

Rational Numbers

What You'll Learn

- **Lessons 5-1 and 5-2** Write fractions as decimals and write decimals as fractions.
- **Lessons 5-3, 5-4, 5-5, and 5-7** Add, subtract, multiply, and divide rational numbers.
- **Lessons 5-6 and 5-9** Use the least common denominator to compare fractions and to solve equations.
- **Lesson 5-8** Use the mean, median, and mode to analyze data.
- **Lesson 5-10** Find the terms of arithmetic and geometric sequences.

Key Vocabulary

- rational number (p. 205)
- algebraic fraction (p. 211)
- multiplicative inverse (p. 215)
- measures of central tendency (p. 238)
- sequence (p. 249)

Why It's Important

Rational numbers are the numbers used most often in the real world. They include fractions, decimals, and integers. Understanding rational numbers is important in understanding and analyzing real-world occurrences, such as changes in barometric pressure during a storm.

You will compare the barometric pressure before and after a storm in Lesson 5-9.

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 5.

For Lessons 5-1 through 5-4

Multiply and Divide Integers

Find each product or quotient. If necessary, round to the nearest tenth.

(For review, see Lessons 2-4 and 2-5.)

1. $3 \div 5$

2. $-1 \div 8$

3. $2 \cdot 17$

4. $-12 \cdot 3$

5. $-2 \div (-9)$

6. $-4(-6)$

7. $5(-15)$

8. $4 \div (-15)$

9. $-24 \div 14$

For Lesson 5-5

Simplify Fractions Using the GCF

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*. (For review, see Lesson 4-5.)

10. $\frac{5}{40}$

11. $\frac{12}{20}$

12. $\frac{14}{39}$

13. $\frac{36}{50}$

For Lessons 5-8 through 5-10

Add and Subtract Integers

Find each sum or difference. (For review, see Lessons 2-2 and 2-3.)

14. $4 + (-9)$

15. $-10 + 16$

16. $20 - 12$

17. $19 - 32$

18. $7 + (-5)$

19. $26 - 11$

20. $(-3) + (-8)$

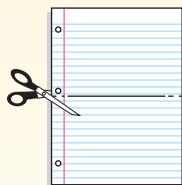
21. $-1 - (-10)$

FOLDABLES™ Study Organizer

Rational Numbers Make this Foldable to help you organize your notes. Begin with three sheets of $8\frac{1}{2}$ " by 11" paper.

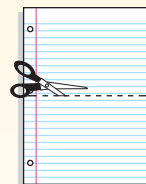
Step 1 Fold and Cut Two Sheets

Fold each sheet in half from top to bottom. Cut along the fold from the edges to the margin.



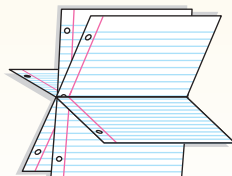
Step 2 Fold and Cut the Other Sheet

Fold in half from top to bottom. Cut along fold between the margins.



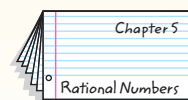
Step 3 Fold

Insert the first two sheets through the third sheet and align the folds.



Step 4 Label

Label each page with a lesson number and title. Write the chapter title on the front.



Reading and Writing As you read and study the chapter, fill the journal with notes, diagrams, and examples for rational numbers.

What You'll Learn

- Write fractions as terminating or repeating decimals.
- Compare fractions and decimals.

Vocabulary

- terminating decimal
- mixed number
- repeating decimal
- bar notation
- period

How


were fractions used to determine the size of the first coins?

In the 18th century, a silver dollar contained \$1 worth of silver. The sizes of all other coins were based on this coin.

- A half dollar contained half the silver of a silver dollar. What was it worth?
- Write the decimal value of each coin in the table.
- Order the fractions in the table from least to greatest. (*Hint*: Use the values of the coins.)

Coin	Fraction of Silver of \$1 Coin
quarter-dollar (quarter)	$\frac{1}{4}$
10-cent (dime)	$\frac{1}{10}$
half-dime* (nickel)	$\frac{1}{20}$

* In 1866, nickels were enlarged for convenience


Reading Math**Terminating**

Everyday Meaning: bringing to an end

Math Meaning: a decimal whose digits end

WRITE FRACTIONS AS DECIMALS Any fraction $\frac{a}{b}$, where $b \neq 0$, can be written as a decimal by dividing the numerator by the denominator. So, $\frac{a}{b} = a \div b$. If the division ends, or terminates, when the remainder is zero, the decimal is a **terminating decimal**.

Example 1 Write a Fraction as a Terminating Decimal

Write $\frac{3}{8}$ as a decimal.

Method 1 Use paper and pencil.

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.000} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

Division ends when the remainder is 0.

0.375 is a terminating decimal.

Method 2 Use a calculator.

$$3 \div 8 \text{ ENTER } .375$$

$$\frac{3}{8} = 0.375$$

A **mixed number** such as $3\frac{1}{2}$ is the sum of a whole number and a fraction. Mixed numbers can also be written as decimals.

Example 2 Write a Mixed Number as a Decimal

Write $3\frac{1}{2}$ as a decimal.

$$\begin{aligned} 3\frac{1}{2} &= 3 + \frac{1}{2} && \text{Write as the sum of an integer and a fraction.} \\ &= 3 + 0.5 && \frac{1}{2} = 0.5 \\ &= 3.5 && \text{Add.} \end{aligned}$$

Not all fractions can be written as terminating decimals.

$$\begin{array}{r} \frac{2}{3} \rightarrow \begin{array}{r} 0.666 \\ 3 \overline{)2.000} \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \end{array} \end{array}$$

The number 6 repeats.

The remainder after each step is 2.

CHECK 2 \div 3 ENTER .666666667 \checkmark The last digit is rounded.

So, $\frac{2}{3} = 0.666666666\dots$. This decimal is called a **repeating decimal**. You can use **bar notation** to indicate that the 6 repeats forever.

$$0.666666666\dots = 0.\overline{6} \quad \text{The digit 6 repeats, so place a bar over the 6.}$$

The **period** of a repeating decimal is the digit or digits that repeat. So, the period of $0.\overline{6}$ is 6.

Decimal	Bar Notation	Period
0.13131313...	$0.\overline{13}$	13
6.855555...	$6.\overline{85}$	5
19.1724724...	$19.\overline{1724}$	724

✓ Concept Check Is $0.\overline{75}$ a terminating or a repeating decimal? Explain.

Example 3 Write Fractions as Repeating Decimals

a. Write $-\frac{6}{11}$ as a decimal.

$$-\frac{6}{11} \rightarrow \begin{array}{r} 0.5454\dots \\ 11 \overline{)6.0000\dots} \end{array} \quad \text{The digits 54 repeat.}$$

$$\text{So, } -\frac{6}{11} = -0.\overline{54}.$$

b. Write $\frac{2}{15}$ as a decimal.

$$\frac{2}{15} \rightarrow \begin{array}{r} 0.1333\dots \\ 15 \overline{)2.0000\dots} \end{array} \quad \text{The digit 3 repeats.}$$

$$\text{So, } \frac{2}{15} = 0.1\overline{3}.$$

Study Tip

Mental Math

It will be helpful to memorize the following list of fraction-decimal equivalents.

$$\frac{1}{2} = 0.5 \quad \frac{3}{4} = 0.75$$

$$\frac{1}{3} = 0.\overline{3} \quad \frac{2}{5} = 0.4$$

$$\frac{1}{4} = 0.25 \quad \frac{3}{5} = 0.6$$

$$\frac{1}{5} = 0.2 \quad \frac{4}{5} = 0.8$$

$$\frac{2}{3} = 0.\overline{6}$$



COMPARE FRACTIONS AND DECIMALS It may be easier to compare numbers when they are written as decimals.

Example 4 Compare Fractions and Decimals

Replace \bullet with $<$, $>$, or $=$ to make $\frac{3}{5} \bullet 0.75$ a true sentence.

$\frac{3}{5} \bullet 0.75$ Write the sentence.

$0.6 \bullet 0.75$ Write $\frac{3}{5}$ as a decimal.



$0.6 < 0.75$ In the tenths place, $6 < 7$.



On a number line, 0.6 is to the left of 0.75, so $\frac{3}{5} < 0.75$.

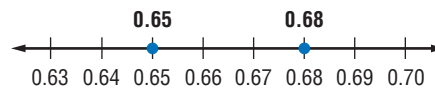
Example 5 Compare Fractions to Solve a Problem

BREAKFAST In a survey of students, $\frac{13}{20}$ of the boys and $\frac{17}{25}$ of the girls make their own breakfast. Of those surveyed, do a greater fraction of boys or girls make their own breakfast?

Write the fractions as decimals and then compare the decimals.

boys: $\frac{13}{20} = 0.65$

girls: $\frac{17}{25} = 0.68$



On a number line, 0.65 is to the left of 0.68. Since $0.65 < 0.68$, $\frac{13}{20} < \frac{17}{25}$. So, a greater fraction of girls make their own breakfast.

Check for Understanding

Concept Check

- Describe the steps you should take to order $\frac{5}{8}$, 0.8, and $\frac{3}{5}$.
- Explain how 0.5 and $0.\bar{5}$ are different. Which is greater?
- OPEN ENDED** Give an example of a repeating decimal whose period is 14.

Guided Practice

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal.

4. $\frac{7}{8}$

5. $2\frac{2}{25}$

6. $-\frac{5}{9}$

7. $\frac{4}{15}$

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

8. $\frac{9}{10} \bullet 0.90$

9. $0.3 \bullet \frac{1}{3}$

10. $\frac{1}{4} \bullet \frac{1}{3}$

11. $-\frac{3}{4} \bullet -\frac{7}{8}$

Application

- TRUCKS** Of all the passenger trucks sold each year in the United States, $\frac{1}{5}$ are pickups and 0.17 are SUVs. Are more SUVs or pickups sold? Explain. **Source:** U.S. Department of Energy, EPA

Practice and Apply

Homework Help

For Exercises	See Examples
13–20	1, 2
21–28	3
32–43	4
46, 47	5

Extra Practice
See page 733.

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal.

- | | | | |
|---------------------|--------------------|---------------------|---------------------|
| 13. $\frac{1}{5}$ | 14. $\frac{3}{20}$ | 15. $\frac{8}{25}$ | 16. $-\frac{5}{8}$ |
| 17. $7\frac{3}{10}$ | 18. $1\frac{1}{2}$ | 19. $5\frac{1}{8}$ | 20. $-3\frac{3}{4}$ |
| 21. $\frac{1}{9}$ | 22. $-\frac{2}{9}$ | 23. $-\frac{5}{11}$ | 24. $\frac{4}{11}$ |
| 25. $\frac{1}{6}$ | 26. $\frac{7}{15}$ | 27. $\frac{5}{16}$ | 28. $\frac{7}{12}$ |

29. **ANIMALS** A marlin can swim $\frac{5}{6}$ mile in one minute. Write $\frac{5}{6}$ as a decimal rounded to the nearest hundredth.

30. **COMPUTERS** In a survey, 17 students out of 20 said they use a home PC as a reference source for school projects. Write 17 out of 20 as a decimal.

Source: NPD Online Research

31. Order $\frac{7}{8}$, 0.8, and $\frac{7}{9}$ from least to greatest.

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

- | | | | |
|-------------------------------------|----------------------------------|--|---------------------------------------|
| 32. $0.3 \bullet \frac{1}{4}$ | 33. $\frac{5}{8} \bullet 0.65$ | 34. $\frac{2}{5} \bullet 0.4$ | 35. $\frac{1}{3} \bullet \frac{1}{2}$ |
| 36. $\frac{1}{5} \bullet 0.\bar{5}$ | 37. $1\frac{1}{20} \bullet 1.01$ | 38. $\frac{7}{8} \bullet \frac{8}{9}$ | 39. $3\frac{4}{9} \bullet 3.\bar{4}$ |
| 40. $6.18 \bullet 6\frac{1}{5}$ | 41. $-0.75 \bullet -\frac{7}{9}$ | 42. $0.3\bar{4} \bullet \frac{34}{99}$ | 43. $-2\frac{1}{12} \bullet -2.09$ |

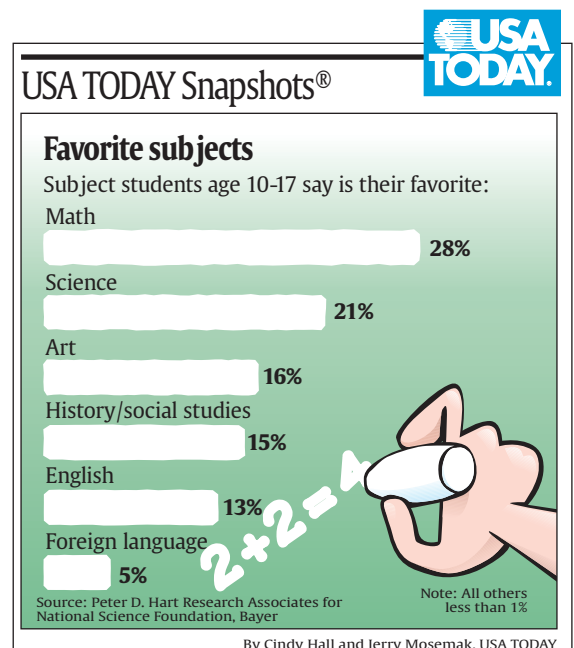
44. On a number line, would $\frac{11}{15}$ be graphed to the right or to the left of $\frac{3}{4}$? Explain.

45. Find a terminating and a repeating decimal between $\frac{1}{6}$ and $\frac{8}{9}$. Explain how you found them.

SCHOOL For Exercises 46 and 47, use the graphic at the right and the information below.

28% = 0.28	21% = 0.21
16% = 0.16	15% = 0.15
13% = 0.13	5% = 0.05

46. Did more or less than one-fourth of the students surveyed choose math as their favorite subject? Explain.
47. Suppose $\frac{1}{7}$ of the students in your class choose English as their favorite subject. How does this compare to the results of the survey? Explain.



48. **CRITICAL THINKING**

- a. Write the prime factorization of each denominator in the fractions listed below.

$$\frac{1}{2'} \frac{1}{3'} \frac{1}{4'} \frac{1}{5'} \frac{1}{6'} \frac{1}{8'} \frac{1}{9'} \frac{1}{10'} \frac{1}{12'} \frac{1}{15'} \frac{1}{20}$$

- b. Write the decimal equivalent of each fraction.
 c. **Make a conjecture** relating prime factors of denominators and the decimal equivalents of fractions.

49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How were fractions used to determine the size of the first coins?

Include the following in your answer:

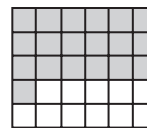
- a description of the first coins, and
- an explanation of why decimals rather than fractions are used in money exchange today.



50. Which decimal is equivalent to $\frac{1}{100}$?
 (A) 0.001 (B) 0.01 (C) 0.1 (D) $0.\bar{1}$

51. Write the shaded portion of the figure at the right as a decimal.

- (A) 0.6 (B) $0.\bar{6}$
 (C) 0.63 (D) $0.6\bar{3}$



Maintain Your Skills

Mixed Review

Write each number in scientific notation. (Lesson 4-8)

52. 854,000,000 53. 0.077 54. 0.00016 55. 925,000

Write each expression using a positive exponent. (Lesson 4-7)

56. 10^{-5} 57. $(-2)^{-7}$ 58. x^{-4} 59. y^{-3}

60. **ALGEBRA** Write $(a \cdot a \cdot a)(a \cdot a)$ using an exponent. (Lesson 4-2)

61. **TRANSPORTATION** A car can travel an average of 464 miles on one tank of gas. If the tank holds 16 gallons of gasoline, how many miles per gallon does it get? (Lesson 3-7)

ALGEBRA Solve each equation. Check your solution. (Lesson 3-4)

62. $4n = 32$ 63. $-64 = 2t$
 64. $\frac{a}{5} = -9$ 65. $-8 = \frac{x}{-7}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Simplify each fraction.

(To review **simplifying fractions**, see Lesson 4-5.)

66. $\frac{4}{30}$ 67. $\frac{5}{65}$ 68. $\frac{36}{60}$ 69. $\frac{12}{18}$
 70. $\frac{21}{24}$ 71. $\frac{16}{28}$ 72. $\frac{32}{48}$ 73. $\frac{125}{1000}$



5-2

Rational Numbers

What You'll Learn

- Write rational numbers as fractions.
- Identify and classify rational numbers.

Vocabulary

- rational number

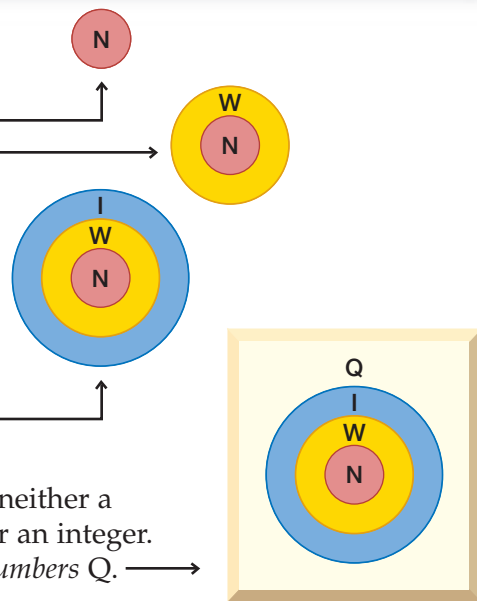
How are rational numbers related to other sets of numbers?

The solution of $2x = 4$ is 2. It is a member of the set of *natural numbers* $N = \{1, 2, 3, \dots\}$.

The solution of $x + 3 = 3$ is 0. It is a member of the set of *whole numbers* $W = \{0, 1, 2, 3, \dots\}$.

The solution of $x + 5 = 2$ is -3 . It is a member of the set of *integers* $I = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.

The solution of $2x = 3$ is $\frac{3}{2}$, which is neither a natural number, a whole number, nor an integer. It is a member of the set of *rational numbers* Q .



Rational numbers include fractions and decimals as well as natural numbers, whole numbers, and integers.

- Is 7 a natural number? a whole number? an integer?
- How do you know that 7 is also a rational number?
- Is every natural number a rational number? Is every rational number a natural number? Give an explanation or a counterexample to support your answers.

Reading Math

Rational

Root Word: Ratio

A *ratio* is the comparison of two quantities by division. Recall that

$$\frac{a}{b} = a \div b, \text{ where } b \neq 0.$$

WRITE RATIONAL NUMBERS AS FRACTIONS A number that can be written as a fraction is called a **rational number**. Some examples of rational numbers are shown below.

$$0.75 = \frac{3}{4} \quad -0.\bar{3} = -\frac{1}{3} \quad 28 = \frac{28}{1} \quad 1\frac{1}{4} = \frac{5}{4}$$

Example 1 Write Mixed Numbers and Integers as Fractions

- Write $5\frac{2}{3}$ as a fraction.
- Write -3 as a fraction.

$$5\frac{2}{3} = \frac{17}{3} \quad \text{Write } 5\frac{2}{3} \text{ as an improper fraction.}$$

$$-3 = \frac{-3}{1} \text{ or } -\frac{3}{1}$$

Concept Check Explain why any integer n is a rational number.

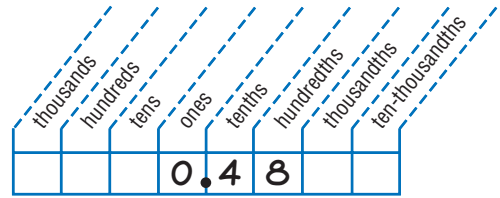
Terminating decimals are rational numbers because they can be written as a fraction with a denominator of 10, 100, 1000, and so on.

Example 2 Write Terminating Decimals as Fractions

Write each decimal as a fraction or mixed number in simplest form.

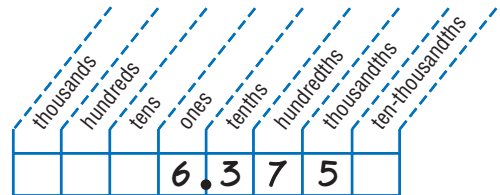
a. 0.48

$$\begin{aligned} 0.48 &= \frac{48}{100} && \text{0.48 is 48 hundredths.} \\ &= \frac{12}{25} && \text{Simplify. The GCF of 48 and 100 is 4.} \end{aligned}$$



b. 6.375

$$\begin{aligned} 6.375 &= 6\frac{375}{1000} && \text{6.375 is 6 and 375 thousandths.} \\ &= 6\frac{3}{8} && \text{Simplify. The GCF of 375 and 1000 is 125.} \end{aligned}$$



Reading Math

Decimals

Use the word *and* to represent the decimal point.

