each probability. Express each probability as a fraction and as a percent.

8. $P(\text{sum of } 2 \text{ or } 6)$
9. $P(\text{even or odd sum})$
10. Refer to Example 3 on page 311. Find the experimental probability of getting heads for the experiment.

Application

11. **FOOD** Maresha took a sample from a package of jellybeans and found that 30% of the beans were red. Suppose there are 250 jellybeans in the package. How many can she expect to be red?



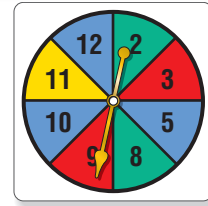
Practice and Apply

Homework Help

For Exercises	See Examples
12–34, 35, 36	1, 2
37	3
	4

Extra Practice
See page 739.

A spinner like the one shown is used in a game. Determine the probability of each outcome if the spinner is equally likely to land on each section. Express each probability as a fraction and as a percent.



- | | | |
|-----------------------------|---------------------------------|------------------------------|
| 12. $P(8)$ | 13. $P(\text{red})$ | 14. $P(\text{even})$ |
| 15. $P(\text{prime})$ | 16. $P(\text{greater than } 5)$ | 17. $P(\text{less than } 2)$ |
| 18. $P(\text{blue or } 11)$ | 19. $P(\text{not yellow})$ | 20. $P(\text{not red})$ |

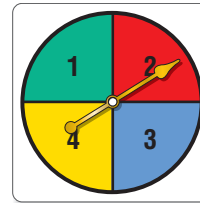
There are 2 red marbles, 4 blue marbles, 7 green marbles, and 5 yellow marbles in a bag. Suppose one marble is selected at random. Find the probability of each outcome. Express each probability as a fraction and as a percent.

- | | | |
|----------------------------|-------------------------------------|--------------------------------|
| 21. $P(\text{blue})$ | 22. $P(\text{yellow})$ | 23. $P(\text{not green})$ |
| 24. $P(\text{purple})$ | 25. $P(\text{red or blue})$ | 26. $P(\text{blue or yellow})$ |
| 27. $P(\text{not orange})$ | 28. $P(\text{not blue or not red})$ | |

29. What is the probability that a calendar is randomly turned to the month of January or April?

30. Find the probability that today is November 31.

Suppose two spinners like the ones shown are spun. Find the probability of each outcome. (*Hint: Make a table to show the sample space as in Example 2 on page 311.*)



- | | |
|---------------------------|------------------------------------|
| 31. $P(2, 7)$ | 32. $P(\text{even, even})$ |
| 33. $P(\text{sum of } 9)$ | 34. $P(2, \text{greater than } 5)$ |

DRIVING For Exercises 35 and 36, use the following information and the table shown. The table shows the approximate number of licensed automobile drivers in the United States in a certain year. An automobile company is conducting a telephone survey using a list of licensed drivers.

Age	Drivers (millions)
19 and under	9
20–29	34
30–39	41
40–49	37
50–59	24
60–69	18
70 and over	17
Total	180

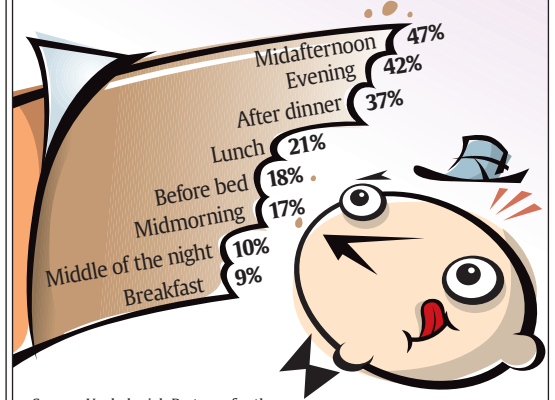
Source: U.S. Department of Transportation

35. Find the probability that a driver will be 19 years old or younger. Express the answer as a decimal rounded to the nearest hundredth and as a percent.
36. What is the probability that a randomly chosen driver will be 40–49 years old? Write the answer as a decimal rounded to the nearest hundredth and as a percent.

USA TODAY Snapshots®

Chocolate cravings

Time of day that adults say they crave chocolate:



Source: Yankelovich Partners for the American Boxed Chocolate Manufacturers

By Cindy Hall and Alejandro Gonzalez, USA TODAY

37. **FOOD** Refer to the graph. Out of 1200 people, how many would you expect to say they crave chocolate after dinner?
38. **CRITICAL THINKING** In the English language, 13% of the letters used are E's. Suppose you are guessing the letters in a two-letter word of a puzzle. Would you guess an E? Explain.
39. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can probability help you make predictions?

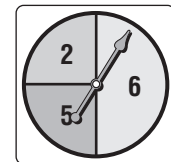
Include the following in your answer:

- an explanation telling the probability of choosing each letter tile, and
- an example of how you can use probability to make predictions.



40. What is the probability of spinning an even number on the spinner shown?

- (A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$



Maintain Your Skills

Mixed Review

41. Find the percent of change from 32 feet to 79 feet. Round to the nearest tenth, if necessary. Then state whether the percent of change is a *percent of increase* or a *percent of decrease*. (Lesson 6-8)

Solve each problem using an equation. Round to the nearest tenth.

(Lesson 6-7)

42. 7 is what percent of 32? 43. What is 28.5% of 84?

ALGEBRA Find each product or quotient. Express your answer in exponential form. (Lesson 4-6)

44. $7^2 \cdot 7^3$ 45. $x^4 \cdot 2x$ 46. $\frac{8^{12}}{8^8}$ 47. $\frac{36n^4}{14n^2}$

Web Quest Internet Project

Kids Gobbling Empty Calories

It is time to complete your project. Use the information and data you have gathered to prepare a brochure or Web page about the nutritional value of fast-food meals. Include the total Calories, grams of fat, and amount of sodium for five meals that a typical student would order from at least three fast-food restaurants.



www.pre-alg.com/webquest



Graphing Calculator Investigation

A Follow-Up of Lesson 6-9

Probability Simulation

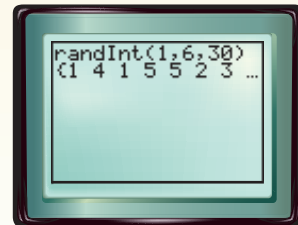
A random number generator can simulate a probability experiment. From the simulation, you can calculate experimental probabilities. Repeating a simulation may result in different probabilities since the numbers generated are different each time.

Example Generate 30 random numbers from 1 to 6, simulating 30 rolls of a number cube.

- Access the random number generator.
- Enter 1 as a lower bound and 6 as an upper bound for 30 trials.

KEYSTROKES: MATH ◀ 5 1 , 6 , 30) ENTER

A set of 30 numbers ranging from 1 to 6 appears. Use the right arrow key to see the next number in the set. Record all 30 numbers, as a column, on a separate sheet of paper.



Exercises

1. Record how often each number on the number cube appeared.
 - a. Find the experimental probability of each number.
 - b. Compare the experimental probabilities with the theoretical probabilities.
2. Repeat the simulation of rolling a number cube 30 times. Record this second set of numbers in a column next to the first set of numbers. Each pair of 30 numbers represents a roll of two number cubes. Find the sum for each of the 30 pairs of rolls.
 - a. Find the experimental probability of each sum.
 - b. Compare the experimental probability with the theoretical probabilities.
3. Design an experiment to simulate 30 spins of a spinner that has equal sections colored red, white, and blue.
 - a. Find the experimental probability of each color.
 - b. Compare the experimental probabilities with the theoretical probabilities.
4. Suppose you play a game where there are three containers, each with ten balls numbered 0 to 9. Pick three numbers and then use the random number generator to simulate the game. Score 2 points if one number matches, 16 points if two numbers match, and 32 points if all three numbers match. Note: numbers can appear more than once.
 - a. Play the game if the order of your numbers *does not* matter. Total your score for 10 simulations.
 - b. Now play the game if the order of the numbers *does* matter. Total your score for 10 simulations.
 - c. With which game rules did you score more points?