- Read 0.375 as three hundred seventy-five thousandths.
- Read 300.075 as three hundred *and* seventy-five thousandths.

Any repeating decimal can be written as a fraction, so repeating decimals are also rational numbers.

Study Tip

Repeating Decimals

When *two* digits repeat, multiply each side by 100. Then subtract N from $100N$ to eliminate the repeating part.

Example 3 Write Repeating Decimals as Fractions

Write $0.\overline{8}$ as a fraction in simplest form.

$$\begin{aligned} N &= 0.888\dots && \text{Let } N \text{ represent the number.} \\ 10N &= 10(0.888\dots) && \text{Multiply each side by 10} \\ 10N &= 8.888\dots && \text{because one digit repeats.} \end{aligned}$$

Subtract N from $10N$ to eliminate the repeating part, $0.888\dots$.

$$\begin{aligned} 10N &= 8.888\dots \\ -(N &= 0.888\dots) \\ \hline 9N &= 8 && 10N - N = 10N - 1N \text{ or } 9N \\ \frac{9N}{9} &= \frac{8}{9} && \text{Divide each side by 9.} \\ N &= \frac{8}{9} && \text{Simplify.} \end{aligned}$$

Therefore, $0.\overline{8} = \frac{8}{9}$.

CHECK $8 \div 9 \text{ [ENTER] } .888888889 \checkmark$

IDENTIFY AND CLASSIFY RATIONAL NUMBERS All rational numbers can be written as terminating or repeating decimals. Decimals that are neither terminating nor repeating, such as the numbers below, are called *irrational* because they cannot be written as fractions. *You will learn more about irrational numbers in Chapter 9.*

$\pi = 3.141592654\dots \rightarrow$ The digits do not repeat.

$4.232232223\dots \rightarrow$ The same block of digits do not repeat.

The following model can help you classify rational numbers.

Concept Summary
Rational Numbers

- **Words** A rational number is any number that can be expressed as the quotient $\frac{a}{b}$ of two integers, a and b , where $b \neq 0$.
- **Model**

Example 4

Classify Numbers

Identify all sets to which each number belongs.

a. -6

-6 is an integer and a rational number.

b. $2\frac{4}{5}$

Because $2\frac{4}{5} = \frac{14}{5}$, it is a rational number. It is neither a whole number nor an integer.

c. $0.914114111\dots$

This is a nonterminating, nonrepeating decimal. So, it is not a rational number.

Check for Understanding

Concept Check

1. Define *rational number* in your own words.
2. **OPEN ENDED** Give an example of a number that is not rational. Explain why it is not rational.

Guided Practice

Write each number as a fraction.

3. $-2\frac{1}{3}$

4. 10

Write each decimal as a fraction or mixed number in simplest form.

5. 0.8

6. 6.35

7. $-0.\overline{7}$

8. $0.\overline{45}$

Identify all sets to which each number belongs.

9. -5

10. 6.05

Application

11. **MEASUREMENT** A *micron* is a unit of measure that is approximately 0.000039 inch. Express this as a fraction.



Practice and Apply

Homework Help

For Exercises	See Examples
12–15	1
16–21, 28–33	2
22–27	3
34–41	4

Extra Practice
See page 733.

Write each number as a fraction.

12. $5\frac{2}{3}$

13. $-1\frac{4}{7}$

14. -21

15. 60

Write each decimal as a fraction or mixed number in simplest form.

16. 0.4

17. 0.09

18. 5.22

19. 1.68

20. 0.625

21. 8.004

22. $0.\overline{2}$

23. $-0.333\dots$

24. $4.\overline{5}$

25. $5.\overline{6}$

26. $0.\overline{32}$

27. $2.\overline{25}$

28. **WHITE HOUSE** The White House covers an area of 0.028 square mile. What fraction of a square mile is this?

29. **RECYCLING** In 1999, 0.06 of all recycled newspapers were used to make tissues. What fraction is this?

GEOGRAPHY Africa makes up $\frac{1}{5}$ of all the land on Earth. Use the table to find the fraction of Earth's land that is made up by other continents.

Write each fraction in simplest form.

30. Antarctica

31. Asia

32. Europe

33. North America

Continent	Decimal Portion of Earth's Land
Antarctica	0.095
Asia	0.295
Europe	0.07
North America	0.16

Source: *Incredible Comparisons*

Identify all sets to which each number belongs.

34. 4

35. -7

36. $-2\frac{5}{8}$

37. $\frac{6}{3}$

38. 15.8

39. $9.0202020\dots$

40. $1.2345\dots$

41. $30.151151115\dots$

42. Write 125 thousandths as a fraction in simplest form.

43. Express *two hundred and nineteen hundredths* as a fraction or mixed number in simplest form.

Determine whether each statement is *sometimes*, *always*, or *never* true. Explain by giving an example or a counterexample.

44. An integer is a rational number.

45. A rational number is an integer.

46. A whole number is not a rational number.

47. **MANUFACTURING** A garbage bag has a thickness of 0.8 mil. This is 0.0008 inch. What fraction of an inch is this?

48. **GEOMETRY** Pi (π) to six decimal places has a value of 3.141592. Pi is often estimated as $\frac{22}{7}$. Is the estimate for π greater than or less than the actual value of π ? Explain.

More About . . .



Recycling

The portions of recycled newspapers used for other purposes are shown below.

Newsprint: 0.34

Exported for recycling: 0.22

Paperboard: 0.17

Other products: 0.18

Source: American Forest and Paper Association, Newspaper Association of America

49. **MACHINERY** Will a steel peg 2.37 inches in diameter fit in a $2\frac{3}{8}$ -inch diameter hole? How do you know?
50. **CRITICAL THINKING** Show that $0.999\dots = 1$.
51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

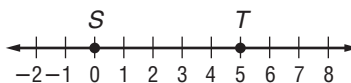
How are rational numbers related to other sets of numbers?

Include the following in your answer:

- examples of numbers that belong to more than one set, and
- examples of numbers that are only rational.



52. There are infinitely many ? between S and T on the number line.



- (A) rational numbers (B) integers
 (C) whole numbers (D) natural numbers
53. Express 0.56 as a fraction in simplest form.
- (A) $\frac{56}{100}$ (B) $\frac{28}{50}$ (C) $\frac{14}{25}$ (D) $\frac{7}{12}$

Maintain Your Skills

Mixed Review Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal. (Lesson 5-1)

54. $\frac{2}{5}$ 55. $-7\frac{4}{5}$ 56. $-\frac{13}{20}$ 57. $2\frac{5}{9}$

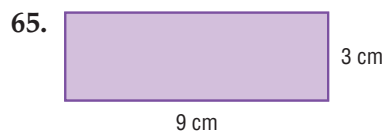
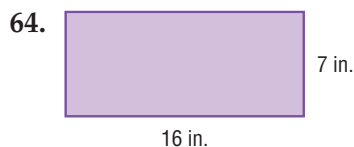
Write each number in standard form. (Lesson 4-8)

58. 2×10^3 59. 3.05×10^6 60. 7.4×10^{-4} 61. 1.681×10^{-2}

62. **ALGEBRA** Write $\frac{12n^2}{3an}$ in simplest form. (Lesson 4-5)

63. Use place value and exponents to express 483 in expanded form. (Lesson 4-2)

Find the perimeter and area of each rectangle. (Lesson 3-7)



Use the Distributive Property to rewrite each expression. (Lesson 3-1)

66. $3(5 + 9)$ 67. $(8 + 1)2$ 68. $6(b - 5)$ 69. $(x + 4)7$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Estimate each product. (To review *estimating products*, see page 717.)

Example: $-3\frac{1}{4} \cdot 5\frac{7}{8} \approx -3 \cdot 6$ or -18

70. $1\frac{2}{3} \cdot 4\frac{1}{8}$ 71. $-5\frac{2}{5} \cdot 3\frac{4}{5}$ 72. $2\frac{1}{4} \cdot 2\frac{1}{9}$
 73. $6\frac{7}{8} \cdot 1\frac{9}{10}$ 74. $9\frac{1}{8} \cdot (-4\frac{3}{4})$ 75. $15\frac{5}{7} \cdot 2\frac{1}{3}$

5-3

Multiplying Rational Numbers

What You'll Learn

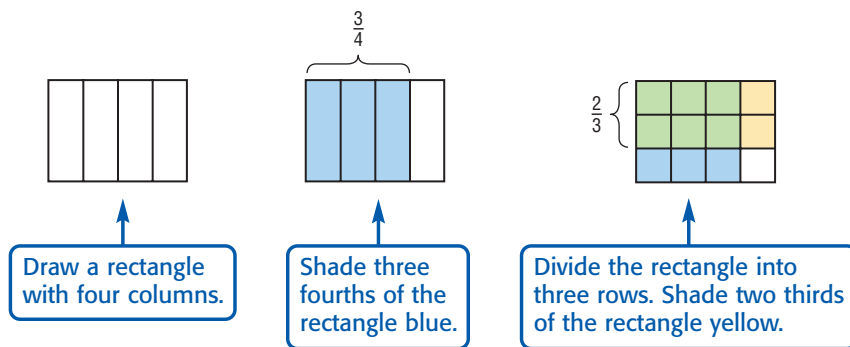
- Multiply fractions.
- Use dimensional analysis to solve problems.

Vocabulary

- dimensional analysis

How is multiplying fractions related to areas of rectangles?

To find $\frac{2}{3} \cdot \frac{3}{4}$, think of using an area model to find $\frac{2}{3}$ of $\frac{3}{4}$.



- a. The overlapping green area represents the product of $\frac{2}{3}$ and $\frac{3}{4}$. What is the product?

Use an area model to find each product.

b. $\frac{1}{2} \cdot \frac{1}{3}$

c. $\frac{3}{5} \cdot \frac{1}{4}$

d. $\frac{3}{4} \cdot \frac{1}{3}$

- e. What is the relationship between the numerators and denominators of the factors and the numerator and denominator of the product?

MULTIPLY FRACTIONS These and other similar models suggest the following rule for multiplying fractions.

Key Concept

Multiplying Fractions

- **Words** To multiply fractions, multiply the numerators and multiply the denominators.
- **Symbols** $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$, where $b, d \neq 0$
- **Example** $\frac{1}{3} \cdot \frac{2}{5} = \frac{1 \cdot 2}{3 \cdot 5}$ or $\frac{2}{15}$

Example 1 Multiply Fractions

Find $\frac{2}{3} \cdot \frac{3}{4}$. Write the product in simplest form.

$$\frac{2}{3} \cdot \frac{3}{4} = \frac{2 \cdot 3}{3 \cdot 4}$$

← Multiply the numerators.

← Multiply the denominators.

$$= \frac{6}{12} \text{ or } \frac{1}{2}$$

Simplify. The GCF of 6 and 12 is 6.

Study Tip

Look Back

To review **GCF**, see Lesson 4-4.

If the fractions have common factors in the numerators and denominators, you can simplify before you multiply.

Example 2 Simplify Before Multiplying

Find $\frac{4}{7} \cdot \frac{1}{6}$. Write the product in simplest form.

$$\begin{aligned} \frac{4}{7} \cdot \frac{1}{6} &= \frac{\cancel{4}^2}{7} \cdot \frac{1}{\cancel{6}_3} && \text{Divide 4 and 6 by their GCF, 2.} \\ &= \frac{2 \cdot 1}{7 \cdot 3} && \text{Multiply the numerators and multiply the denominators.} \\ &= \frac{2}{21} && \text{Simplify.} \end{aligned}$$

Study Tip

Negative Fractions

$-\frac{5}{12}$ can be written as $\frac{-5}{12}$ or as $\frac{5}{-12}$.

Example 3 Multiply Negative Fractions

Find $-\frac{5}{12} \cdot \frac{3}{8}$. Write the product in simplest form.

$$\begin{aligned} -\frac{5}{12} \cdot \frac{3}{8} &= \frac{-5}{\cancel{12}_4} \cdot \frac{\cancel{3}^1}{8} && \text{Divide 3 and 12 by their GCF, 3.} \\ &= \frac{-5 \cdot 1}{4 \cdot 8} && \text{Multiply the numerators and multiply the denominators.} \\ &= -\frac{5}{32} && \text{Simplify.} \end{aligned}$$

Example 4 Multiply Mixed Numbers

Find $1\frac{2}{5} \cdot 2\frac{1}{2}$. Write the product in simplest form. Estimate: $1 \cdot 3 = 3$

$$\begin{aligned} 1\frac{2}{5} \cdot 2\frac{1}{2} &= \frac{7}{5} \cdot \frac{5}{2} && \text{Rename } 1\frac{2}{5} \text{ as } \frac{7}{5} \text{ and rename } 2\frac{1}{2} \text{ as } \frac{5}{2}. \\ &= \frac{\cancel{7}^1}{5} \cdot \frac{\cancel{5}^1}{2} && \text{Divide by the GCF, 5.} \\ &= \frac{7 \cdot 1}{1 \cdot 2} && \text{Multiply.} \\ &= \frac{7}{2} \text{ or } 3\frac{1}{2} && \text{Simplify.} \end{aligned}$$

ALGEBRA CONNECTION

Algebraic fractions are multiplied in the same manner as numeric fractions.

Example 5 Multiply Algebraic Fractions

Find $\frac{2a}{b} \cdot \frac{b^2}{d}$. Write the product in simplest form.

$$\begin{aligned} \frac{2a}{b} \cdot \frac{b^2}{d} &= \frac{2a}{\cancel{b}} \cdot \frac{\cancel{b}^1 \cdot b}{d} && \text{The GCF of } b \text{ and } b^2 \text{ is } b. \\ &= \frac{2ab}{d} && \text{Simplify.} \end{aligned}$$

DIMENSIONAL ANALYSIS **Dimensional analysis** is the process of including units of measurement when you compute. You can use dimensional analysis to check whether your answers are reasonable.

Example 6 Use Dimensional Analysis

SPACE TRAVEL The landing speed of the space shuttle is about 216 miles per hour. How far does the shuttle travel in $\frac{1}{3}$ hour during landing?

Words Distance equals the rate multiplied by the time.

Variables Let d = distance, r = rate, and t = time.

Formula $d = rt$ Write the formula.

$$d = 216 \text{ miles per hour} \cdot \frac{1}{3} \text{ hour} \quad \text{Include the units.}$$

$$= \frac{\overset{72}{\cancel{216}} \text{ miles}}{\underset{1}{\cancel{\text{hour}}} \text{ hour}} \cdot \frac{1}{\underset{3}{\cancel{3}} \text{ hour}} \text{ hour} \quad \text{Divide by the common factors and units.}$$

$$= 72 \text{ miles} \quad \text{Simplify.}$$

The space shuttle travels 72 miles in $\frac{1}{3}$ hour during landing.

CHECK The problem asks for the distance. When you divide the common units, the answer is expressed in miles. So, the answer is reasonable.

 **Concept Check** What is the final unit when you multiply feet per second by seconds?

Check for Understanding

Concept Check

- OPEN ENDED** Choose two rational numbers whose product is a number between 0 and 1.
- FIND THE ERROR** Terrence and Marie are finding $\frac{5}{24} \cdot \frac{18}{25}$.

Terrence

$$\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{24}}} \cdot \frac{\overset{3}{\cancel{18}}}{\underset{5}{\cancel{25}}} = \frac{3}{20}$$

Marie

$$\frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{24}}} \cdot \frac{\overset{9}{\cancel{18}}}{\underset{5}{\cancel{25}}} = \frac{9}{20}$$

Who is correct? Explain your reasoning.

Guided Practice

Find each product. Write in simplest form.

3. $\frac{1}{4} \cdot \frac{3}{5}$

4. $\frac{1}{2} \left(-\frac{5}{6}\right)$

5. $-\frac{2}{3} \left(-\frac{5}{6}\right)$

6. $7 \left(\frac{8}{21}\right)$

7. $3\frac{1}{4} \cdot \frac{2}{11}$

8. $-5\frac{1}{3} \cdot 3\frac{3}{8}$

ALGEBRA Find each product. Write in simplest form.

9. $\frac{2}{x} \cdot \frac{3x}{7}$

10. $\frac{a}{b} \cdot \frac{5b}{c}$

11. $\frac{4t}{9r} \cdot \frac{18r}{t^2}$

Application

- TRAVEL** A car travels 65 miles per hour for $3\frac{1}{2}$ hours. What is the distance traveled? Use the formula $d = rt$ and show how you can divide by the common units.

Practice and Apply

Homework Help

For Exercises See Examples

13–24	1–3
25–33	4
37–42	5
47–50	6

Extra Practice
See page 734.

Find each product. Write in simplest form.

- | | | |
|--|---|--|
| 13. $\frac{6}{7} \cdot \frac{2}{7}$ | 14. $\frac{4}{9} \cdot \frac{2}{3}$ | 15. $\frac{1}{5} \left(-\frac{1}{8}\right)$ |
| 16. $-\frac{3}{4} \cdot \frac{3}{5}$ | 17. $\frac{5}{9} \cdot \frac{8}{25}$ | 18. $-\frac{1}{2} \left(-\frac{2}{7}\right)$ |
| 19. $\frac{2}{5} \cdot \frac{5}{6}$ | 20. $\frac{8}{9} \cdot \frac{27}{28}$ | 21. $\frac{3}{4} \left(-\frac{1}{3}\right)$ |
| 22. $-\frac{7}{8} \cdot \frac{2}{5}$ | 23. $\frac{3}{5} \cdot \frac{15}{24}$ | 24. $\frac{3}{32} \cdot \frac{24}{39}$ |
| 25. $2 \cdot \frac{7}{12}$ | 26. $\frac{6}{15}(-3)$ | 27. $6\frac{2}{3} \cdot \frac{1}{2}$ |
| 28. $\frac{5}{12} \cdot 3\frac{1}{9}$ | 29. $2\frac{2}{6} \cdot 6\frac{2}{7}$ | 30. $3\frac{1}{3} \cdot 2\frac{5}{8}$ |
| 31. $-6\frac{2}{3} \left(-1\frac{1}{2}\right)$ | 32. $1\frac{3}{7} \left(-9\frac{4}{5}\right)$ | 33. $-1\frac{1}{4} \cdot 3\frac{5}{9}$ |

MEASUREMENT Complete.

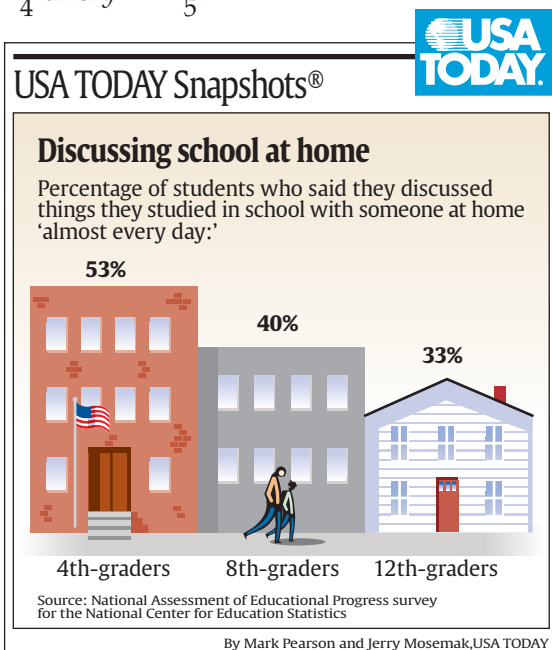
- | | |
|---|---|
| 34. $\underline{\quad}$ feet = $\frac{5}{6}$ mile
(Hint: 1 mile = 5280 feet) | 35. $\underline{\quad}$ ounces = $\frac{3}{8}$ pound
(Hint: 1 pound = 16 ounces) |
| 36. $\frac{2}{3}$ hour = $\underline{\quad}$ minutes | 37. $\frac{3}{4}$ yard = $\underline{\quad}$ inches |

ALGEBRA Find each product. Write in simplest form.

- | | | |
|--|--|---|
| 38. $\frac{4a}{5} \cdot \frac{3}{a}$ | 39. $\frac{3x}{y} \cdot \frac{9y}{x}$ | 40. $\frac{12}{jk} \cdot \frac{3k}{4}$ |
| 41. $\frac{8}{c} \cdot \frac{c^2}{11}$ | 42. $\frac{n}{18} \cdot \frac{6}{n^4}$ | 43. $\frac{x}{2z} \cdot \frac{2z^3}{3}$ |
| 44. ALGEBRA Evaluate x^2 if $x = -\frac{1}{2}$. | | |
| 45. ALGEBRA Evaluate $(xy)^2$ if $x = \frac{3}{4}$ and $y = -\frac{4}{5}$. | | |

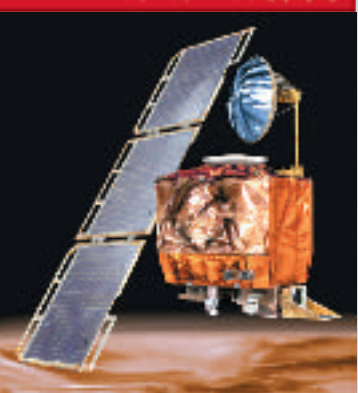
SCHOOL For Exercises 46 and 47, use the graphic at the right.

46. Five-eighths of an eighth grade class are boys. Predict approximately what fraction of the eighth graders are boys who talk about school at home.
(Hint: $40\% = \frac{2}{5}$)
47. In a 12th grade class, five-ninths of the students are girls. Predict about what fraction of twelfth graders are girls who talk about school at home.
(Hint: $33\% = \frac{33}{100}$)



48. **GARDENING** Jamal's lawn is $\frac{2}{3}$ of an acre. If $7\frac{1}{2}$ bags of fertilizer are needed for 1 acre, how much will he need to fertilize his lawn?

More About . . .



Converting Measures

In 1998, a Mars probe was lost because scientists did not convert a customary measure of force to a metric measure.

Source: *Newsday*

CONVERTING MEASURES Use dimensional analysis and the fractions in the table to find each missing measure.

Conversion Factors	
Customary → Metric	Metric → Customary
$\frac{2.54 \text{ cm}}{1 \text{ in.}}$	$\frac{0.39 \text{ in.}}{1 \text{ cm}}$
$\frac{1.61 \text{ km}}{1 \text{ mi}}$	$\frac{0.62 \text{ mi}}{1 \text{ km}}$
$\frac{0.09 \text{ m}^2}{1 \text{ ft}^2}$	$\frac{10.76 \text{ ft}^2}{1 \text{ m}^2}$

49. 5 inches = centimeters
 50. 10 kilometers = miles
 51. 26.3 centimeters = inches
 52. $8\frac{2}{3}$ square feet = square meters

53. **CRITICAL THINKING** Use the digits 3, 4, 5, 6, 8, and 9 to make true sentences.

a. $\frac{\square}{\square} \times \frac{\square}{\square} = \frac{6}{5}$

b. $\frac{\square}{\square} \times \frac{\square}{\square} = \frac{5}{8}$

54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is multiplying fractions related to areas of rectangles?

Include the following in your answer:

- an area model of a multiplication problem involving fractions, and
- an explanation of how rectangles can be used to show multiplication of fractions.



Standardized Test Practice

A B C D

55. The product of $\frac{8}{15}$ and $\frac{3}{8}$ is a number

- (A) between 0 and 1. (B) between 1 and 2.
 (C) between 2 and 3. (D) greater than 3.

56. What is the equivalent length of a chain that is 52 feet long?

- (A) 4 yards 5 feet (B) 4.5 yards
 (C) 17 yards 1 foot (D) 17.1 yards

Extending the Lesson

Evaluate each expression if $n = 2$ and $p = -4$.

57. $5n^{-3}$ 58. $7 \cdot 3^p$ 59. $13n^{-1}p^{-2}$

Maintain Your Skills

Mixed Review

Write each decimal as a fraction or mixed number in simplest form.

(Lesson 5-2)

60. 0.18 61. -0.2 62. 3.04 63. $0.\bar{7}$

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal. (Lesson 5-1)

64. $\frac{17}{20}$ 65. $\frac{1}{6}$ 66. $2\frac{2}{11}$ 67. $-4\frac{7}{8}$

68. **ALGEBRA** What is the product of x^2 and x^4 ? (Lesson 4-6)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find the GCF of each pair of monomials.

(To review the GCF of monomials, see Lesson 4-4.)

69. $8n, 16n$ 70. $5ab, 8b$ 71. $12t, 10t$
 72. $2rs, 3rs$ 73. $9k, 27$ 74. $4p^2, 6p$



5-4

Dividing Rational Numbers

What You'll Learn

- Divide fractions using multiplicative inverses.
- Use dimensional analysis to solve problems.

Vocabulary

- multiplicative inverses
- reciprocals

How is dividing by a fraction related to multiplying?

The model shows $4 \div \frac{1}{3}$. Each of the 4 circles is divided into $\frac{1}{3}$ -sections.



There are twelve $\frac{1}{3}$ -sections, so $4 \div \frac{1}{3} = 12$. Another way to find the number of sections is $4 \times 3 = 12$.

Use a model to find each quotient. Then write a related multiplication problem.

a. $2 \div \frac{1}{3}$

b. $4 \div \frac{1}{2}$

c. $3 \div \frac{1}{4}$

d. Make a conjecture about how dividing by a fraction is related to multiplying.

DIVIDE FRACTIONS Rational numbers have all of the properties of whole numbers and integers. Another property is shown by $\frac{1}{3} \cdot \frac{3}{1} = 1$. Two numbers whose product is 1 are called **multiplicative inverses** or **reciprocals**.

Key Concept**Inverse Property of Multiplication**

- **Words** The product of a number and its multiplicative inverse is 1.
- **Symbols** For every number $\frac{a}{b}$, where $a, b \neq 0$, there is exactly one number $\frac{b}{a}$ such that $\frac{a}{b} \cdot \frac{b}{a} = 1$.
- **Example** $\frac{3}{4}$ and $\frac{4}{3}$ are multiplicative inverses because $\frac{3}{4} \cdot \frac{4}{3} = 1$.

Study Tip**Reading Math**

Multiplicative inverse and *reciprocal* are different terms for the same concept. They may be used interchangeably.

Example 1 Find Multiplicative Inverses

Find the multiplicative inverse of each number.

a. $-\frac{3}{8}$

$$-\frac{3}{8} \left(-\frac{8}{3} \right) = 1 \quad \text{The product is 1.}$$

The multiplicative inverse or reciprocal of $-\frac{3}{8}$ is $-\frac{8}{3}$.

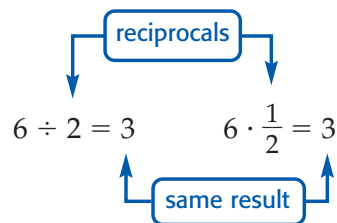
b. $2\frac{1}{5}$

$$2\frac{1}{5} = \frac{11}{5} \quad \text{Write as an improper fraction.}$$

$$\frac{11}{5} \cdot \frac{5}{11} = 1 \quad \text{The product is 1.}$$

The reciprocal of $2\frac{1}{5}$ is $\frac{5}{11}$.

Dividing by 2 is the same as multiplying by $\frac{1}{2}$, its multiplicative inverse. This is true for any rational number.



Key Concept

Dividing Fractions

- **Words** To divide by a fraction, multiply by its multiplicative inverse.
- **Symbols** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$, where $b, c, d \neq 0$
- **Example** $\frac{1}{4} \div \frac{5}{7} = \frac{1}{4} \cdot \frac{7}{5}$ or $\frac{7}{20}$

Concept Check Does every rational number have a multiplicative inverse? Explain.

Example 2

Divide by a Fraction

Find $\frac{1}{3} \div \frac{5}{9}$. Write the quotient in simplest form.

$$\begin{aligned} \frac{1}{3} \div \frac{5}{9} &= \frac{1}{3} \cdot \frac{9}{5} && \text{Multiply by the multiplicative inverse of } \frac{5}{9}, \frac{9}{5}. \\ &= \frac{1}{\cancel{3}} \cdot \frac{\overset{3}{9}}{5} && \text{Divide 3 and 9 by their GCF, 3.} \\ &= \frac{3}{5} && \text{Simplify.} \end{aligned}$$

Study Tip

Dividing By a Whole Number

When dividing by a whole number, always rename it as an improper fraction first. Then multiply by its reciprocal.

Example 3

Divide by a Whole Number

Find $\frac{5}{8} \div 6$. Write the quotient in simplest form.

$$\begin{aligned} \frac{5}{8} \div 6 &= \frac{5}{8} \div \frac{6}{1} && \text{Write 6 as } \frac{6}{1}. \\ &= \frac{5}{8} \cdot \frac{1}{6} && \text{Multiply by the multiplicative inverse of } \frac{6}{1}, \frac{1}{6}. \\ &= \frac{5}{48} && \text{Multiply the numerators and multiply the denominators.} \end{aligned}$$

Example 4

Divide by a Mixed Number

Find $-7\frac{1}{2} \div 2\frac{1}{10}$. Write the quotient in simplest form.

$$\begin{aligned} -7\frac{1}{2} \div 2\frac{1}{10} &= -\frac{15}{2} \div \frac{21}{10} && \text{Rename the mixed numbers as improper fractions.} \\ &= -\frac{15}{2} \cdot \frac{10}{21} && \text{Multiply by the multiplicative inverse of } \frac{21}{10}, \frac{10}{21}. \\ &= -\frac{\overset{5}{15}}{\underset{1}{2}} \cdot \frac{\overset{5}{10}}{\underset{7}{21}} && \text{Divide out common factors.} \\ &= -\frac{25}{7} \text{ or } -3\frac{4}{7} && \text{Simplify.} \end{aligned}$$




You can divide algebraic fractions in the same way that you divide numerical fractions.

Example 5 *Divide by an Algebraic Fraction*

Find $\frac{3xy}{4} \div \frac{2x}{8}$. Write the quotient in simplest form.

$$\begin{aligned} \frac{3xy}{4} \div \frac{2x}{8} &= \frac{3xy}{4} \cdot \frac{8}{2x} && \text{Multiply by the multiplicative inverse of } \frac{2x}{8}, \frac{8}{2x}. \\ &= \frac{\overset{1}{\cancel{3}xy} \cdot \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{2}x}} && \text{Divide out common factors.} \\ &= \frac{6y}{2} \text{ or } 3y && \text{Simplify.} \end{aligned}$$

More About . . .



Cheerleading

In the 1930s, both men and women cheerleaders began wearing cheerleading uniforms.

Source: www.umn.edu

DIMENSIONAL ANALYSIS Dimensional analysis is a useful way to examine the solution of division problems.

Example 6 *Use Dimensional Analysis*

CHEERLEADING How many cheerleading uniforms can be made with $22\frac{3}{4}$ yards of fabric if each uniform requires $\frac{7}{8}$ -yard?

To find how many uniforms, divide $22\frac{3}{4}$ by $\frac{7}{8}$.

$$\begin{aligned} 22\frac{3}{4} \div \frac{7}{8} &= 22\frac{3}{4} \cdot \frac{8}{7} && \text{Multiply by the reciprocal of } \frac{7}{8}, \frac{8}{7}. \\ &= \frac{91}{4} \cdot \frac{8}{7} && \text{Write } 22\frac{3}{4} \text{ as an improper fraction.} \\ &= \frac{\overset{13}{\cancel{91}} \cdot \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \cdot \underset{1}{\cancel{7}}} && \text{Divide out common factors.} \\ &= 26 && \text{Simplify.} \end{aligned}$$

So, 26 uniforms can be made.

CHECK Use dimensional analysis to examine the units.

$$\begin{aligned} \text{yards} \div \frac{\text{yards}}{\text{uniform}} &= \cancel{\text{yards}} \cdot \frac{\text{uniform}}{\cancel{\text{yards}}} && \text{Divide out the units.} \\ &= \text{uniform} && \text{Simplify.} \end{aligned}$$

The result is expressed as uniforms. This agrees with your answer of 26 uniforms.

Check for Understanding

- Concept Check**
1. Explain how reciprocals are used in division of fractions.
 2. **OPEN ENDED** Write a division expression that can be simplified by using the multiplicative inverse $\frac{7}{5}$.

Guided Practice Find the multiplicative inverse of each number.

3. $\frac{4}{5}$

4. -16

5. $3\frac{1}{8}$

Find each quotient. Write in simplest form.

6. $\frac{1}{2} \div \frac{6}{7}$

7. $-\frac{2}{3} \div \left(-\frac{5}{6}\right)$

8. $\frac{7}{9} \div \frac{2}{3}$

9. $7\frac{1}{3} \div 5$

10. $-\frac{8}{9} \div 3\frac{1}{5}$

11. $2\frac{1}{6} \div \left(-1\frac{1}{5}\right)$

ALGEBRA Find each quotient. Write in simplest form.

12. $\frac{14}{n} \div \frac{1}{n}$

13. $\frac{ab}{4} \div \frac{b}{6}$

14. $\frac{x^2}{5} \div \frac{ax}{2}$

Application 15. **CARPENTRY** How many boards, each 2 feet 8 inches long, can be cut from a board 16 feet long if there is no waste?

Practice and Apply

Homework Help

For Exercises

16–21
22–39
40–45
46, 47

See Examples

1
2–4
5
6

Extra Practice
See page 734.

Find the multiplicative inverse of each number.

16. $\frac{6}{11}$

17. $-\frac{1}{5}$

18. -7

19. 24

20. $5\frac{1}{4}$

21. $-3\frac{2}{9}$

Find each quotient. Write in simplest form.

22. $\frac{1}{4} \div \frac{3}{5}$

23. $\frac{2}{9} \div \frac{1}{4}$

24. $-\frac{1}{2} \div \frac{5}{6}$

25. $\frac{6}{11} \div \left(-\frac{4}{5}\right)$

26. $\frac{8}{9} \div \frac{4}{3}$

27. $\frac{7}{8} \div \frac{14}{15}$

28. $\frac{3}{4} \div \frac{3}{4}$

29. $\frac{2}{9} \div \left(-\frac{2}{9}\right)$

30. $\frac{3}{5} \div \frac{5}{9}$

31. $\frac{3}{10} \div \frac{1}{5}$

32. $12 \div \frac{4}{9}$

33. $-8 \div \frac{4}{5}$

34. $-\frac{5}{8} \div (-4)$

35. $6\frac{2}{3} \div 5$

36. $-1\frac{1}{9} \div \frac{2}{3}$

37. $-\frac{2}{3} \div \left(-\frac{1}{3}\right)$

38. $3\frac{3}{10} \div 1\frac{5}{6}$

39. $7\frac{1}{2} \div \left(-1\frac{1}{5}\right)$

ALGEBRA Find each quotient. Write in simplest form.

40. $\frac{a}{7} \div \frac{a}{42}$

41. $\frac{10}{3x} \div \frac{5}{2x}$

42. $\frac{c}{8} \div \frac{cd}{5}$

43. $\frac{5s}{t} \div \frac{6rs}{t}$

44. $\frac{k^3}{9} \div \frac{k}{24}$

45. $\frac{2s}{t^2} \div \frac{st^3}{8}$

46. **COOKING** How many $\frac{1}{4}$ -pound hamburgers can be made from $2\frac{3}{4}$ pounds of ground beef?

47. **SEWING** How many 9-inch ribbons can be cut from $1\frac{1}{2}$ yards of ribbon?

ALGEBRA Evaluate each expression.

48. $m \div n$ if $m = -\frac{8}{9}$ and $n = \frac{7}{18}$

49. $r^2 \div s^2$ if $r = -\frac{3}{4}$ and $s = 1\frac{1}{3}$

More About . . .



Travel

Trains in Europe can travel faster than 125 miles per hour.

Source: www.geography.about.com

For Exercises 50 and 51, solve each problem. Then check your answer using dimensional analysis.

50. **TRAVEL** How long would it take a train traveling 80 miles per hour to go 280 miles?
51. **FOOD** The average young American woman drinks $1\frac{1}{2}$ cans of cola each day. At this rate, in how many days would it take to drink a total of 12 cans? **Source:** U.S. Department of Agriculture
52. **CRITICAL THINKING**
- Divide $\frac{3}{4}$ by $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, and $\frac{1}{12}$.
 - What happens to the quotient as the value of the divisor decreases?
 - Make a conjecture** about the quotient when you divide $\frac{3}{4}$ by fractions that increase in value. Test your conjecture.
53. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is dividing by a fraction related to multiplying?

Include the following in your answer:

- a model of a whole number divided by a fraction, and
- an explanation of how division of fractions is related to multiplication.

Standardized Test Practice

A B C D

54. Carla baby-sits for $2\frac{1}{4}$ hours and earns \$11.25. What is her rate?
 (A) \$4.00/h (B) \$5.50/h (C) \$4.50/h (D) \$5.00/h
55. What is $\frac{3}{10}$ divided by $1\frac{4}{5}$?
 (A) $\frac{1}{2}$ (B) $\frac{3}{8}$ (C) $\frac{1}{6}$ (D) $\frac{27}{50}$

Maintain Your Skills

Mixed Review

Find each product. Write in simplest form. (Lesson 5-3)

56. $\frac{3}{5} \cdot \frac{1}{3}$ 57. $\frac{2}{9} \cdot \frac{15}{16}$ 58. $2\frac{4}{5} \cdot \frac{3}{8}$ 59. $-\frac{5}{12} \cdot 1\frac{1}{7}$

Identify all sets to which each number belongs. (Lesson 5-2)

60. 16 61. $-2.8888\dots$ 62. $0.\overline{9}$ 63. $5.121221222\dots$

64. Write the prime factorization of 150. Use exponents for repeated factors.
(Lesson 4-3)

65. **ALGEBRA** Solve $3x - 5 = 16$. (Lesson 3-5)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write each improper fraction as a mixed number in simplest form. (To review *simplifying fractions*, see Lesson 4-5.)

66. $\frac{9}{4}$ 67. $\frac{8}{7}$ 68. $\frac{17}{2}$
 69. $\frac{25}{4}$ 70. $\frac{24}{5}$ 71. $\frac{22}{6}$
 72. $\frac{15}{6}$ 73. $\frac{30}{18}$ 74. $\frac{18}{15}$

Adding and Subtracting Like Fractions

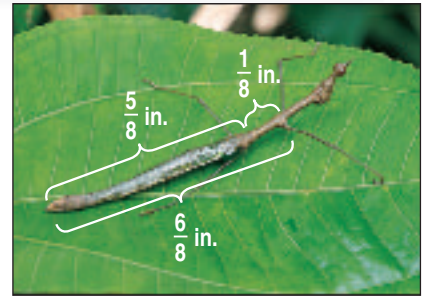
What You'll Learn

- Add like fractions.
- Subtract like fractions.

Why are fractions important when taking measurements?

Measures of different parts of an insect are shown in the diagram. The sum of the parts is $\frac{6}{8}$ inch. Use a ruler to find each measure.

- a. $\frac{1}{8}$ in. + $\frac{3}{8}$ in. b. $\frac{3}{8}$ in. + $\frac{4}{8}$ in.
 c. $\frac{4}{8}$ in. + $\frac{4}{8}$ in. d. $\frac{6}{8}$ in. - $\frac{3}{8}$ in.



ADD LIKE FRACTIONS Fractions with the same denominator are called *like fractions*. The rule for adding like fractions is stated below.