Vocabulary and Concept Check

base (p. 288)	percent of decrease (p. 306)	scale (p. 276)
compound interest (p. 303)	percent of increase (p. 305)	scale drawing (p. 276)
cross products (p. 270)	percent proportion (p. 288)	scale factor (p. 277)
discount (p. 299)	probability (p. 310)	scale model (p. 276)
experimental probability (p. 311)	proportion (p. 270)	simple event (p. 310)
outcome (p. 310)	random (p. 309)	simple interest (p. 300)
part (p. 288)	rate (p. 265)	theoretical probability (p. 311)
percent (p. 281)	ratio (p. 264)	unbiased (p. 309)
percent equation (p. 298)	sample (p. 309)	unit rate (p. 265)
percent of change (p. 304)	sample space (p. 311)	

Complete each sentence with the correct term.

1. A statement of equality of two ratios is called a(n) _____.
2. A(n) _____ is a ratio that compares a number to 100.
3. The ratio of a length on a scale drawing to the corresponding length on the real object is called the _____.
4. The set of all possible outcomes is the _____.
5. _____ is what actually occurs when conducting a probability experiment.

Lesson-by-Lesson Review

6-1 Ratios and Rates

See pages
264–268.

Concept Summary

- A ratio is a comparison of two numbers by division.
- A rate is a ratio of two measurements having different units of measure.
- A rate that is simplified so that it has a denominator of 1 is called a unit rate.

Example

Express the ratio *2 meters to 35 centimeters* as a fraction in simplest form.

$$\begin{aligned} \frac{2 \text{ meters}}{35 \text{ centimeters}} &= \frac{200 \text{ centimeters}}{35 \text{ centimeters}} && \text{Convert 2 meters to centimeters.} \\ &= \frac{40 \text{ centimeters}}{7 \text{ centimeters}} \text{ or } \frac{40}{7} && \text{Divide the numerator and denominator by the GCF, 5.} \end{aligned}$$

Exercises Express each ratio as a fraction in simplest form.

See Examples 1 and 2 on pages 264 and 265.

6. 9 students out of 33 students
7. 12 hits out of 16 times at bat
8. 30 hours to 18 hours
9. 5 quarts to 5 gallons
10. 10 inches to 4 feet
11. 2 tons to 1800 pounds

6-2 Using ProportionsSee pages
270–274.**Concept Summary**

- A proportion is an equation stating two ratios are equal.
- If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Example

Solve $\frac{3}{7} = \frac{15}{x}$.

$$\frac{3}{7} = \frac{15}{x} \quad \text{Write the proportion.}$$

$$3 \cdot x = 7 \cdot 15 \quad \text{Cross products}$$

$$3x = 105 \quad \text{Multiply.}$$

$$\frac{3x}{3} = \frac{105}{3} \quad \text{Divide each side by 3.}$$

$$x = 35 \quad \text{The solution is 35.}$$

Exercises Solve each proportion. See Example 2 on page 271.

12. $\frac{n}{12} = \frac{4}{3}$

13. $\frac{21}{x} = \frac{84}{120}$

14. $\frac{9}{7} = \frac{22.5}{y}$

15. $\frac{5}{7.5} = \frac{0.6}{k}$

6-3 Scale Drawings and ModelsSee pages
276–280.**Concept Summary**

- A scale drawing or a scale model represents an object that is too large or too small to be drawn or built at actual size.
- The ratio of a length on a scale drawing or model to the corresponding length on the real object is called the scale factor.

Example

A scale drawing shows a pond that is 1.75 inches long. The scale on the drawing is 0.25 inch = 1 foot. What is the length of the actual pond?

$$\begin{array}{l} \text{drawing length} \rightarrow \\ \text{actual length} \rightarrow \end{array} \frac{0.25 \text{ in.}}{1 \text{ ft}} = \frac{1.75 \text{ in.}}{x \text{ ft}} \quad \begin{array}{l} \leftarrow \text{drawing length} \\ \leftarrow \text{actual length} \end{array}$$

$$0.25 \cdot x = 1 \cdot 1.75 \quad \text{Find the cross products.}$$

$$0.25x = 1.75 \quad \text{Simplify.}$$

$$x = 7 \quad \text{Divide each side by 0.25.}$$

The actual length of the pond is 7 feet.