Key Concept

Adding Like Fractions

- **Words** To add fractions with like denominators, add the numerators and write the sum over the denominator.
- **Symbols** $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, where $c \neq 0$
- **Example** $\frac{1}{5} + \frac{2}{5} = \frac{1+2}{5}$ or $\frac{3}{5}$

Example 1 Add Fractions

Find $\frac{3}{7} + \frac{5}{7}$. Write the sum in simplest form. **Estimate:** $0 + 1 = 1$

$$\begin{aligned} \frac{3}{7} + \frac{5}{7} &= \frac{3+5}{7} && \text{The denominators are the same. Add the numerators.} \\ &= \frac{8}{7} \text{ or } 1\frac{1}{7} && \text{Simplify and rename as a mixed number.} \end{aligned}$$

Study Tip

Alternative Method
You can also stack the mixed numbers vertically to find the sum.

$$\begin{array}{r} 6\frac{5}{8} \\ + 1\frac{1}{8} \\ \hline 7\frac{6}{8} \text{ or } 7\frac{3}{4} \end{array}$$

Example 2 Add Mixed Numbers

Find $6\frac{5}{8} + 1\frac{1}{8}$. Write the sum in simplest form. **Estimate:** $7 + 1 = 8$


$$\begin{aligned} 6\frac{5}{8} + 1\frac{1}{8} &= (6 + 1) + \left(\frac{5}{8} + \frac{1}{8}\right) && \text{Add the whole numbers and fractions separately.} \\ &= 7 + \frac{5+1}{8} && \text{Add the numerators.} \\ &= 7\frac{6}{8} \text{ or } 7\frac{3}{4} && \text{Simplify.} \end{aligned}$$

SUBTRACT LIKE FRACTIONS The rule for subtracting fractions with like denominators is similar to the rule for addition.

Key Concept

Subtracting Like Fractions

- **Words** To subtract fractions with like denominators, subtract the numerators and write the difference over the denominator.
- **Symbols** $\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$, where $c \neq 0$
- **Example** $\frac{5}{7} - \frac{1}{7} = \frac{5-1}{7}$ or $\frac{4}{7}$

 **Concept Check** How is the rule for subtracting fractions with like denominators similar to the rule for adding fractions with like denominators?

Example 3 Subtract Fractions

Find $\frac{9}{20} - \frac{13}{20}$. Write the difference in simplest form. **Estimate:** $\frac{1}{2} - 1 = -\frac{1}{2}$

$$\begin{aligned}\frac{9}{20} - \frac{13}{20} &= \frac{9-13}{20} && \text{The denominators are the same. Subtract the numerators.} \\ &= \frac{-4}{20} \text{ or } -\frac{1}{5} && \text{Simplify.}\end{aligned}$$

You can write the mixed numbers as improper fractions before adding or subtracting.

Example 4 Subtract Mixed Numbers

Evaluate $a - b$ if $a = 9\frac{1}{6}$ and $b = 5\frac{2}{6}$. **Estimate:** $9 - 5 = 4$

$$\begin{aligned}a - b &= 9\frac{1}{6} - 5\frac{2}{6} && \text{Replace } a \text{ with } 9\frac{1}{6} \text{ and } b \text{ with } 5\frac{2}{6}. \\ &= \frac{55}{6} - \frac{32}{6} && \text{Write the mixed numbers as improper fractions.} \\ &= \frac{23}{6} && \text{Subtract the numerators.} \\ &= 3\frac{5}{6} && \text{Simplify.}\end{aligned}$$

ALGEBRA CONNECTION

You can use the same rules for adding or subtracting like algebraic fractions as you did for adding or subtracting like numerical fractions.

Example 5 Add Algebraic Fractions

Find $\frac{n}{8} + \frac{5n}{8}$. Write the sum in simplest form.

$$\begin{aligned}\frac{n}{8} + \frac{5n}{8} &= \frac{n+5n}{8} && \text{The denominators are the same. Add the numerators.} \\ &= \frac{6n}{8} && \text{Add the numerators.} \\ &= \frac{3n}{4} && \text{Simplify.}\end{aligned}$$

Check for Understanding

Concept Check

1. Draw a model to show the sum $\frac{2}{7} + \frac{4}{7}$.
2. **OPEN ENDED** Write a subtraction expression in which the difference of two fractions is $\frac{18}{25}$.
3. **FIND THE ERROR** Kayla and Ethan are adding $-2\frac{1}{8}$ and $-4\frac{3}{8}$.

Kayla

$$-2\frac{1}{8} + \left(-4\frac{3}{8}\right) = -\frac{17}{8} + \left(-\frac{35}{8}\right)$$

$$= -\frac{52}{8} \text{ or } -6\frac{1}{2}$$

Ethan

$$-2\frac{1}{8} + \left(-4\frac{3}{8}\right) = \frac{17}{8} + \left(-\frac{35}{8}\right)$$

$$= -\frac{18}{8} \text{ or } -2\frac{1}{4}$$

Who is correct? Explain your reasoning.

Guided Practice

Find each sum or difference. Write in simplest form.

4. $\frac{1}{7} + \frac{5}{7}$
5. $\frac{11}{14} - \frac{3}{14}$
6. $\frac{3}{10} + \frac{3}{10}$
7. $-\frac{1}{8} - \frac{5}{8}$
8. $-2\frac{4}{5} + \left(-\frac{2}{5}\right)$
9. $7\frac{1}{8} - \left(-1\frac{3}{8}\right)$
10. **ALGEBRA** Evaluate $x + y$ if $x = 2\frac{4}{9}$ and $y = 8\frac{7}{9}$.

ALGEBRA Find each sum or difference. Write in simplest form.

11. $\frac{6r}{11} + \frac{2r}{11}$
12. $\frac{19}{a} - \frac{12}{a}, a \neq 0$
13. $\frac{5}{3x} - \frac{6}{3x}, x \neq 0$

Application

14. **MEASUREMENT** Hoai was $62\frac{1}{8}$ inches tall at the end of school in June. He was $63\frac{7}{8}$ inches tall in September. How much did he grow during the summer?

Practice and Apply

Homework Help

For Exercises	See Examples
15–24	1–3
25–38, 45, 46	4
39–44	5

Extra Practice
See page 734.

Find each sum or difference. Write in simplest form.

15. $\frac{2}{5} + \frac{1}{5}$
16. $\frac{10}{11} - \frac{8}{11}$
17. $\frac{17}{18} - \frac{5}{18}$
18. $\frac{3}{10} + \frac{7}{10}$
19. $\frac{1}{12} + \left(-\frac{7}{12}\right)$
20. $\frac{9}{20} - \left(-\frac{7}{20}\right)$
21. $-\frac{3}{4} + \left(-\frac{3}{4}\right)$
22. $-\frac{13}{16} + \left(-\frac{9}{16}\right)$
23. $-\frac{7}{9} + \frac{5}{9}$
24. $-\frac{17}{20} + \frac{9}{20}$
25. $7\frac{2}{5} + 4\frac{2}{5}$
26. $-4\frac{5}{8} - \frac{3}{8}$
27. $5\frac{7}{9} - \left(-3\frac{5}{9}\right)$
28. $2\frac{5}{12} + \left(-2\frac{7}{12}\right)$
29. $2\frac{3}{8} - 1\frac{5}{8}$
30. $8\frac{9}{10} - 6\frac{1}{10}$
31. $7\frac{4}{7} - 2\frac{5}{7}$
32. $-8\frac{6}{11} - \left(-2\frac{5}{11}\right)$
33. Find $12\frac{7}{8} - 7\frac{3}{8} + 2\frac{5}{8}$.
34. Find $5\frac{5}{6} + 3\frac{5}{6} - 2\frac{1}{6}$.

Career Choices



Carpenter

Carpenters must be able to make precise measurements and know how to add and subtract fractional measures.

Online Research

For information about a career as a carpenter, visit: www.pre-alg.com/careers

ALGEBRA Evaluate each expression if $x = \frac{8}{15}$, $y = 2\frac{1}{15}$, and $z = \frac{11}{15}$. Write in simplest form.

35. $x + y$

36. $z + y$

37. $z - x$

38. $y - x$

ALGEBRA Find each sum or difference. Write in simplest form.

39. $\frac{x}{8} + \frac{4x}{8}$

40. $\frac{3r}{10} + \frac{3r}{10}$

41. $\frac{12}{m} - \frac{9}{m}, m \neq 0$

42. $\frac{10a}{3b} - \frac{7a}{3b}, b \neq 0$

43. $5\frac{4}{7}c - 3\frac{1}{7}c$

44. $-2\frac{1}{6}y + 8\frac{5}{6}y$

45. **CARPENTRY** A 5-foot long kitchen countertop is to be installed between two walls that are $54\frac{5}{8}$ inches apart. How much of the countertop must be cut off so that it fits between the walls?

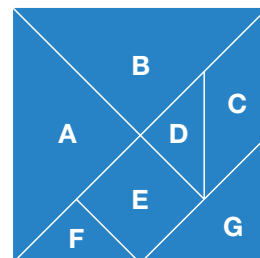
46. **SEWING** Chumani is making a linen suit. The portion of the pattern envelope that shows the yards of fabric needed for different sizes is shown at the right. If Chumani is making a size 6 jacket and skirt from 45-inch fabric, how much fabric should she buy?

Size	(6)	(8)	(10)
JACKET			
45"	$2\frac{5}{8}$	$2\frac{3}{4}$	$2\frac{3}{4}$
60"	2	2	2
SKIRT			
45"	$\frac{7}{8}$	$1\frac{1}{8}$	$1\frac{1}{8}$
60"	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$

47. **GARDENING** Melanie's flower garden has a perimeter of 25 feet. She plans to add 2 feet 9 inches to the width and 3 feet 9 inches to the length. What is the new perimeter in feet?

48. **CRITICAL THINKING** The 7-piece puzzle at the right is called a *tangram*.

- If the value of the entire puzzle is 1, what is the value of each piece?
- How much is $A + B$?
- How much is $F + D$?
- How much is $C + E$?
- Which pieces each equal the sum of E and G ?



49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are fractions important when taking measurements?

Include the following in your answer:

- the fraction of an inch that each mark on a ruler or tape measure represents, and
- some real-world examples in which fractional measures are used.

50. Find $\frac{13}{20} - \frac{7}{20}$. Write in simplest form.
 (A) $\frac{6}{10}$ (B) $\frac{3}{5}$ (C) $\frac{6}{20}$ (D) $\frac{3}{10}$
51. A piece of wood is $1\frac{9}{16}$ inches thick. A layer of padding $\frac{15}{16}$ inch thick is placed on top. What is the total thickness of the wood and the padding?
 (A) $2\frac{1}{2}$ in. (B) $1\frac{1}{2}$ in. (C) $1\frac{24}{16}$ in. (D) $1\frac{3}{8}$ in.

Maintain Your Skills

Mixed Review Find each quotient. Write in simplest form. (Lesson 5-4)

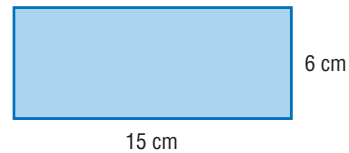
52. $\frac{1}{6} \div \frac{3}{4}$ 53. $-\frac{5}{8} \div \frac{1}{3}$ 54. $\frac{2}{5} \div 1\frac{1}{2}$

Find each product. Write in simplest form. (Lesson 5-3)

55. $\frac{2}{5} \cdot \frac{3}{4}$ 56. $\frac{1}{6} \cdot \left(-\frac{8}{9}\right)$ 57. $\frac{4}{7} \cdot 2\frac{1}{3}$

58. Find the product of $4y^2$ and $8y^5$. (Lesson 4-6)

59. **GEOMETRY** Find the perimeter and area of the rectangle.
 (Lesson 3-7)



Getting Ready for the Next Lesson

PREREQUISITE SKILL Use exponents to write the prime factorization of each number or monomial. (To review **prime factorization**, see Lesson 4-3.)

60. 60 61. 175 62. 112
 63. $12n$ 64. $24s^2$ 65. $42a^2b$

Practice Quiz 1

Lessons 5-1 through 5-5

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal. (Lesson 5-1)

1. $\frac{4}{25}$ 2. $-\frac{2}{9}$ 3. $3\frac{1}{8}$

Write each decimal as a fraction or mixed number in simplest form.

(Lesson 5-2)

4. -6.75 5. 0.12 6. 0.5555...

Simplify each expression. (Lessons 5-3, 5-4, and 5-5)

7. $\frac{5}{18} \cdot \frac{4}{15}$ 8. $\frac{7}{8} \div \left(-\frac{1}{4}\right)$ 9. $\frac{11}{12} - \frac{6}{12}$

10. **ALGEBRA** Find $6\frac{3}{5}a + 2\frac{4}{5}a$. Write in simplest form. (Lesson 5-5)



Factors and Multiples

Many words used in mathematics are also used in everyday language. You can use the everyday meaning of these words to better understand their mathematical meaning. The table shows both meanings of the words *factor* and *multiple*.

Term	Everyday Meaning	Mathematical Meaning
factor	something that contributes to the production of a result <ul style="list-style-type: none"> • The weather was not a <i>factor</i> in the decision. • The type of wood is one <i>factor</i> that contributes to the cost of the table. 	one of two or more numbers that are multiplied together to form a product
multiple	involving more than one or shared by many <ul style="list-style-type: none"> • <i>multiple</i> births • <i>multiple</i> ownership 	the product of a quantity and a whole number

Source: Merriam Webster's Collegiate Dictionary

When you count by 2, you are listing the multiples of 2. When you count by 3, you are listing the multiples of 3, and so on, as shown in the table below.

Number	Factors	Multiples
2	1, 2	2, 4, 6, 8, ...
3	1, 3	3, 6, 9, 12, ...
4	1, 2, 4	4, 8, 12, 16, ...

Notice that the mathematical meaning of each word is related to the everyday meaning. The word *multiple* means many, and in mathematics, a number has infinitely many multiples.

Reading to Learn

- Write your own rule for remembering the difference between *factor* and *multiple*.
- RESEARCH** Use the Internet or a dictionary to find the everyday meaning of each word listed below. Compare them to the mathematical meanings of *factor* and *multiple*. Note the similarities and differences.
 - factotum
 - multicultural
 - multimedia
- Make lists of other words that have the prefixes fact- or multi-. Determine what the words in each list have in common.



5-6

Least Common Multiple

What You'll Learn

- Find the least common multiple of two or more numbers.
- Find the least common denominator of two or more fractions.

Vocabulary

- multiple
- common multiples
- least common multiple (LCM)
- least common denominator (LCD)

How can you use prime factors to find the least common multiple?

A voter voted for both president and senator in the year 2000.

- List the next three years in which the voter can vote for president.
- List the next three years in which the voter can vote for senator.
- What will be the next year in which the voter has a chance to vote for both president and senator?

Candidate	Length of Term (years)
President	4
Senator	6



LEAST COMMON MULTIPLE A **multiple** of a number is a product of that number and a whole number.

Sometimes numbers have some of the same multiples. These are called **common multiples**.

Some common multiples of 4 and 6 are 0, 12, and 24.

multiples of 4: 0, 4, 8, 12, 16, 20, 24, 28, ...

multiples of 6: 0, 6, 12, 18, 24, 30, 36, 42, ...

Multiples of 5	
$5 \cdot 0 = 0$	
$5 \cdot 1 = 5$	
$5 \cdot 2 = 10$	
$5 \cdot 3 = 15$	
⋮	⋮

The least of the *nonzero* common multiples is called the **least common multiple (LCM)**. So, the LCM of 4 and 6 is 12.

When numbers are large, an easier way of finding the least common multiple is to use prime factorization. The LCM is the smallest product that contains the prime factors of each number.

Example 1 Find the LCM

Find the LCM of 108 and 240.

Number	Prime Factorization	Exponential Form
108	$2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$	$2^2 \cdot 3^3$
240	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$	$2^4 \cdot 3 \cdot 5$

Study Tip


Prime Factors

If a prime factor appears in both numbers, use the factor with the greatest exponent.

The prime factors of both numbers are 2, 3, and 5. Multiply the greatest power of 2, 3, and 5 appearing in either factorization.

$$\begin{aligned}\text{LCM} &= 2^4 \cdot 3^3 \cdot 5 \\ &= 2160\end{aligned}$$

So, the LCM of 108 and 240 is 2160.

 **Concept Check** What is the LCM of 6 and 12?



The LCM of two or more monomials is found in the same way as the LCM of two or more numbers.

Example 2 The LCM of Monomials

Find the LCM of $18xy^2$ and $10y$.

$$\begin{aligned}18xy^2 &= 2 \cdot 3^2 \cdot x \cdot y^2 \\ 10y &= 2 \cdot 5 \cdot y\end{aligned}$$

$$\begin{aligned}\text{LCM} &= 2 \cdot 3^2 \cdot 5 \cdot x \cdot y^2 && \text{Multiply the greatest power of each prime factor.} \\ &= 90xy^2\end{aligned}$$

The LCM of $18xy^2$ and $10y$ is $90xy^2$.

LEAST COMMON DENOMINATOR The **least common denominator (LCD)** of two or more fractions is the LCM of the denominators.

Example 3 Find the LCD

Find the LCD of $\frac{5}{9}$ and $\frac{11}{21}$.

$$\begin{aligned}9 &= 3^2 && \leftarrow \text{Write the prime factorization of 9 and 21.} \\ 21 &= 3 \cdot 7 && \leftarrow \text{Highlight the greatest power of each prime factor.}\end{aligned}$$

$$\begin{aligned}\text{LCM} &= 3^2 \cdot 7 && \text{Multiply.} \\ &= 63\end{aligned}$$

The LCD of $\frac{5}{9}$ and $\frac{11}{21}$ is 63.



The LCD for algebraic fractions can also be found.


Example 4 Find the LCD of Algebraic Fractions

Find the LCD of $\frac{5}{12b^2}$ and $\frac{3}{8ab}$.

$$\begin{aligned}12b^2 &= 2^2 \cdot 3 \cdot b^2 \\ 8ab &= 2^3 \cdot a \cdot b\end{aligned}$$

$$\text{LCM} = 2^3 \cdot 3 \cdot a \cdot b^2 \text{ or } 24ab^2$$

Thus, the LCD of $\frac{5}{12b^2}$ and $\frac{3}{8ab}$ is $24ab^2$.

 **Concept Check** What is the LCD of $\frac{1}{x^2}$ and $\frac{1}{xy}$?



One way to compare fractions is to write them using the LCD. We can multiply the numerator and the denominator of a fraction by the same number, because it is the same as multiplying the fraction by 1.

Example 5 Compare Fractions

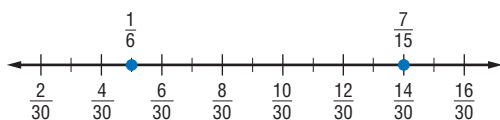
Replace \bullet with $<$, $>$, or $=$ to make $\frac{1}{6} \bullet \frac{7}{15}$ a true statement.

The LCD of the fractions is $2 \cdot 3 \cdot 5$ or 30. Rewrite the fractions using the LCD and then compare the numerators.

$$\frac{1}{6} = \frac{1 \cdot 5}{2 \cdot 3 \cdot 5} = \frac{5}{30} \quad \text{Multiply the fraction by } \frac{5}{5} \text{ to make the denominator 30.}$$

$$\frac{7}{15} = \frac{7 \cdot 2}{3 \cdot 5 \cdot 2} = \frac{14}{30} \quad \text{Multiply the fraction by } \frac{2}{2} \text{ to make the denominator 30.}$$

Since $\frac{5}{30} < \frac{14}{30}$, then $\frac{1}{6} < \frac{7}{15}$.



$\frac{1}{6}$ is to the left of $\frac{7}{15}$ on the number line.

Check for Understanding

Concept Check

1. **Compare and contrast** least common multiple (LCM) and least common denominator (LCD).
2. **OPEN ENDED** Write two fractions whose least common denominator (LCD) is 35.

Guided Practice

Find the least common multiple (LCM) of each pair of numbers or monomials.

- | | | |
|-----------|-----------|---------------|
| 3. 6, 8 | 4. 7, 9 | 5. 10, 14 |
| 6. 12, 30 | 7. 16, 24 | 8. $36ab, 4b$ |

Find the least common denominator (LCD) of each pair of fractions.

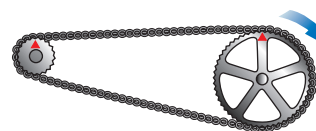
- | | | |
|-------------------------------|---------------------------------|-------------------------------------|
| 9. $\frac{1}{2}, \frac{3}{8}$ | 10. $\frac{2}{3}, \frac{7}{10}$ | 11. $\frac{2}{25x}, \frac{13}{20x}$ |
|-------------------------------|---------------------------------|-------------------------------------|

Replace each \bullet with $<$, $>$, or $=$ to make a true statement.

- | | | |
|--|---|---------------------------------------|
| 12. $\frac{1}{4} \bullet \frac{3}{16}$ | 13. $\frac{10}{45} \bullet \frac{2}{9}$ | 14. $\frac{5}{7} \bullet \frac{7}{9}$ |
|--|---|---------------------------------------|

Application

15. **CYCLING** The front bicycle gear has 52 teeth and the back gear has 20 teeth. How many revolutions must each gear make for them to align again as shown? (*Hint*: First, find the number of teeth. Then divide to find the final answers.)



back gear

front gear

Practice and Apply

Homework Help

For Exercises	See Examples
16–29	1
30–33	2
34–39	3
40, 41	4
44–54	5

Extra Practice
See page 735.

Find the least common multiple (LCM) of each set of numbers or monomials.

- | | | |
|----------------------|------------------|-----------------------|
| 16. 4, 10 | 17. 20, 12 | 18. 2, 9 |
| 19. 16, 3 | 20. 15, 75 | 21. 21, 28 |
| 22. 14, 28 | 23. 20, 50 | 24. 18, 32 |
| 25. 24, 32 | 26. 10, 20, 40 | 27. 7, 21, 84 |
| 28. 9, 12, 15 | 29. 45, 30, 35 | 30. $20c$, $12c$ |
| 31. $16a^2$, $14ab$ | 32. $7x$, $12x$ | 33. $75n^2$, $25n^4$ |

Find the least common denominator (LCD) of each pair of fractions.

- | | | | |
|---------------------------------|---------------------------------|------------------------------------|--------------------------------------|
| 34. $\frac{1}{4}, \frac{7}{8}$ | 35. $\frac{8}{15}, \frac{1}{3}$ | 36. $\frac{4}{5}, \frac{1}{2}$ | 37. $\frac{2}{5}, \frac{6}{7}$ |
| 38. $\frac{4}{9}, \frac{5}{12}$ | 39. $\frac{3}{8}, \frac{5}{6}$ | 40. $\frac{1}{3t}, \frac{4}{5t^2}$ | 41. $\frac{7}{8cd}, \frac{5}{16c^2}$ |

42. **PLANETS** The table shows the number of Earth years it takes for some of the planets to revolve around the Sun. Find the least common multiple of the revolution times to determine approximately how often these planets align.

Planet	Revolution Time (Earth Years)
Jupiter	12
Saturn	30
Uranus	84

43. **AUTO RACING** One driver can circle a one-mile track in 30 seconds. Another driver takes 20 seconds. If they both start at the same time, in how many seconds will they be together again at the starting line?

Replace each \bullet with $<$, $>$, or $=$ to make a true statement.

- | | | |
|--|--|--|
| 44. $\frac{1}{2} \bullet \frac{5}{12}$ | 45. $\frac{7}{9} \bullet \frac{5}{6}$ | 46. $\frac{3}{5} \bullet \frac{4}{7}$ |
| 47. $\frac{21}{100} \bullet \frac{1}{5}$ | 48. $\frac{17}{34} \bullet \frac{1}{2}$ | 49. $\frac{12}{17} \bullet \frac{36}{51}$ |
| 50. $\frac{8}{9} \bullet \frac{19}{21}$ | 51. $-\frac{9}{11} \bullet -\frac{5}{6}$ | 52. $-\frac{14}{15} \bullet -\frac{9}{10}$ |

53. **ANIMALS** Of all the endangered species in the world, $\frac{7}{39}$ of the reptiles and $\frac{5}{9}$ of the amphibians are in the United States. Is there a greater fraction of endangered reptiles or amphibians in the U.S.?

 **Online Research Data Update** How many endangered species are in the United States today? Visit www.pre-alg.com/data_update to learn more.

54. **TELEPHONES** Eleven out of 20 people in Chicago, Illinois, have cell phones, and $\frac{14}{25}$ of the people in Anchorage, Alaska, have cell phones. In which city do a greater fraction of people have cell phones? **Source:** Polk Research
55. Find two composite numbers between 10 and 20 whose least common multiple (LCM) is 36.

More About . . .



Planets

A spacecraft's launch date depends on planet alignment because the gravitational forces could affect the spacecraft's flight path.

Source: Memphis Space Center



Determine whether each statement is *sometimes*, *always*, or *never* true. Give an example to support your answer.

56. The LCM of three numbers is one of the numbers.
 57. If two numbers do not contain any factors in common, then the LCM of the two numbers is 1.
 58. The LCM of two numbers is greater than the GCF of the numbers.

Study Tip

Relatively Prime

Recall that two numbers that are relatively prime have a GCF of 1.

59. CRITICAL THINKING

- a. If two numbers are relatively prime, what is their LCM? Give two examples and explain your reasoning.
 b. Determine whether the LCM of two whole numbers is *always*, *sometimes*, or *never* a multiple of the GCF of the same two numbers. Explain.

60. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use prime factors to find the least common multiple?

Include the following in your answer:

- a definition of least common multiple, and
- a description of the steps you take to find the LCM of two or more numbers.

61. Find the least common multiple of $12a^2b$ and $9ac$.

(A) $36a^2b$ (B) $36a^2bc$ (C) $3a^2bc$ (D) $3abc$

62. A $\frac{7}{8}$ -inch wrench is too large to tighten a bolt. Of these, which is the next smaller size?

(A) $\frac{3}{4}$ -inch (B) $\frac{5}{8}$ -inch (C) $\frac{7}{16}$ -inch (D) $\frac{13}{16}$ -inch

Standardized Test Practice

A B C D

Maintain Your Skills

Mixed Review Find each sum or difference. Write in simplest form. (Lesson 5-5)

63. $\frac{7}{8} - \frac{3}{8}$ 64. $3\frac{9}{11} - \frac{5}{11}$ 65. $\frac{13}{14} + \frac{3}{14}$ 66. $2\frac{5}{6} + 4\frac{1}{6}$

ALGEBRA Find each quotient. Write in simplest form. (Lesson 5-4)

67. $\frac{3}{n} \div \frac{1}{n}$ 68. $\frac{x}{8} \div \frac{x}{6}$ 69. $\frac{ac}{5} \div \frac{c}{d}$ 70. $\frac{6k}{7m} \div \frac{3}{14m}$

71. **ALGEBRA** Translate the sum of 7 and two times a number is 11 into an equation. Then find the number. (Lesson 3-6)

ALGEBRA Solve each equation. Check your solution. (Lessons 3-3 and 3-4)

72. $9 = x - 4$ 73. $\frac{a}{-2} = 10$ 74. $-5c = -105$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Estimate each sum. (To review *estimating with fractions*, see page 716.)

75. $\frac{3}{8} + \frac{3}{4}$ 76. $\frac{9}{10} + \frac{14}{15}$ 77. $\frac{4}{7} + 2\frac{1}{5}$
 78. $5\frac{7}{8} + \frac{2}{3}$ 79. $8\frac{3}{11} + 7\frac{2}{9}$ 80. $20\frac{5}{16} + 6\frac{1}{9}$



Algebra Activity

A Follow-Up of Lesson 5-6

Juniper Green

Juniper Green is a game that was invented by a teacher in England.

Getting Ready

This game is for two people, so students should divide into pairs.

Rules of the Game

- The first player selects an even number from the hundreds chart and circles it with a colored marker.
- The next player selects any remaining number that is a factor or multiple of this number and circles it.
- Players continue taking turns circling numbers, as shown below.
- When a player cannot select a number or circles a number incorrectly, then the game is over and the other player wins.



1st move
Player 1 circles 42.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

2nd move
Player 2 circles 7 because it is a factor of 42.

3rd move
Player 1 circles 70 because it is a multiple of 7.

Analyze the Strategies

Play the game several times and then answer the following questions.

1. Why do you think the first player must select an even number? Explain.
2. Describe the kinds of moves that were made just before the game was over.

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Adding and Subtracting Unlike Fractions

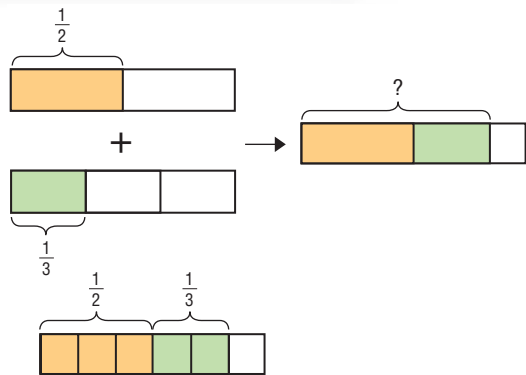
What You'll Learn

- Add unlike fractions.
- Subtract unlike fractions.

How can the LCM be used to add and subtract fractions with different denominators?

The sum $\frac{1}{2} + \frac{1}{3}$ is modeled at the right. We can use the LCM to find the sum.

- What is the LCM of the denominators?
- If you partition the model into six parts, what fraction of the model is shaded?
- How many parts are $\frac{1}{2}$? $\frac{1}{3}$?
- Describe a model that you could use to add $\frac{1}{3}$ and $\frac{1}{4}$. Then use it to find the sum.



ADD UNLIKE FRACTIONS Fractions with different denominators are called *unlike fractions*. In the activity above, you used the LCM of the denominators to rename the fractions. You can use any common denominator.

Key Concept

Adding Unlike Fractions

- **Words** To add fractions with unlike denominators, rename the fractions with a common denominator. Then add and simplify.
- **Example** $\frac{1}{3} + \frac{2}{5} = \frac{1}{3} \cdot \frac{5}{5} + \frac{2}{5} \cdot \frac{3}{3}$
 $= \frac{5}{15} + \frac{6}{15}$ or $\frac{11}{15}$


Example 1 Add Unlike Fractions

Find $\frac{1}{4} + \frac{2}{3}$.

$$\frac{1}{4} + \frac{2}{3} = \frac{1}{4} \cdot \frac{3}{3} + \frac{2}{3} \cdot \frac{4}{4} \quad \text{Use } 4 \cdot 3 \text{ or } 12 \text{ as the common denominator.}$$

$$= \frac{3}{12} + \frac{8}{12} \quad \text{Rename each fraction with the common denominator.}$$

$$= \frac{11}{12} \quad \text{Add the numerators.}$$

 **Concept Check** Name a common denominator of $\frac{5}{9}$ and $\frac{4}{5}$.

You can rename unlike fractions using any common denominator. However, it is usually simpler to use the least common denominator.

Example 2 Add Fractions

Find $\frac{3}{8} + \frac{7}{12}$. **Estimate:** $\frac{1}{2} + \frac{1}{2} = 1$

$$\frac{3}{8} + \frac{7}{12} = \frac{3}{8} \cdot \frac{3}{3} + \frac{7}{12} \cdot \frac{2}{2} \quad \text{The LCD is } 2^3 \cdot 3 \text{ or } 24.$$

$$= \frac{9}{24} + \frac{14}{24} \quad \text{Rename each fraction with the LCD.}$$

$$= \frac{23}{24} \quad \text{Add the numerators.}$$

Example 3 Add Mixed Numbers

Find $1\frac{2}{9} + (-2\frac{1}{3})$. Write in simplest form. **Estimate:** $1 + (-2) = -1$

$$1\frac{2}{9} + (-2\frac{1}{3}) = \frac{11}{9} + (-\frac{7}{3}) \quad \text{Write the mixed numbers as improper fractions.}$$

$$= \frac{11}{9} + (-\frac{7}{3}) \cdot \frac{3}{3} \quad \text{Rename } -\frac{7}{3} \text{ using the LCD, 9.}$$

$$= \frac{11}{9} + (-\frac{21}{9}) \quad \text{Simplify.}$$

$$= \frac{-10}{9} \quad \text{Add the numerators.}$$

$$= -1\frac{1}{9} \quad \text{Simplify.}$$

Study Tip

Negative Signs

When adding or subtracting a negative fraction, place the negative sign in the numerator.

SUBTRACT UNLIKE FRACTIONS The rule for subtracting fractions with unlike denominators is similar to the rule for addition.

Key Concept

Subtracting Unlike Fractions

- **Words** To subtract fractions with unlike denominators, rename the fractions with a common denominator. Then subtract and simplify.

• **Example**
$$\frac{6}{7} - \frac{2}{3} = \frac{6}{7} \cdot \frac{3}{3} - \frac{2}{3} \cdot \frac{7}{7}$$

$$= \frac{18}{21} - \frac{14}{21} \text{ or } \frac{4}{21}$$

Example 4 Subtract Fractions

Find $\frac{5}{21} - \frac{6}{7}$.

$$\frac{5}{21} - \frac{6}{7} = \frac{5}{21} - \frac{6}{7} \cdot \frac{3}{3} \quad \text{The LCD is 21.}$$

$$= \frac{5}{21} - \frac{18}{21} \quad \text{Rename } \frac{6}{7} \text{ using the LCD.}$$

$$= \frac{-13}{21} \text{ or } -\frac{13}{21} \quad \text{Subtract the numerators.}$$



Example 5 Subtract Mixed Numbers

Find $6\frac{1}{2} - 4\frac{1}{5}$. Write in simplest form.

$$6\frac{1}{2} - 4\frac{1}{5} = \frac{13}{2} - \frac{21}{5}$$

Write the mixed numbers as improper fractions.

$$= \frac{13}{2} \cdot \frac{5}{5} - \frac{21}{5} \cdot \frac{2}{2}$$

Rename the fractions using the LCD.

$$= \frac{65}{10} - \frac{42}{10}$$

Simplify.

$$= \frac{23}{10} \text{ or } 2\frac{3}{10}$$

Subtract.

Example 6 Use Fractions to Solve a Problem

HOUSES The diagram shows a cross-section of an outside wall. How thick is the wall?

Explore You know the measure of each layer of the wall.

Plan Add the measures to find the total thickness of the wall. Estimate your answer.

$$\frac{1}{2} + 5\frac{1}{2} + 1 + 1 = 8$$

Solve $\frac{5}{8} + 5\frac{1}{2} + \frac{3}{4} + \frac{7}{8} = \frac{5}{8} + 5\frac{4}{8} + \frac{6}{8} + \frac{7}{8}$

Rename the fractions with the LCD, 8.

$$= 5\frac{22}{8}$$

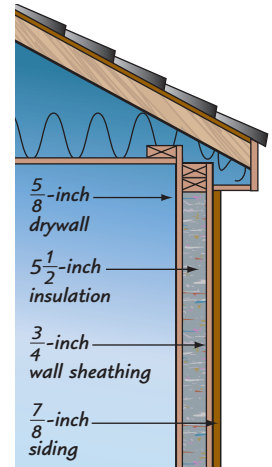
Add the like fractions.

$$= 7\frac{6}{8} \text{ or } 7\frac{3}{4} \text{ in.}$$

Simplify.

The wall is $7\frac{3}{4}$ inches thick.

Examine Since $7\frac{3}{4}$ is close to 8, the answer is reasonable.

**Check for Understanding****Concept Check**

- Describe the first step in adding or subtracting fractions with unlike denominators.
- OPEN ENDED** Write a real-world problem that you could solve by subtracting $2\frac{1}{8}$ from $15\frac{3}{4}$.
- FIND THE ERROR** José and Daniel are finding $\frac{9}{10} + \frac{7}{12}$.

José

$$\frac{9}{10} + \frac{7}{12} = \frac{9}{10} \cdot \frac{12}{12} + \frac{7}{12} \cdot \frac{10}{10}$$

Daniel

$$\frac{9}{10} + \frac{7}{12} = \frac{9+7}{10+12}$$

Who is correct? Explain your reasoning.

Guided Practice Find each sum or difference. Write in simplest form.

4. $\frac{1}{10} + \frac{1}{3}$ 5. $-\frac{1}{6} + \frac{7}{18}$ 6. $\frac{1}{4} - \frac{2}{3}$
 7. $-\frac{7}{10} - \frac{2}{15}$ 8. $6\frac{4}{5} + (-1\frac{3}{4})$ 9. $-9\frac{3}{4} - (-5\frac{1}{2})$

Application 10. **SEWING** Jessica needs $1\frac{5}{8}$ yards of fabric to make a skirt and $3\frac{1}{2}$ yards to make a coat. How much fabric does she need in all?

Practice and Apply

Homework Help

For Exercises	See Examples
11–28	1–5
31–34	6

Extra Practice
See page 735.

Find each sum or difference. Write in simplest form.

11. $\frac{3}{5} + \frac{3}{10}$ 12. $\frac{9}{26} + \frac{3}{13}$ 13. $\frac{3}{7} + (-\frac{1}{4})$
 14. $-\frac{5}{8} + (-\frac{1}{3})$ 15. $\frac{7}{8} - (-\frac{3}{16})$ 16. $-\frac{2}{5} - \frac{7}{8}$
 17. $\frac{3}{4} - \frac{5}{8}$ 18. $\frac{5}{7} + (-\frac{10}{21})$ 19. $-\frac{1}{2} + \frac{3}{8}$
 20. $-\frac{2}{3} + \frac{7}{12}$ 21. $1\frac{2}{5} - \frac{1}{3}$ 22. $\frac{7}{8} + 4\frac{1}{24}$
 23. $-6\frac{2}{3} - \frac{8}{9}$ 24. $2\frac{16}{30} - \frac{7}{15}$ 25. $-4\frac{1}{6} + (-7\frac{11}{18})$
 26. $3\frac{1}{2} - (-7\frac{1}{3})$ 27. $-19\frac{3}{8} - (-4\frac{3}{4})$ 28. $-3\frac{2}{5} - (-2\frac{4}{7})$

29. **ALGEBRA** Evaluate $x - y$ if $x = 4\frac{7}{18}$ and $y = 1\frac{1}{12}$.

30. **ALGEBRA** Solve $\frac{64}{143} - \frac{21}{208} = a$.

31. **EARTH SCIENCE** Did you know that water has a greater density than ice? Use the information in the table to find how much more water weighs per cubic foot.

1 Cubic Foot	Weight (lb)
water	$62\frac{1}{2}$
ice	$56\frac{9}{10}$

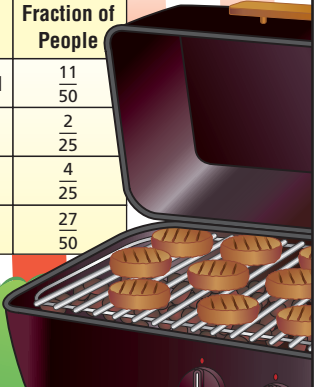
32. **GRILLING** Use the table to find the fraction of people who grill two, three, or four times per month.

33. **PUBLISHING** The length of a page in a yearbook is 10 inches. The top margin is $\frac{1}{2}$ inch, and the bottom margin is $\frac{3}{4}$ inch. What is the length of the page inside the margins?

34. **VOTING** In the class election, Murray received $\frac{1}{3}$ of the votes and Sara received $\frac{2}{5}$ of the votes. Makayla received the rest. What fraction of the votes did Makayla receive?

Barbecuing in the Summer

Times Per Month	Fraction of People
Less than 1	$\frac{11}{50}$
1	$\frac{2}{25}$
2-3	$\frac{4}{25}$
4	$\frac{27}{50}$



Source: American Plastics Council

35. **CRITICAL THINKING** A set of measuring cups has measures of 1 cup, $\frac{3}{4}$ cup, $\frac{1}{2}$ cup, $\frac{1}{3}$ cup, and $\frac{1}{4}$ cup. How could you get $\frac{1}{6}$ cup of milk by using these measures?
36. **CRITICAL THINKING** Do you think the rational numbers are closed under *addition*, *subtraction*, *multiplication*, or *division*? Explain.
37. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can the LCM be used to add and subtract fractions with different denominators?

Include the following in your answer:

- an example using the LCM, and
- an explanation of how prime factorization is a helpful way to add and subtract fractions that have different denominators.



38. For an art project, Halle needs $11\frac{3}{8}$ inches of red ribbon and $6\frac{7}{9}$ inches of white ribbon. Which is the best estimate for the total amount of ribbon that she needs?
- (A) 18 in. (B) 26 in. (C) 10 in. (D) 8 in.
39. How much less is $\frac{6}{15}$ than $9\frac{1}{2}$?
- (A) $9\frac{27}{30}$ (B) $9\frac{1}{10}$ (C) $1\frac{11}{30}$ (D) $9\frac{3}{10}$

Extending the Lesson

More than a thousand years ago, the Greeks wrote all fractions as the sum of *unit fractions*. A unit fraction is a fraction that has a numerator of 1, such as $\frac{1}{5}$, $\frac{1}{7}$, or $\frac{1}{4}$. Express each fraction below as the sum of two different unit fractions.