Exercises On the model of a ship, the scale is 1 inch = 12 feet. Find the actual length of each room. See Example 1 on page 277.

	Room	Model Length
16.	Stateroom	0.9 in.
17.	Galley	3.8 in.
18.	Gym	6.0 in.



6-4 Fractions, Decimals, and Percents

See pages
281–285.

Concept Summary

- A percent is a ratio that compares a number to 100.
- Fractions, decimals, and percents are all different ways to represent the same number.

Examples

1 Express 60% as a fraction in simplest form and as a decimal.

$$60\% = \frac{60}{100} \text{ or } \frac{3}{5} \quad 60\% = \underbrace{60\%}_{0.6} \text{ or } 0.6$$

2 Express 0.38 as a percent.

$$0.38 = 0.38 \text{ or } 38\%$$

3 Express $\frac{5}{8}$ as a percent.

$$\frac{5}{8} = 0.625 \text{ or } 62.5\%$$

Exercises Express each percent as a fraction or mixed number in simplest form and as a decimal. See Examples 1 and 3 on pages 281 and 282.

- | | | | |
|----------|----------|-----------|----------|
| 19. 35% | 20. 42% | 21. 8% | 22. 19% |
| 23. 120% | 24. 250% | 25. 62.5% | 26. 8.8% |

Express each decimal or fraction as a percent. Round to the nearest tenth percent, if necessary. See Examples 2, 4, and 5 on pages 282 and 283.

- | | | | |
|-------------------|---------------------|--------------------|----------------------|
| 27. 0.24 | 28. 0.03 | 29. 0.452 | 30. 1.9 |
| 31. $\frac{2}{5}$ | 32. $\frac{13}{22}$ | 33. $\frac{6}{80}$ | 34. $\frac{77}{225}$ |

6-5 Using the Percent Proportion

See pages
288–292.

Concept Summary

- If a is the part, b is the base, and p is the percent, then $\frac{a}{b} = \frac{p}{100}$.

Example

Forty-eight is 32% of what number?

$$\frac{a}{b} = \frac{p}{100} \rightarrow \frac{48}{b} = \frac{32}{100} \quad \text{Replace } a \text{ with } 48 \text{ and } p \text{ with } 32.$$

$$48 \cdot 100 = b \cdot 32 \quad \text{Find the cross products.}$$

$$4800 = 32b \quad \text{Simplify.}$$

$$150 = b \quad \text{Divide each side by } 32.$$

So, 48 is 32% of 150.

Exercises Use the percent proportion to solve each problem.

See Examples 1–6 on pages 288–290.

- | | |
|-------------------------------|-------------------------------|
| 35. 18 is what percent of 45? | 36. What percent of 60 is 39? |
| 37. 23 is 92% of what number? | 38. What is 74% of 110? |
| 39. What is 80% of 62.5? | 40. 36 is 15% of what number? |

6-6 Finding Percents Mentally

See pages
293–297.

Concept Summary

- When working with common percents like 10%, 20%, 25%, and 50%, it is helpful to use the fraction form of the percent.

Examples

1 Find 20% of \$45 mentally.

$$\begin{aligned} 20\% \text{ of } 45 &= \frac{1}{5} \text{ of } 45 && \text{Think: } 20\% = \frac{1}{5}. \\ &= 9 && \text{Think: } \frac{1}{5} \text{ of } 45 \text{ is } 9. \end{aligned}$$

So, 20% of \$45 is \$9.

2 Estimate 32% of 150.

$$\begin{aligned} 32\% \text{ is about } 33\frac{1}{3}\% \text{ or } \frac{1}{3}. \\ \frac{1}{3} \text{ of } 150 \text{ is } 50. \end{aligned}$$

So, 32% of 150 is about 50.

Exercises Find the percent of each number mentally.

See Example 1 on pages 293 and 294.

- | | | |
|---------------|-----------------------------|---------------|
| 41. 50% of 86 | 42. 20% of 55 | 43. 25% of 36 |
| 44. 40% of 75 | 45. $33\frac{1}{3}\%$ of 24 | 46. 90% of 60 |

Estimate. Explain which method you used to estimate.

See Example 2 on pages 294 and 295.

- | | | |
|---------------|----------------------------|-----------------|
| 47. 48% of 32 | 48. 67% of 30 | 49. 20% of 51 |
| 50. 25% of 27 | 51. $\frac{1}{3}\%$ of 304 | 52. 147% of 200 |

6-7 Using Percent Equations

See pages
298–302.

Concept Summary

- The percent equation is an equivalent form of the percent proportion in which the percent is written as a decimal.
- Part = Percent · Base, where percent is in decimal form.

Example

119 is 85% of what number?

The part is 119, and the percent is 85%. Let n represent the base.

$$119 = 0.85n \quad \text{Write } 85\% \text{ as the decimal } 0.85.$$

$$\frac{119}{0.85} = \frac{0.85n}{0.85} \quad \text{Divide each side by } 0.85.$$

$$140 = n \quad \text{So, } 119 \text{ is } 85\% \text{ of } 140.$$

Exercises Solve each problem using the percent equation.

See Examples 1–3 on pages 298 and 299.

- | | |
|-------------------------------|---------------------------------|
| 53. 24 is what percent of 50? | 54. 70 is 40% of what number? |
| 55. What is 90% of 105? | 56. What is 12.5% of 68? |
| 57. 56 is 28% of what number? | 58. 35.7 is what percent of 17? |

6-8 Percent of Change

See pages
304–308.

Concept Summary

- A percent of increase tells how much an amount has increased in relation to the original amount. (The percent will be positive.)
- A percent of decrease tells how much an amount has decreased in relation to the original amount. (The percent will be negative.)

Example

Find the percent of change from 36 pounds to 14 pounds.

$$\begin{aligned} \text{percent of change} &= \frac{\text{new weight} - \text{original weight}}{\text{original weight}} && \text{Write the ratio.} \\ &= \frac{14 - 36}{36} && \text{Substitution} \\ &= \frac{-22}{36} && \text{Subtraction} \\ &\approx -0.611 \text{ or } -61.1\% && \text{Simplify.} \end{aligned}$$

The percent of decrease is about 61.1%.

Exercises Find the percent of change. Round to the nearest tenth, if necessary. Then state whether each change is a *percent of increase* or a *percent of decrease*. See Examples 1, 2, and 4 on pages 304–306.

59. from 40 ft to 12 ft 60. from 80 cm to 96 cm
61. from 29 min to 54 min 62. from 80 lb to 77 lb

6-9 Probability and Predictions

See pages
310–314.

Concept Summary

- The probability of an event is a ratio that compares the number of favorable outcomes to the number of possible outcomes.

Example

Suppose a number cube is rolled. Find the probability of rolling a 5 or 6.

Favorable outcomes: 5 and 6.

Possible outcomes: 1, 2, 3, 4, 5, and 6.

$$\begin{aligned} P(5 \text{ or } 6) &= \frac{\text{number of favorable outcomes}}{\text{number of possible outcomes}} \\ &= \frac{2}{6} \text{ or } \frac{1}{3} \end{aligned} \quad \text{So, the probability of rolling a 5 or 6 is } \frac{1}{3} \text{ or } 33\frac{1}{3}\%$$

Exercises There are 2 blue marbles, 5 red marbles, and 8 green marbles in a bag. One marble is selected at random. Find the probability of each outcome. See Examples 1 and 2 on page 311.

63. $P(\text{red})$ 64. $P(\text{green})$ 65. $P(\text{blue or green})$
66. $P(\text{not blue})$ 67. $P(\text{yellow})$ 68. $P(\text{green, red, or blue})$

Vocabulary and Concepts

1. Explain the difference between a ratio and a rate.
2. Describe how to express a fraction as a percent.

Skills and Applications

Express each ratio as a fraction in simplest form.

3. 15 girls out of 40 students
4. 6 feet to 3 yards

Express each ratio as a unit rate. Round to the nearest tenth or cent.

5. 145 miles in 3 hours
6. \$245 for 9 tickets
7. Convert 15 miles per hour to x feet per minute.
8. What value of y makes $\frac{8.4}{y} = \frac{1.2}{1.1}$ a proportion?

Express each percent as a fraction or mixed number in simplest form and as a decimal.