40. $\frac{7}{12}$ 41. $\frac{3}{5}$ 42. $\frac{2}{9}$

Maintain Your Skills

Mixed Review

Find the LCD of each pair of fractions. (Lesson 5-6)

43. $\frac{4}{9}$, $\frac{7}{12}$ 44. $\frac{3}{15t}$, $\frac{2}{5t}$ 45. $\frac{1}{3n}$, $\frac{7}{6n^3}$

Find each sum or difference. Write in simplest form. (Lesson 5-5)

46. $\frac{4}{7} + \frac{6}{7}$ 47. $2\frac{3}{4} + 6\frac{3}{4}$ 48. $\frac{7}{8} - \frac{5}{8}$
49. $3\frac{2}{5} - \frac{3}{5}$ 50. $4\frac{1}{6} + 5\frac{5}{6}$ 51. $8 - 6\frac{1}{5}$

52. Write the prime factorization of 124. (Lesson 4-3)

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each sum. (To review *adding integers*, see Lesson 2-2.)

53. $24 + (-12) + 15$ 54. $(-2) + 5 + (-3)$
55. $4 + (-9) + (-9) + 5$ 56. $-10 + (-9) + (-11) + (-8)$





Algebra Activity

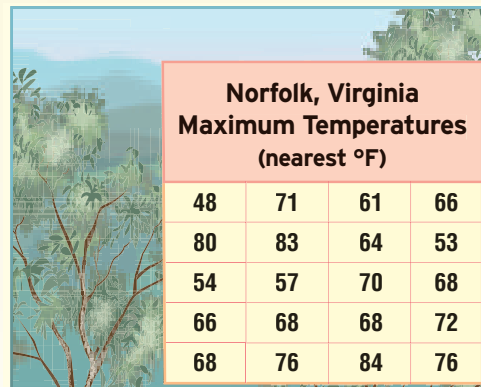
A Preview of Lesson 5-8

Analyzing Data

Often, it is useful to describe or represent a set of data by using a single number. The table shows the daily maximum temperatures for twenty days during a recent April in Norfolk, Virginia.

One number to describe this data set might be 68. Some reasons for choosing this number are listed below.

- It occurs four times, more often than any other number.
- If the numbers are arranged in order from least to greatest, 68 falls in the center of the data set.



48	71	61	66
80	83	64	53
54	57	70	68
66	68	68	72
68	76	84	76

There is an equal number of data above and below 68.

48 53 54 57 61 64 66 66 68 68 68 68 70 71 72 76 76 80 83 84

So, if you wanted to describe a typical high temperature for Norfolk during April, you could say 68°F.

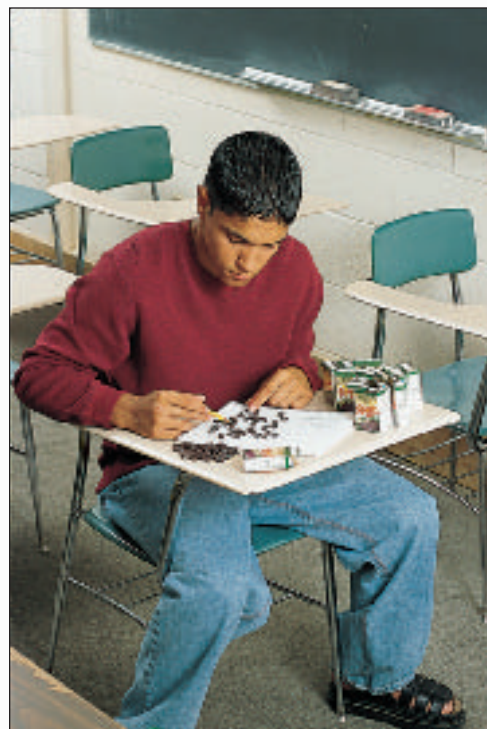
Collect the Data

Collect a group of data. Use one of the suggestions below, or use your own method.

- Research data about the weather in your city or in another city, such as temperatures, precipitation, or wind speeds.
- Find a graph or table of data in the newspaper or a magazine. Some examples include financial data, population data, and so on.
- Conduct a survey to gather some data about your classmates.
- Count the number of raisins in a number of small boxes.

Analyze the Data

1. Choose a number that best describes all of the data in the set.
2. Explain what your number means, and explain which method you used to choose your number.
3. Describe how your number might be useful in real life.



5-8 Measures of Central Tendency

What You'll Learn

- Use the mean, median, and mode as measures of central tendency.
- Analyze data using mean, median, and mode.

Vocabulary

- measures of central tendency
- mean
- median
- mode

How are measures of central tendency used in the real world?

The *Iditarod* is a 1150-mile dogsled race across Alaska. The winning times for 1973–2000 are shown in the table.

- Which number appears most often?
- If you list the data in order from least to greatest, which number is in the middle?
- What is the sum of all the numbers divided by 28? If necessary, round to the nearest tenth.
- If you had to give one number that best represents the winning times, which would you choose? Explain.

Winning Times (days)						
20	21	15	19	17	15	15
14	12	16	13	13	18	12
11	11	11	11	13	11	11
11	9	9	9	9	10	9



Source: Anchorage Daily News

MEAN, MEDIAN, AND MODE When you have a list of numerical data, it is often helpful to use one or more numbers to represent the whole set. These numbers are called **measures of central tendency**. You will study three types.

Study Tip

Mean, Median, Mode

- The mean and median do not have to be part of the data set.
- If there is a mode, it is always a member of the data set.

Key Concept

Measures of Central Tendency

Statistic	Definition
mean	the sum of the data divided by the number of items in the data set
median	the middle number of the ordered data, or the mean of the middle two numbers
mode	the number or numbers that occur most often

Example 1 Find the Mean, Median, and Mode

SPORTS The heights of the players on the girls' basketball team are shown in the chart. Find the mean, median, and mode.

Height of Players (cm)		
130	154	148
155	172	153
160	162	140
149	151	150

$$\begin{aligned}
 \text{mean} &= \frac{\text{sum of heights}}{\text{number of players}} \\
 &= \frac{130 + 154 + 148 + \dots + 150}{12} \\
 &= \frac{1824}{12} \text{ or } 152
 \end{aligned}$$

The mean height is 152 centimeters.

To find the median, order the numbers from least to greatest.

130, 140, 148, 149, 150, 151, 153, 154, 155, 160, 162, 172

$$\frac{151 + 153}{2} = 152$$

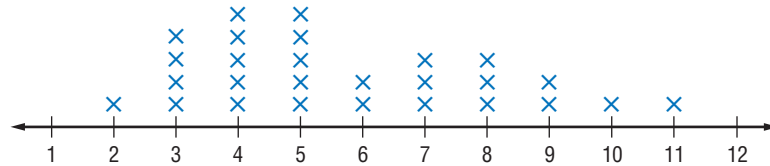
There is an even number of items. Find the mean of the two middle numbers.

The median height is 152.

There is no mode because each number in the set occurs once.

Example 2 Use a Line Plot

HURRICANES The line plot shows the number of Atlantic hurricanes that occurred each year from 1974–2000. Find the mean, median, and mode.



Source: Colorado State/Tropical Prediction Center

$$\text{mean} = \frac{2 + 3(4) + 4(5) + 5(5) + 6(2) + 7(3) + 8(3) + 9(2) + 10 + 11}{27} \approx 5.7$$

There are 27 numbers. So the middle number, which is the median, is the 14th number, or 5.

You can see from the graph that 4 and 5 both occur most often in the data set. So there are two modes, 4 and 5.

✓ Concept Check If 4 were added to the data set, what would be the new mode?

A number in a set of data that is much greater or much less than the rest of the data is called an *extreme value*. An extreme value can affect the mean of the data.

Example 3 Find Extreme Values that Affect the Mean

NUTRITION The table shows the number of Calories per serving of each vegetable. Identify an extreme value and describe how it affects the mean.

Vegetable	Calories	Vegetable	Calories
asparagus	14	cauliflower	10
beans	30	celery	17
bell pepper	20	corn	66
broccoli	25	lettuce	9
cabbage	17	spinach	9
carrots	28	zucchini	17

The data value 66 appears to be an extreme value.

Calculate the mean with and without the extreme value to find how it affects the mean.

mean with extreme value

$$\frac{\text{sum of values}}{\text{number of values}} = \frac{262}{12}$$

$$\approx 21.8$$

mean without extreme value

$$\frac{\text{sum of values}}{\text{number of values}} = \frac{196}{11}$$

$$\approx 17.8$$

The extreme value increases the mean by $21.8 - 17.8$ or about 4.

ANALYZE DATA You can use measures of central tendency to analyze data.

Example 4 Use Mean, Median, and Mode to Analyze Data

HOURLY PAY Compare and contrast the central tendencies of the salaries for the two stores. Based on the averages, which store pays its employees better?

Hourly Salaries (\$)	
Sports Superstore	Extreme Sports
7, 24, 7, 6, 8, 8, 8, 6	8, 9, 10, 10, 9, 8, 10, 10

Sports Superstore

mean:

$$= \frac{7 + 24 + 7 + 6 + 8 + 8 + 8 + 6}{8}$$

$$= \$9.25$$

median: 6, 6, 7, 7, 8, 8, 8, 24

$$\frac{7 + 8}{2} \text{ or } \$7.50$$

mode: \$8

Extreme Sports

mean:

$$= \frac{8 + 9 + 10 + 10 + 9 + 8 + 10 + 10}{8}$$

$$= \$9.25$$

median: 8, 8, 9, 9, 10, 10, 10, 10

$$\frac{9 + 10}{2} \text{ or } \$9.50$$

mode: \$10

The \$24 per hour salary at Sports Superstore is an extreme value that increases the mean salary. However, the employees at Extreme Sports are generally better paid, as shown by the higher median and mode salaries.

If you know the value of the mean, you can work backward to find a missing value in the data set.



Example 5 Work Backward

Grid-In Test Item

Francisca needs an average score of 92 on five quizzes to earn an A. The mean of her first four scores was 91. What is the lowest score that she can receive on the fifth quiz to earn an A?

Read the Test Item To find the lowest score, write an equation to find the sum of the first four scores. Then write an equation to find the fifth score.

Solve the Test Item

Step 1 Find the sum of the first four scores x .

$$\text{mean of first four scores} \rightarrow 91 = \frac{x}{4} \leftarrow \text{sum of first four scores}$$

$$(91)4 = \left(\frac{x}{4}\right)4 \quad \text{Multiply each side by 4.}$$

$$364 = x \quad \text{Simplify.}$$

Step 2 Find the fifth score y .

$$\text{mean} = \frac{\text{sum of the first four scores} + \text{fifth score}}{5} \quad \text{Write an equation.}$$

$$92 = \frac{364 + y}{5} \quad \text{Substitution}$$

$$460 = 364 + y \quad \text{Multiply each side by 5 and simplify.}$$

$$96 = y \quad \text{Subtract 364 from each side and simplify.}$$

Test-Taking Tip

Substituting Check that your answer satisfies the conditions of the original problem.



Check for Understanding

Concept Check

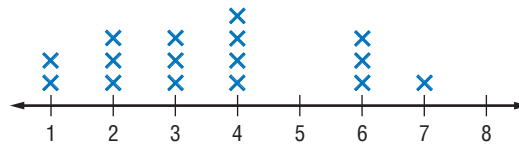
1. **Explain** which measure of central tendency is most affected by an extreme value.
2. **OPEN ENDED** Write a set of data with at least four numbers that has a mean of 8 and a median that is *not* 8.

Guided Practice

Find the mean, median, and mode for each set of data. If necessary, round to the nearest tenth.

3. 4, 5, 7, 3, 9, 11, 23, 37
4. 7.2, 3.6, 9.0, 5.2, 7.2, 6.5, 3.6

5.



Application

VACATIONS For Exercises 6–8, use the table.

6. Find the mean, median, and mode.
7. Identify any extreme values and describe how they affect the mean.
8. Which statistic would you say best represents the data? Explain.
9. Brad's average for five quizzes is 86. If he wants to have an average of 88 for six quizzes, what is the lowest score he can receive on his sixth quiz?

Annual Vacation Days	
Country	Number of Days
Brazil	34
Canada	26
France	37
Germany	35
Italy	42
Japan	25
Korea	25
United Kingdom	28
United States	13

Source: World Tourism Organization

Standardized Test Practice

A B C D

Practice and Apply

Homework Help

For Exercises

10–13	1
14, 15	2
16	3
17, 18	4

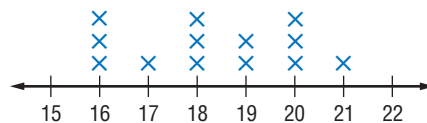
See Examples

Extra Practice
See page 735.

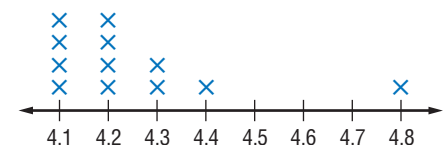
Find the mean, median, and mode for each set of data. If necessary, round to the nearest tenth.

10. 41, 37, 43, 43, 36
11. 2, 8, 16, 21, 3, 8, 9, 7, 6
12. 14, 6, 8, 10, 9, 5, 7, 13
13. 7.5, 7.1, 7.4, 7.6, 7.4, 9.0, 7.9, 7.1

14.



15.



16. **BASKETBALL** Refer to the cartoon at the right. Which measure of central tendency would make opponents believe that the height of the team is much taller than it really is? Explain.
17. **TESTS** Which measure of central tendency best summarizes the test scores shown below? Explain.
97, 99, 95, 89, 99, 100, 87, 85, 89, 92, 96, 95, 60, 97, 85



WebQuest

Using measures of central tendency can help you analyze the data from fast-food restaurants. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

18. **SALARIES** The graph shows the mean and median salaries of baseball players from 1983 to 2000. Explain why the mean is so much greater than the median.

19. **CRITICAL THINKING** A real estate guide lists the “average” home prices for counties in your state. Do you think the mean, median, or mode would be the most useful average for homebuyers? Explain.

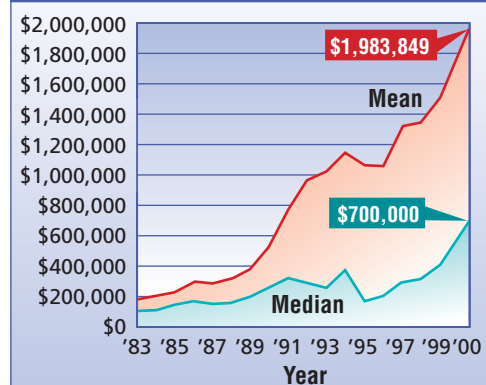
20. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are measures of central tendency used in the real world?

Include the following in your answer:

- examples of real-life data from home or school that can be described using the mean, median, or mode, and
- one or more newspaper articles in which averages are used.

Baseball Salaries from 1983-2000



Source: USA TODAY research

21. If 18 were added to the data set below, which statement is true?
16, 14, 22, 16, 16, 18, 15, 25

- (A) The mode increases. (B) The mean decreases.
(C) The mean increases. (D) The median increases.

22. Jonelle's tips as a waitress are shown in the table. On Friday, her tips were \$74. Which measure of central tendency will change the *most* as a result?

- (A) mean (B) median
(C) mode (D) no measure

Day	Tips
Monday	\$36
Tuesday	\$32
Wednesday	\$40
Thursday	\$36

Maintain Your Skills

Mixed Review Find each sum or difference. Write in simplest form. (Lesson 5-7)

23. $9\frac{2}{3} + \frac{1}{6}$

24. $\frac{7}{8} - \frac{3}{10}$

25. $-2\frac{3}{4} - 1\frac{1}{8}$

Replace each \bullet with $<$, $>$, or $=$ to make a true statement. (Lesson 5-6)

26. $\frac{1}{2} \bullet \frac{5}{12}$

27. $\frac{16}{50} \bullet \frac{9}{30}$

28. $\frac{4}{5} \bullet \frac{48}{60}$

ALGEBRA Solve each equation. Check your solution. (Lessons 3-3 and 3-4)

29. $y - 5 = 13$

30. $10 = 14 + n$

31. $-4w = 20$

Getting Ready for the Next Lesson

PREREQUISITE SKILL Find each quotient. If necessary, round to the nearest tenth. (To review **dividing decimals**, see page 715.)

32. $25.6 \div 3$

33. $37 \div 4.7$

34. $30.5 \div 11.2$

35. $46.8 \div 15.6$



Graphing Calculator Investigation

A Follow-Up of Lesson 5-8

Mean and Median

A graphing calculator is able to perform operations on large data sets efficiently. You can use a TI-83 Plus graphing calculator to find the mean and median of a set of data.

SURVEYS Fifteen seventh graders were surveyed and asked what was their weekly allowance (in dollars). The results of the survey are shown at the right.

20	10	5	5	10
15	5	10	5	10
5	5	5	5	5

Find the mean and median allowance.

Step 1 Enter the data.

- Clear any existing lists.

KEYSTROKES:

- Enter the allowances as L1.

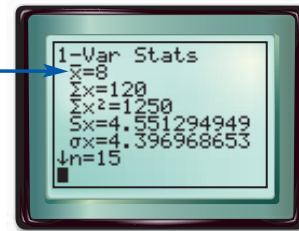
KEYSTROKES: 20 10 ... 5

Step 2 Find the mean and median.

- Display a list of statistics for the data.

KEYSTROKES:

The first value, \bar{x} , is the mean.



Use the down arrow key to locate "Med." The median allowance is \$5 and the mean allowance is \$8.

Exercises

Clear list L1 and find the mean and median of each data set. Round decimal answers to the nearest hundredth.

- 6.4, 5.6, 7.3, 1.2, 5.7, 8.9
- 23, -13, -16, -21, -15, -34, -22
- 123, 423, 190, 289, 99, 178, 156, 217, 217
- 8.4, 2.2, -7.3, -5.3, 6.7, -4.3, 5.1, 1.3, -1.1, -3.2, 2.2, 2.9, 1.4, 68
- Look back at the medians found. When is the median a member of the data set?
- Refer to Exercise 4.
 - Which statistic better represents the data, the mean or median? Explain.
 - Suppose the number 68 should have been 6.8. Recalculate the mean and median. Is there a significant difference between the first pair of values and the second pair?
 - When there is an error in one of the data values, which statistic is less likely to be affected? Why?



Solving Equations with Rational Numbers

What You'll Learn

- Solve equations containing rational numbers.

How are reciprocals used in solving problems involving music?

Musical sounds are made by vibrations. If n represents the number of vibrations for middle C, then the approximate vibrations for the other notes going up the scale are given below.

Notes	Middle C	D	E	F	G	A	B	C
Number of Vibrations	n	$\frac{9}{8}n$	$\frac{5}{4}n$	$\frac{4}{3}n$	$\frac{3}{2}n$	$\frac{5}{3}n$	$\frac{15}{8}n$	$\frac{2}{1}n$

- A guitar string vibrates 440 times per second to produce the A above middle C. Write an equation to find the number of vibrations per second to produce middle C. If you multiply each side by 3, what is the result?
- How would you solve the second equation you wrote in part a?
- How can you combine the steps in parts a and b into one step?
- How many vibrations per second are needed to produce middle C?

SOLVE ADDITION AND SUBTRACTION EQUATIONS You can solve rational number equations the same way you solved equations with integers.

Study Tip

Look Back

To review **solving equations**, see Lessons 3-3 and 3-4.

Example 1 Solve by Using Addition

Solve $2.1 = t - 8.5$. Check your solution.

$$2.1 = t - 8.5 \quad \text{Write the equation.}$$

$$2.1 + 8.5 = t - 8.5 + 8.5 \quad \text{Add 8.5 to each side.}$$

$$10.6 = t \quad \text{Simplify.}$$

CHECK $2.1 = t - 8.5$ Write the original equation.

$$2.1 \stackrel{?}{=} 10.6 - 8.5 \quad \text{Replace } t \text{ with } 10.6.$$

$$2.1 = 2.1 \checkmark \quad \text{Simplify.}$$

Example 2 Solve by Using Subtraction

Solve $x + \frac{3}{5} = \frac{2}{3}$.

$$x + \frac{3}{5} = \frac{2}{3} \quad \text{Write the equation.}$$

$$x + \frac{3}{5} - \frac{3}{5} = \frac{2}{3} - \frac{3}{5} \quad \text{Subtract } \frac{3}{5} \text{ from each side.}$$

$$x = \frac{2}{3} - \frac{3}{5} \quad \text{Simplify.}$$

$$x = \frac{10}{15} - \frac{9}{15} \text{ or } \frac{1}{15} \quad \text{Rename the fractions using the LCD and subtract.}$$

Study Tip

Reciprocals

Recall that dividing by a fraction is the same as multiplying by its multiplicative inverse.

SOLVE MULTIPLICATION AND DIVISION EQUATIONS To solve $\frac{1}{2}x = 3$, you can divide each side by $\frac{1}{2}$ or multiply each side by the multiplicative inverse of $\frac{1}{2}$, which is 2.

$$\frac{1}{2}x = 3 \quad \text{Write the equation.}$$

$$2 \cdot \frac{1}{2}x = 2 \cdot 3 \quad \text{Multiply each side by 2.}$$

The product of any number and its multiplicative inverse is 1.

$$1x = 6 \quad \text{Simplify.}$$

$$x = 6$$

Example 3 Solve by Using Division

Solve $-3y = 1.5$. Check your solution.

$$-3y = 1.5 \quad \text{Write the equation.}$$

$$\frac{-3y}{-3} = \frac{1.5}{-3} \quad \text{Divide each side by } -3.$$

$$y = -0.5 \quad \text{Simplify.}$$

CHECK $-3y = 1.5$ Write the original equation.

$$-3(-0.5) \stackrel{?}{=} 1.5 \quad \text{Replace } y \text{ with } -0.5.$$

$$1.5 = 1.5 \checkmark \quad \text{Simplify.}$$

Example 4 Solve by Using Multiplication

a. Solve $5 = \frac{1}{4}y$. Check your solution.

$$5 = \frac{1}{4}y \quad \text{Write the equation.}$$

$$4(5) = 4\left(\frac{1}{4}y\right) \quad \text{Multiply each side by 4.}$$

$$20 = y \quad \text{Simplify.}$$

CHECK $5 = \frac{1}{4}y$ Write the original equation.

$$5 \stackrel{?}{=} \frac{1}{4}(20) \quad \text{Replace } y \text{ with } 20.$$

$$5 = 5 \checkmark \quad \text{Simplify.}$$

b. Solve $\frac{2}{3}x = 7$. Check your solution.

$$\frac{2}{3}x = 7 \quad \text{Write the equation.}$$

$$\frac{3}{2}\left(\frac{2}{3}x\right) = \frac{3}{2}(7) \quad \text{Multiply each side by } \frac{3}{2}.$$

$$x = \frac{21}{2} \quad \text{Simplify.}$$

$$x = 10\frac{1}{2} \quad \text{Simplify. Check the solution.}$$



Concept Check

What is the first step in solving $\frac{1}{8}r = \frac{1}{4}$?



Check for Understanding

Concept Check

1. Name the property of equality that you would use to solve $2 = \frac{3}{4} + x$.
2. **OPEN ENDED** Write an equation that can be solved by multiplying each side by 6.
3. **FIND THE ERROR** Grace and Ling are solving $0.3x = 4.5$.

Grace	Ling
$0.3x = 4.5$	$0.3x = 4.5$
$\frac{0.3x}{3} = \frac{4.5}{3}$	$\frac{0.3x}{0.3} = \frac{4.5}{0.3}$
$x = 1.5$	$x = 15$

Who is correct? Explain your reasoning.

Guided Practice

Solve each equation. Check your solution.

4. $y + 3.5 = 14.9$
5. $b - 5 = 13.7$
6. $\frac{3}{2} = w + \frac{3}{5}$
7. $c - \frac{3}{5} = \frac{5}{6}$
8. $x + \frac{5}{8} = 7\frac{1}{2}$
9. $4\frac{1}{6} = r + 6\frac{1}{4}$
10. $3.5a = 7$
11. $-\frac{1}{6}s = 15$
12. $9 = \frac{3}{4}g$

Application

13. **METEOROLOGY** When a storm struck, the barometric pressure was 28.79 inches. Meteorologists said that the storm caused a 0.36-inch drop in pressure. What was the barometric pressure before the storm?

Practice and Apply

Homework Help

For Exercises	See Examples
14–27, 37–40	1, 2
28–36	3, 4

Extra Practice

See page 736.

Solve each equation. Check your solution.

14. $y + 7.2 = 21.9$
15. $4.7 = a + 7.1$
16. $x - 5.3 = 8.1$
17. $n - 4.72 = 7.52$
18. $t + 3.17 = -3.17$
19. $a - 2.7 = 3.2$
20. $\frac{2}{3} = \frac{1}{8} + b$
21. $m + \frac{7}{12} = -\frac{5}{18}$
22. $g + \frac{2}{3} = 2$
23. $7 = \frac{2}{9} + k$
24. $n - \frac{3}{8} = \frac{1}{6}$
25. $x - \frac{2}{5} = -\frac{8}{15}$
26. $7\frac{1}{3} = c - \frac{4}{5}$
27. $-2 = \frac{3}{10} + f$
28. $\frac{7}{9}k = -\frac{5}{12}$
29. $4.1p = 16.4$
30. $8 = \frac{2}{3}d$
31. $0.4y = 2$
32. $\frac{1}{5}t = 9$
33. $4 = -\frac{1}{8}q$
34. $\frac{1}{3}n = \frac{2}{9}$
35. $\frac{5}{8} = \frac{1}{2}r$
36. $\frac{2}{3}a = 6$
37. $b - 1\frac{1}{2} = 4\frac{1}{4}$
38. $7\frac{1}{2} = r - 5\frac{2}{3}$
39. $3\frac{3}{4} + n = 6\frac{5}{8}$
40. $y + 1\frac{1}{3} = 3\frac{1}{18}$

41. **PUBLISHING** A newspaper is $12\frac{1}{4}$ inches wide and 22 inches long. This is $1\frac{1}{4}$ inches narrower and half an inch longer than the old edition. What were the previous dimensions of the newspaper?



More About . . .

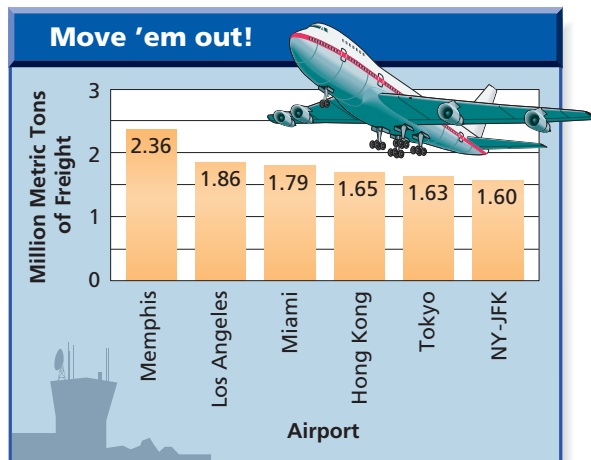


Airports

In 1995, there were 580 million airline passengers. In 2005, there will be an estimated 823 million airline passengers.

Source: Department of Transportation

42. **AIRPORTS** The graph shows the world's busiest cargo airports. What is the difference in cargo handling between Memphis and Tokyo?



Source: Airports Council International

43. **BUSINESS** A store is going out of business. All of the items are marked $\frac{1}{3}$ off the ticketed price. How much would a shirt that was originally priced at \$24.99 cost now?

44. **COOKING** Lucas made $2\frac{1}{2}$ batches of cookies for a bake sale and used $3\frac{3}{4}$ cups of sugar. How much sugar is needed for one batch of cookies?

45. **TRAINS** As a train begins to roll, the cars are "jerked" into motion. Slack is built into the couplings so that the engine does not have to move every car at once. If the slack built into each coupling is 3 inches, how many feet of slack is there between ten freight cars? (*Hint:* Do not include the coupling between the engine and the first car.)

46. **CRITICAL THINKING** The denominator of a fraction is 4 more than the numerator. If both the numerator and denominator are increased by 1, the resulting fraction equals $\frac{1}{2}$. Find the original fraction.

47. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are reciprocals used in solving problems involving music?

Include the following in your answer:

- an example of how fractions are used to compare musical notes, and
- an explanation of how reciprocals are useful in finding the number of vibrations per second needed to produce certain notes.

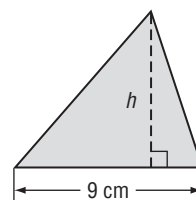


48. Find the value of z in $\frac{5}{6}z = \frac{3}{5}$.

- (A) $\frac{18}{25}$ (B) $\frac{15}{30}$ (C) $\frac{1}{2}$ (D) $1\frac{7}{18}$

49. The area A of the triangle is $33\frac{3}{4}$ square centimeters. Use the formula $A = \frac{1}{2}bh$ to find the height h of the triangle.

- (A) $3\frac{13}{18}$ cm (B) $6\frac{1}{2}$ cm
 (C) $18\frac{13}{18}$ cm (D) $7\frac{1}{2}$ cm



Maintain Your Skills

Mixed Review Find the mean, median, and mode for each set of data. If necessary, round to the nearest tenth. (Lesson 5-8)

50. 2, 8, 5, 18, 3, 5, 6

51. 11, 12, 12, 14, 16, 11, 15

52. 0.9, 0.5, 0.7, 0.4, 0.3, 0.2

53. 56, 77, 60, 60, 72, 100

Find each sum or difference. Write in simplest form. (Lesson 5-7)

54. $\frac{3}{5} + \frac{1}{3}$

55. $\frac{5}{6} - \frac{1}{8}$

56. $-4\frac{1}{4} - \frac{1}{6}$

57. $\frac{5}{9} + \left(-\frac{1}{12}\right)$

58. $-3\frac{3}{4} + \left(-2\frac{1}{8}\right)$

59. $8\frac{9}{10} - 1\frac{1}{6}$

60. **ALGEBRA** Evaluate $a - b$ if $a = 9\frac{5}{6}$ and $b = 1\frac{1}{6}$. (Lesson 5-5)

61. **HEALTH** According to the National Sleep Foundation, teens should get approximately 9 hours of sleep each day. What fraction of the day is this? Write in simplest form. (Lesson 4-5)

ALGEBRA Solve each equation. Check your solution. (Lesson 3-5)

62. $3t - 6 = 15$

63. $8 = \frac{k}{-2} + 5$

64. $9n - 13n = 4$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Divide. If necessary, write as a fraction in simplest form. (To review **dividing integers**, see Lesson 2-5.)

65. $-18 \div 3$

66. $24 \div (-2)$

67. $-20 \div (-4)$

68. $-55 \div (-5)$

69. $-12 \div 36$

70. $9 \div (-81)$

Practice Quiz 2

Lessons 5-6 through 5-9

Find the least common multiple of each set of numbers. (Lesson 5-6)

1. 8, 9

2. 12, 30

3. 2, 10

4. 6, 8

Find each sum or difference. Write in simplest form. (Lesson 5-7)

5. $\frac{3}{4} + \frac{1}{16}$

6. $-\frac{3}{7} + \frac{5}{14}$

7. $1\frac{1}{5} - \left(-\frac{7}{15}\right)$

8. $6\frac{3}{4} - 2\frac{1}{6}$

9. **WEATHER** The low temperatures on March 24 for twelve different cities are recorded at the right. Find the mean, median, and mode. If necessary, round to the nearest tenth. (Lesson 5-8)

Low Temperatures
(°F)

31	29	30	22
45	35	29	30
40	29	31	38

10. **ALGEBRA** Solve $a - 1\frac{1}{3} = 4\frac{1}{6}$. (Lesson 5-9)

Arithmetic and Geometric Sequences

What You'll Learn

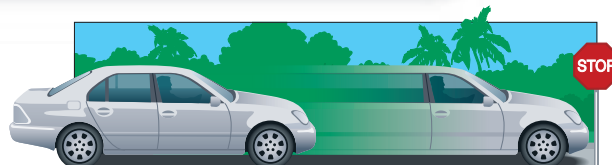
- Find the terms of arithmetic sequences.
- Find the terms of geometric sequences.

Vocabulary

- sequence
- arithmetic sequence
- term
- common difference
- geometric sequence
- common ratio

How can sequences be used to make predictions?

The table shows the distance a car moves during the time it takes to apply the brakes and while braking.



Speed (mph)	Reaction Distance (ft)	Braking Distance (ft)
20	20	20
30	30	45
40	40	80
50	50	125
60	60	180

- What is the reaction distance for a car going 70 mph?
- What is the braking distance for a car going 70 mph?
- What is the difference in reaction distances for every 10-mph increase in speed?
- Describe the braking distance as speed increases.

ARITHMETIC SEQUENCES A **sequence** is an ordered list of numbers. An **arithmetic sequence** is a sequence in which the difference between any two consecutive terms is the same. So, you can find the next term in the sequence by adding the same number to the previous term.

Each number is called a **term** of the sequence.

20, 30, 40, 50, 60, ...

+10 +10 +10 +10

The difference is called the **common difference**.

Example 1 Identify an Arithmetic Sequence

State whether the sequence 8, 5, 2, -1, -4, ... is arithmetic. If it is, state the common difference and write the next three terms.

8, 5, 2, -1, -4 Notice that $5 - 8 = -3$, $2 - 5 = -3$, and so on.

$-3 \quad -3 \quad -3 \quad -3$

The terms have a common difference of -3 , so the sequence is arithmetic. Continue the pattern to find the next three terms.

-4, -7, -10, -13

$-3 \quad -3 \quad -3$

The next three terms of the sequence are -7 , -10 , and -13 .



Example 2 Identify an Arithmetic Sequence

State whether the sequence 1, 2, 4, 7, 11, ... is arithmetic. If it is, write the next three terms of the sequence.

1, 2, 4, 7, 11



The terms do not have a common difference.

The sequence is not arithmetic. However, if the pattern continues, the next three differences will be 5, 6, and 7.

11, 16, 22, 29



The next three terms are 16, 22, and 29.

GEOMETRIC SEQUENCES A **geometric sequence** is a sequence in which the quotient of any two consecutive terms is the same. So, you can find the next term in the sequence by multiplying the previous term by the same number.

1, 4, 16, 64, 256, ...



The quotient is called the **common ratio**.



Concept Check Name the common ratio in the sequence 10, 20, 40, 80, 160,

Example 3 Identify Geometric Sequences

a. State whether the sequence $-2, 6, -18, 54, \dots$ is geometric. If it is, state the common ratio and write the next three terms.

$-2, 6, -18, 54$ Notice that $6 \div (-2) = -3$, $-18 \div 6 = -3$, and

$54 \div (-18) = -3$.

$\times (-3) \times (-3) \times (-3)$

The common ratio is -3 , so the sequence is geometric. Continue the pattern to find the next three terms.

54, $-162, 486, -1458$



The next three terms are $-162, 486$, and -1458 .

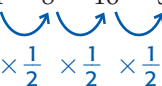
b. State whether the sequence $20, 10, 5, \frac{5}{2}, \frac{5}{4}, \dots$ is geometric. If it is, state the common ratio and write the next three terms.

$20, 10, 5, \frac{5}{2}, \frac{5}{4}$



The common ratio is $\frac{1}{2}$ or 0.5, so the sequence is geometric. Continue the pattern to find the next three terms.

$\frac{5}{4}, \frac{5}{8}, \frac{5}{16}, \frac{5}{32}$



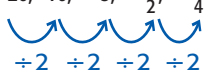
The next three terms are $\frac{5}{8}, \frac{5}{16}$, and $\frac{5}{32}$.

Study Tip

Alternative Method

Multiplying by $\frac{1}{2}$ is the same as dividing by 2, so the following is also true.

$20, 10, 5, \frac{5}{2}, \frac{5}{4}$



Check for Understanding

Concept Check

1. Compare and contrast arithmetic and geometric sequences.
2. **OPEN ENDED** Describe the terms of a geometric sequence whose common ratio is a fraction or decimal between 0 and 1. Then write four terms of such a sequence and name the common ratio.

Guided Practice

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio and write the next three terms of the sequence.

3. 3, 7, 11, 15, ...
4. 1, 3, 9, 27, ...
5. 6, 8, 12, 18, ...
6. 13, 8, 3, -2, ...
7. $3, \frac{8}{3}, \frac{7}{3}, 2, \dots$
8. 48, 12, 3, $\frac{3}{4}, \dots$

Application

9. **CARS** A new car is worth only about 0.82 of its value from the previous year during the first three years. Approximately how much will a \$20,000 car be worth in 3 years? **Source:** www.caprice.com

Practice and Apply

Homework Help

For Exercises 10–29

See Examples 1–3

Extra Practice See page 736.

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio and write the next three terms of the sequence.

10. 2, 5, 8, 11, ...
11. -6, 5, 16, 27, ...
12. $\frac{1}{2}, 1, 2, 4, \dots$
13. 2, 6, 18, 54, ...
14. 18, 11, 4, -3, ...
15. 25, 22, 19, 16, ...
16. $4, 1, \frac{1}{4}, \frac{1}{16}, \dots$
17. $-5, 1, -\frac{1}{5}, \frac{1}{25}, \dots$
18. $\frac{1}{2}, 1, \frac{3}{2}, 2, \dots$
19. $0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \dots$
20. 0.75, 1.5, 2.25, ...
21. 4.5, 4.0, 3.5, 3.0, ...
22. 11, 14, 19, 26, ...
23. 17, 16, 14, 11, ...
24. 24, 12, 6, 3, ...
25. $18, -6, 2, -\frac{2}{3}, \dots$
26. 0.1, 0.3, 0.9, 2.7, ...
27. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$
28. **PHYSICAL SCIENCE** A ball bounces back 0.75 of its height on every bounce. If a ball is dropped from 160 feet, how high does it bounce on the third bounce?
29. **TELEPHONE RATES** For an overseas call, WorldTel charges \$6 for the first minute and then \$3 for each additional minute.
 - a. Is the cost an arithmetic or geometric sequence? Explain.
 - b. How much would a 15-minute call cost?
30. **CRITICAL THINKING** The sum of four numbers in an arithmetic sequence is 42. What could the numbers be? Give two different examples.
31. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can sequences be used to make predictions?

Include the following in your answer:

- a discussion of common difference and common ratio, and
- examples of sequences occurring in nature.



32. State the next term in the sequence 56, 48, 40, 32, ...

(A) 20

(B) 22

(C) 24

(D) 28

33. Which statement is true as the side length of a square increases?

(A) The perimeter values form an arithmetic sequence.

(B) The perimeter values form a geometric sequence.

(C) The area values form an arithmetic sequence.

(D) The area values form a geometric sequence.

Side Length	Perimeter	Area
1	4	1
2	8	4
3	12	9
4	16	16
5	20	25

**Extending
the Lesson**

In an arithmetic sequence, d represents the common difference, a_1 represents the first term, a_2 represents the second term, and so on. For example, in the sequence 6, 9, 12, 15, 18, 21, ..., $a_1 = 6$ and $d = 3$.

34. Use the information in the table to write an expression for finding the n th term of an arithmetic sequence.

Arithmetic Sequence	numbers	6	9	12	15	...	
	symbols	a_1	a_2	a_3	a_4	...	a_n
Expressed in Terms of d and the First Term	numbers	$6 + 0(3)$	$6 + 1(3)$	$6 + 2(3)$	$6 + 3(3)$...	$6 + (n - 1)(3)$
	symbols	$a_1 + 0d$	$a_1 + 1d$	$a_1 + 2d$	$a_1 + 3d$...	?

35. Use the expression you wrote in Exercise 34 to find the 9th term of an arithmetic sequence if $a_1 = 16$ and $d = 5$.