9. 36%
10. 52%
11. 225%
12. 315%
13. 0.6%
14. 0.4%

Express each decimal or fraction as a percent. Round to the nearest tenth percent, if necessary.

15. 0.47
16. 0.025
17. 5.38
18. $\frac{7}{20}$
19. $\frac{30}{22}$
20. $\frac{18}{4000}$

Use the percent proportion to solve each problem.

21. 36 is what percent of 80?
22. 35.28 is 63% of what number?

Estimate.

23. 25% of 82
24. 63% of 77
25. Find the interest on \$2700 that is invested at 4% for $2\frac{1}{2}$ years.
26. Find the discount for a \$135 coat that is on sale at 15% off.
27. Find the percent of change from 175 pounds to 140 pounds. Round to the nearest tenth.
28. There are 3 purple balls, 5 orange balls, and 8 yellow balls in a bowl. Suppose one ball is selected at random. Find $P(\text{orange})$.
29. **DESIGN** A builder is designing a swimming pool that is 8.5 inches in length on the scale drawing. The scale of the drawing is 1 inch = 6 feet. What is the length of the actual swimming pool?

30. **STANDARDIZED TEST PRACTICE** The table lists the reasons shoppers use online customer service. Out of 350 shoppers who own a computer, how many would you expect to say they use online customer service to track packages?

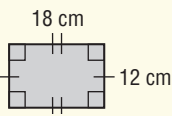
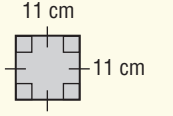
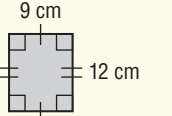
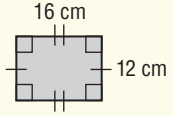
- (A) 189 (B) 84 (C) 19 (D) 154

Reasons	Percent
Track Delivery	54
Product Information	24
Verify Shipping Charges	17
Transaction Help	16

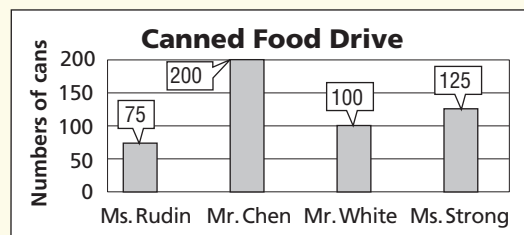


Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

- Evaluate $x - y + z$ if $x = -6$, $y = 9$, and $z = -3$. (Lesson 2-3)
 - (A) 0
 - (B) -6
 - (C) -18
 - (D) 15
- Which figure has an area of 192 cm^2 ? (Lesson 3-7)
 - (A) 
 - (B) 
 - (C) 
 - (D) 
- Which expression is *not* a monomial? (Lesson 4-1)
 - (A) $5(-y)$
 - (B) $8k$
 - (C) $m - n$
 - (D) $2x(-3y)$
- Which fraction represents the ratio 8 apples to $36 \text{ pieces of fruit}$ in simplest form? (Lesson 6-1)
 - (A) $\frac{1}{4}$
 - (B) $\frac{4}{9}$
 - (C) $\frac{2}{9}$
 - (D) $\frac{1}{6}$
- The ratio of girls to boys in a class is 5 to 4. Suppose there are 27 students in the class. How many of the students are girls? (Lesson 6-2)
 - (A) 40
 - (B) 15
 - (C) 12
 - (D) 9
- A scale model of an airplane has a width of 13.5 inches. The scale of the model is 1 inch = 8 feet. What is the width of the actual airplane? (Lesson 6-3)
 - (A) 110 ft
 - (B) 108 ft
 - (C) 104 ft
 - (D) 115 ft

- Randy, Eduardo, and Kelli took a quiz. For every 50 questions on the quiz, Randy answered 47 correctly. Eduardo answered 91% of the questions correctly. For every 10 questions on the quiz, Kelli answered 9 correctly. Who had the highest score?
 - (A) Randy
 - (B) Eduardo
 - (C) Kelli
 - (D) all the same
- The graph shows the amount of canned food collected by the 9th grade classes at Hilltop High School.



Of the total amount of cans collected, what percent did Mr. Chen's class collect?

(Lesson 6-4)

- (A) 25%
 - (B) 33%
 - (C) 40%
 - (D) 50%
- The table shows the average salaries in each of the four major sports for the 1990-91 and 2000-01 seasons. (Lesson 6-8)

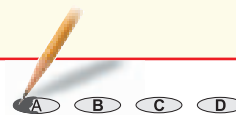
Sport	1990-91	2000-01
Hockey	\$271,000	\$1,400,000
Basketball	823,000	3,530,000
Football	430,000	1,200,000
Baseball	597,537	2,260,000

Source: USA TODAY

Which sport had a percent of increase in average salary of about 325%?

- (A) Hockey
- (B) Basketball
- (C) Football
- (D) Baseball

Test-Taking Tip



Question 8

To find what percent of the cans a class collected, you will first need to find the total number of cans collected by all of the 9th grade classes.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

10. Ana earns \$6.80 per hour when she works on weekdays. She earns twice that amount per hour when she works on weekends. If Ana worked 4 hours on Tuesday, 4 hours on Thursday, and 5 hours on Saturday, then how much did she earn?

(Prerequisite Skill, p. 713)

11. Juan and Julia decided to eat lunch at The Sub Shop. Juan ordered a veggie sub, lemonade, and a cookie. Julia ordered a ham sub, milk, and a cookie. What was the total cost of Juan and Julia's lunch?

Item	Cost
Veggie Sub	\$3.89
Turkey Sub	\$3.79
Ham Sub	\$3.49
Soda	\$1.25
Lemonade	\$1.00
Milk	\$0.79
Cookie	\$0.99

(Prerequisite Skill, p. 713)

12. What number should replace X in this pattern?

$4^0 = 1$
$4^1 = 4$
$4^2 = 16$
$4^3 = 64$
$4^4 = X$

(Lesson 4-2)

13. Find the value of m in $\frac{3}{8}m = \frac{1}{4}$.

(Lesson 5-9)

14. Nakayla purchased a package of 8 hamburger buns for \$1.49. What is the ratio of the cost per hamburger bun? Round to the nearest penny.

(Lesson 6-1)

15. What is 40% of 70?

(Lesson 6-5)

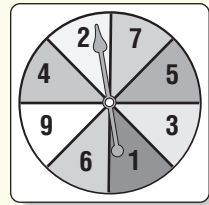
16. Cameron purchased the portable stereo shown. About how much money did he save?

(Lesson 6-7)



17. If you spin the spinner shown at the right, what is the probability that the arrow will stop at an even number?

(Lesson 6-9)

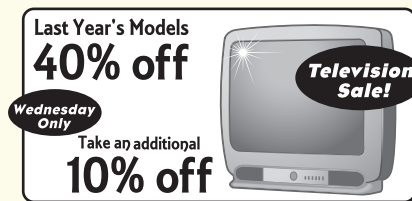


Part 3 Extended Response

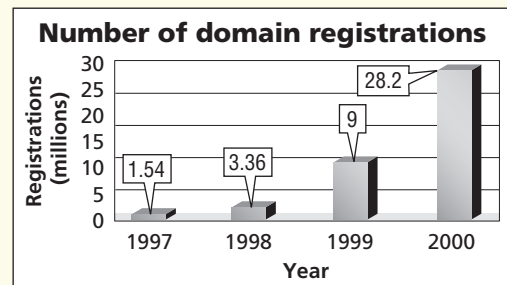
Record your answers on a sheet of paper. Show your work.

18. An electronics store is having a sale on certain models of televisions. Mr. Castillo would like to buy a television that is on sale. This television normally costs \$679.

(Lesson 6-7)



- a. What price, not including tax, will Mr. Castillo pay if he buys the television on Saturday?
- b. What price, not including tax, will Mr. Castillo pay if he buys the television on Wednesday?
- c. How much money will Mr. Castillo save if he buys the television on a Saturday?
19. The graph shows the number of domain registrations for the years 1997–2000.



Source: Network Solutions (VeriSign)

Write a few sentences describing the percent of change in the number of domain registrations from one year to the next.

(Lesson 6-8)