Maintain Your Skills

Mixed Review ALGEBRA Solve each equation. Check your solution. (Lesson 5-9)

36. $\frac{7}{6} = y + \frac{5}{12}$

37. $k - 4.1 = -9.38$

38. $40.3 = 6.2x$

39. $\frac{3}{4}b = 7\frac{1}{2}$

Find the mean, median, and mode for each set of data. If necessary, round to the nearest tenth. (Lesson 5-8)

40. 11, 45, 62, 12, 47, 8, 12, 35

41. 2.3, 3.6, 4.1, 3.6, 2.9, 3.0

42. **ALGEBRA** Find the LCD of $\frac{1}{6a^2}$ and $\frac{5}{9a^3}$. (Lesson 5-6)

43. **HOME REPAIR** Cole is installing shelves in his closet. Because of the shape of the closet, the three shelves measure $34\frac{3}{8}$ inches, $33\frac{7}{8}$ inches, and $34\frac{5}{8}$ inches. What length of lumber does he need to buy? (Lesson 5-5)

Find each product or quotient. Express using exponents. (Lesson 4-6)

44. $3^4 \cdot 3^2$

45. $\frac{b^5}{b^2}$

46. $2x^3(5x^2)$

Find the greatest common factor of each set of numbers. (Lesson 4-4)

47. 36, 42

48. 9, 24

49. 60, 45, 30



Algebra Activity

A Follow-Up of Lesson 5-10

Fibonacci Sequence

A special sequence that is neither arithmetic nor geometric is called the **Fibonacci sequence**. Each term in the sequence is the sum of the previous two terms, beginning with 1.

$$\begin{array}{cccccccc} 1, & 1, & 2, & 3, & 5, & 8, & 13, & \dots \\ & \frown & \frown & \frown & \frown & \frown & & \\ & +1 & +1 & +2 & +3 & +5 & & \end{array}$$

Collect the Data

- Examine an artichoke, a pineapple, a pinecone, or the seeds in the center of a sunflower.
- For each item, count the number of spiral rows and record your data in a table. If possible, count the rows that spiral up from left to right and count the rows that spiral up from right to left. *Note:* It may be helpful to use a marker to keep track of the rows as you are counting.



Analyze the Data

1. What do you notice about the number of rows in each item?
2. Compare your data with the data of the other students. How do they compare?

Make a Conjecture

3. Have a discussion with other students to determine the relationship between the number of rows in sunflowers, pinecones, pineapples, and artichokes and the Fibonacci sequence.

Extend the Activity

Numbers in an arithmetic sequence have a common difference, and numbers in a geometric sequence have a common ratio. The numbers in a Fibonacci sequence have a different kind of pattern.

4. Write the first fifteen terms in the Fibonacci sequence.
5. Use a calculator to divide each term by the previous term. Make a list of the quotients. If necessary, round to seven decimal places.
6. Describe the pattern in the quotients.



7. **RESEARCH** Find the definition of the **golden ratio**. What is the relationship between the numbers in the Fibonacci sequence and the golden ratio?

Vocabulary and Concept Check

arithmetic sequence (p. 249)
bar notation (p. 201)
common difference (p. 249)
common multiples (p. 226)
common ratio (p. 250)
dimensional analysis (p. 212)
Fibonacci sequence (p. 253)
geometric sequence (p. 250)

least common denominator (LCD) (p. 227)
least common multiple (LCM) (p. 226)
mean (p. 238)
measures of central tendency (p. 238)
median (p. 238)
mixed number (p. 200)
mode (p. 238)
multiple (p. 226)

multiplicative inverse (p. 215)
period (p. 201)
rational number (p. 205)
reciprocals (p. 215)
repeating decimal (p. 201)
sequence (p. 249)
term (p. 249)
terminating decimal (p. 200)

Choose the correct term to complete each sentence.

1. A mixed number is an example of a (whole, rational) number.
2. The decimal 0.900 is a (terminating, repeating) decimal.
3. $\frac{x}{15}$ is an example of an (algebraic fraction, integer).
4. The numbers 12, 15, and 18 are (factors, multiples) of 3.
5. To add unlike fractions, rename the fractions using the (LCD, GCF).
6. The product of a number and its multiplicative inverse is (0, 1).
7. To divide by a fraction, multiply the number by the (reciprocal, LCD) of the fraction.
8. The (median, mode) is the number that occurs most often in a set of data.
9. A common difference is found between terms in a(n) (arithmetic, geometric) sequence.

Lesson-by-Lesson Review

5-1

Writing Fractions as Decimals

See pages
200–204.

Concept Summary

- Any fraction or mixed number can be written as a terminating or repeating decimal.

Example

Write $2\frac{7}{10}$ as a decimal.

$$2\frac{7}{10} = 2 + \frac{7}{10}$$

Write as the sum of an integer and a fraction.

$$= 2 + 0.7 \text{ or } 2.7$$

Write $\frac{7}{10}$ as a decimal and add.

Exercises Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal. See Examples 1–3 on pages 200 and 201.

10. $\frac{7}{8}$

11. $\frac{9}{20}$

12. $\frac{2}{3}$

13. $-\frac{7}{15}$

14. $8\frac{3}{25}$

15. $6\frac{4}{11}$

5-2 Rational Numbers

See pages
205–209.

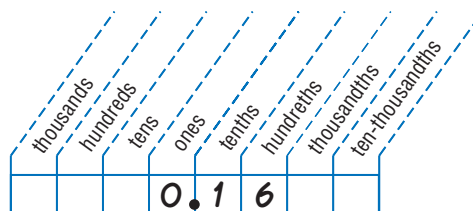
Concept Summary

- Any number that can be written as a fraction is a rational number.
- Decimals that are terminating or repeating are rational numbers.

Example

Write 0.16 as a fraction in simplest form.

$$\begin{aligned} 0.16 &= \frac{16}{100} && 0.16 \text{ is 16 hundredths.} \\ &= \frac{4}{25} && \text{Simplify. The GCF of 16 and 100 is 4.} \end{aligned}$$



Exercises Write each decimal as a fraction or mixed number in simplest form. See Examples 2 and 3 on page 206.

- | | | | |
|----------------------|----------------------|-----------------------|-----------------------|
| 16. 0.23 | 17. 0.6 | 18. -0.05 | 19. 0.125 |
| 20. 2.36 | 21. 4.44 | 22. -8.002 | 23. 0.555... |
| 24. $0.\overline{3}$ | 25. $1.\overline{7}$ | 26. $0.\overline{72}$ | 27. $3.\overline{36}$ |

5-3 Multiplying Rational Numbers

See pages
210–214.

Concept Summary

- To multiply fractions, multiply the numerators and multiply the denominators.
- Dimensional analysis is a useful way to keep track of units while computing.

Example

Find $\frac{4}{5} \cdot 3\frac{1}{3}$. Write the product in simplest form.

$$\begin{aligned} \frac{4}{5} \cdot 3\frac{1}{3} &= \frac{4}{5} \cdot \frac{10}{3} && \text{Rename } 3\frac{1}{3} \text{ as an improper fraction.} \\ &= \frac{4}{\cancel{5}^2} \cdot \frac{\cancel{10}^2}{3} && \text{Divide by the GCF, 5.} \\ &= \frac{8}{3} \text{ or } 2\frac{2}{3} && \text{Multiply and then simplify.} \end{aligned}$$

Exercises Find each product. Write in simplest form.

See Examples 1–5 on pages 210 and 211.

- | | | | |
|--|---------------------------------------|---------------------------------------|--|
| 28. $\frac{2}{3} \cdot \frac{1}{8}$ | 29. $-\frac{7}{15} \cdot \frac{5}{9}$ | 30. $\frac{6}{11} \cdot \frac{2}{15}$ | 31. $8 \cdot \frac{4}{5}$ |
| 32. $-1\frac{5}{6} \cdot 9$ | 33. $\frac{6}{7} \cdot \frac{14}{9}$ | 34. $\frac{8}{9} \cdot \frac{5}{12}$ | 35. $2\frac{1}{4} \cdot \left(-\frac{4}{3}\right)$ |
| 36. $\frac{14}{15} \cdot 3\frac{2}{7}$ | 37. $2\frac{5}{6} \cdot 3\frac{1}{3}$ | 38. $\frac{ab}{4} \cdot \frac{2}{bc}$ | 39. $\frac{x^2}{r} \cdot \frac{r}{x}$ |

5-4 Dividing Rational Numbers

See pages
215–219.

Concept Summary

- The product of a number and its multiplicative inverse or reciprocal is 1.
- To divide by a fraction, multiply by its reciprocal.

Example

Find $\frac{6}{7} \div \frac{3}{4}$. Write the quotient in simplest form.

$$\begin{aligned} \frac{6}{7} \div \frac{3}{4} &= \frac{\cancel{6}^2}{7} \cdot \frac{4}{\cancel{3}_1} && \text{Multiply by the reciprocal of } \frac{3}{4}, \frac{4}{3}. \\ &= \frac{8}{7} \text{ or } 1\frac{1}{7} && \text{Multiply and then simplify.} \end{aligned}$$

Exercises Find each quotient. Write in simplest form.

See Examples 2–5 on pages 216 and 217.

40. $\frac{4}{9} \div \frac{1}{3}$	41. $\frac{2}{5} \div \left(-\frac{1}{15}\right)$	42. $\frac{6}{13} \div \frac{8}{9}$	43. $5 \div \left(-1\frac{1}{3}\right)$
44. $\frac{11}{18} \div 4\frac{1}{2}$	45. $-3\frac{3}{5} \div \frac{6}{7}$	46. $\frac{n}{8} \div \frac{n}{32}$	47. $\frac{2}{7x} \div \frac{3}{2}$

5-5 Adding and Subtracting Like Fractions

See pages
220–224.

Concept Summary

- To add like fractions, add the numerators and write the sum over the denominator.
- To subtract like fractions, subtract the numerators and write the sum over the denominator.

Example

Find $3\frac{7}{8} + 9\frac{3}{8}$. Write the sum in simplest form.

Estimate: $4 + 9 = 13$

$$\begin{aligned} 3\frac{7}{8} + 9\frac{3}{8} &= (3 + 9) + \left(\frac{7}{8} + \frac{3}{8}\right) && \text{Add the whole numbers and fractions separately.} \\ &= 12 + \frac{7+3}{8} && \text{The denominators are the same. Add the numerators.} \\ &= 12\frac{10}{8} && \text{Simplify.} \\ &= 13\frac{2}{8} \text{ or } 13\frac{1}{4} && \text{Simplify.} \end{aligned}$$

Exercises Find each sum or difference. Write in simplest form.

See Examples 1–3 and 5 on pages 220 and 221.

48. $\frac{5}{18} + \frac{11}{18}$	49. $\frac{7}{9} + \left(-\frac{2}{9}\right)$	50. $\frac{19}{20} - \frac{17}{20}$	51. $1\frac{16}{21} - \frac{9}{21}$
52. $-\frac{12}{17} + \frac{10}{17}$	53. $8\frac{3}{10} - 5\frac{7}{10}$	54. $\frac{7t}{15} + \frac{t}{15}$	55. $\frac{5}{3x} - \frac{1}{3x}$



5-6 Least Common MultipleSee pages
226–230.**Concept Summary**

- The LCM of two numbers is the least nonzero multiple common to both numbers.
- To compare fractions with unlike denominators, write the fractions using the LCD and compare the numerators.

ExampleReplace \bullet with $<$, $>$, or $=$ to make $\frac{7}{15} \bullet \frac{5}{9}$ a true statement.The LCD is $3^2 \cdot 5$ or 45. Rewrite the fractions using the LCD.

$$\frac{7}{15} \cdot \frac{3}{3} = \frac{21}{45} \qquad \frac{5}{9} \cdot \frac{5}{5} = \frac{25}{45}$$

Since $21 < 25$, $\frac{21}{45} < \frac{25}{45}$. So, $\frac{7}{15} < \frac{5}{9}$.**Exercises** Find the least common multiple (LCM) of each pair of numbers or monomials. See Examples 1 and 2 on pages 226 and 227.

56. 4, 18

57. 24, 20

58. $4a$, $6a$

59. $7c^2$, $21c$

Replace each \bullet with $<$, $>$, or $=$ to make a true statement.

See Example 5 on page 228.

60. $\frac{3}{8} \bullet \frac{5}{12}$

61. $\frac{2}{9} \bullet \frac{4}{15}$

62. $\frac{5}{20} \bullet \frac{1}{4}$

63. $\frac{3}{7} \bullet \frac{8}{21}$

5-7 Adding and Subtracting Unlike FractionsSee pages
232–236.**Concept Summary**

- To add or subtract fractions with unlike denominators, rename the fractions with the LCD. Then add or subtract.

ExampleFind $\frac{7}{9} - \frac{5}{12}$.

$$\frac{7}{9} - \frac{5}{12} = \frac{7}{9} \cdot \frac{4}{4} - \frac{5}{12} \cdot \frac{3}{3} \quad \text{The LCD is } 3^2 \cdot 2^2 \text{ or } 36.$$

$$= \frac{28}{36} - \frac{15}{36} \quad \text{Rename the fractions using the LCD.}$$

$$= \frac{13}{36} \quad \text{Subtract the like fractions.}$$

Exercises Find each sum or difference. Write in simplest form.

See Examples 2–5 on pages 233 and 234.

64. $\frac{1}{3} + \frac{5}{6}$

65. $\frac{11}{12} + \frac{3}{4}$

66. $\frac{7}{8} - \frac{5}{6}$

67. $3\frac{7}{12} - \frac{3}{4}$

68. $-\frac{3}{7} + \left(-\frac{11}{14}\right)$

69. $1\frac{2}{5} - \left(-\frac{1}{3}\right)$

70. $5\frac{1}{2} - 2\frac{2}{3}$

71. $-2\frac{1}{6} + 5\frac{1}{3}$



- Extra Practice, see pages 733–736.
- Mixed Problem Solving, see page 762.

5-8 Measures of Central Tendency

See pages
238–242.

Concept Summary

- The mean, median, and mode can be used to describe sets of data.

Example Find the mean, median, and mode of 8, 4, 2, 2, and 10.

$$\text{mean: } \frac{8 + 4 + 2 + 2 + 10}{5} \text{ or } 5.2 \qquad \text{median: } 4 \qquad \text{mode: } 2$$

Exercises Find the mean, median, and mode for each set of data. If necessary, round to the nearest tenth. See Example 1 on page 238.

72. 4, 5, 7, 3, 9, 11, 23, 37 73. 3.6, 7.2, 9.0, 5.2, 7.2, 6.5, 3.6

5-9 Solving Equations with Rational Numbers

See pages
244–248.

Concept Summary

- To solve an equation, use inverse operations to isolate the variable.

Example Solve $1.6x = 8$. Check your solution.

$1.6x = 8$	Write the equation.	CHECK	$1.6x = 8$	Write the equation.
$\frac{1.6x}{1.6} = \frac{8}{1.6}$	Divide each side by 1.6.		$1.6(5) \stackrel{?}{=} 8$	Replace x with 5.
$x = 5$	Simplify.		$8 = 8 \checkmark$	Simplify.

Exercises Solve each equation. Check your solution.

See Examples 1–4 on pages 244 and 245.

74. $\frac{1}{2} = a + \frac{3}{8}$ 75. $x - 1.5 = 1.75$ 76. $0.2t = 6$ 77. $2 = -\frac{4}{5}n$

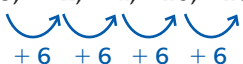
5-10 Arithmetic and Geometric Sequences

See pages
249–252.

Concept Summary

- In an arithmetic sequence, the terms have a common difference.
- In a geometric sequence, the terms have a common ratio.

Example State whether $-8, -2, 4, 10, 16, \dots$ is *arithmetic, geometric, or neither*. If it is arithmetic or geometric, write the next three terms.

$-8, -2, 4, 10, 16$ The common difference is 6, so the sequence is arithmetic.


 $+6 \quad +6 \quad +6 \quad +6$ The next three terms are $16 + 6$ or 22, $22 + 6$ or 28, and $28 + 6$ or 34.

Exercises State whether each sequence is *arithmetic, geometric, or neither*. If it is arithmetic or geometric, state the common difference or common ratio and write the next three terms of the sequence. See Examples 1–3 on pages 249 and 250.

78. 4, 9, 14, 19, ... 79. 1, 3, 9, 27, ... 80. $32, 8, 2, \frac{1}{2}, \dots$ 81. $-6, -5, -2, 3, \dots$

Vocabulary and Concepts

1. State the difference between a terminating and a repeating decimal.
2. Describe how to add fractions with unlike denominators.
3. Define *geometric sequence*.

Skills and Applications

Write each fraction or mixed number as a decimal. Use a bar to show a repeating decimal.

4. $\frac{9}{20}$

5. $-\frac{7}{8}$

6. $4\frac{2}{9}$

Write each decimal as a fraction or mixed number in simplest form.

7. 0.24

8. 5.06

9. $-2.\bar{3}$

Replace each \bullet with $<$, $>$, or $=$ to make a true sentence.

10. $0.6 \bullet \frac{2}{3}$

11. $-1\frac{5}{8} \bullet -1.6$

Find the least common denominator (LCD) of each pair of fractions.

12. $\frac{5}{6}, \frac{2}{9}$

13. $\frac{9}{4a^2}, \frac{2}{3ab}$

Find each product, quotient, sum, or difference. Write in simplest form.

14. $\frac{5}{8} \cdot \frac{6}{11}$

15. $\frac{5}{8} + \frac{1}{8}$

16. $\frac{7}{9} \div \frac{4}{15}$

17. $3\frac{5}{6} + 1\frac{2}{9}$

18. $\frac{ab}{9} \div \frac{b}{3}$

19. $\frac{11x}{3y} - \frac{8x}{3y}$

For Exercises 20 and 21, use the data set {20.5, 18.6, 16.3, 4.8, 19.1, 17.3, 20.5}.

20. Find the mean, median, and mode. If necessary, round to the nearest tenth.
21. Identify an extreme value and describe how it affects the mean.

Solve each equation. Check your solution.

22. $x + 4.3 = 9.8$

23. $12 = 0.75x$

24. $3.1m = 12.4$

25. $p - \frac{4}{5} = \frac{2}{3}$

26. $-\frac{3}{8} = \frac{5}{3}a$

27. $y - 2\frac{1}{4} = 1\frac{5}{6}$

State whether each sequence is *arithmetic*, *geometric*, or *neither*. If it is arithmetic or geometric, state the common difference or common ratio and write the next three terms of the sequence.

28. 2, 8, 32, 128, ...

29. 5.5, 4.9, 4.3, 3.7, ...

30. 1, 2, 4, 7, ...

31. **TRAVEL** Max drives 6 hours at an average rate of 65 miles per hour. What is the distance Max travels? Use $d = rt$.
32. **SEWING** Allie needs $4\frac{2}{3}$ yards of lace to finish sewing the edges of a blanket. She only has $\frac{3}{4}$ of that amount. How much lace does Allie have?
33. **STANDARDIZED TEST PRACTICE** Write the sum of $-6\frac{1}{4}$ and $-9\frac{3}{20}$.

(A) $-15\frac{2}{5}$

(B) $-15\frac{1}{4}$

(C) $-16\frac{2}{5}$

(D) $-3\frac{1}{5}$



Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

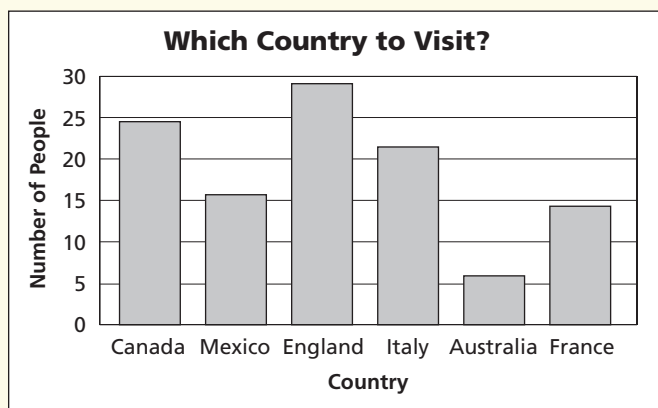
1. Kenzie paid for a CD with a \$20 bill. She received 3 dollars, 3 dimes, and 2 pennies in change. How much did she pay for the CD?

(Prerequisite Skill, p. 707)

- (A) \$16.68 (B) \$16.88
(C) \$17.68 (D) \$17.88

2. A survey of 110 people asked which country they would most like to visit. The bar graph shows the data. How many people chose Canada, England, or Australia as the country they would most like to visit?

(Prerequisite Skill, pp. 722–723)



- (A) 52 (B) 58
(C) 68 (D) 74


Test-Taking Tip

(A) (B) (C) (D)

Question 2

When an item includes a graph, scan the graph to see what kind of information it includes and how the information is organized. Don't try to memorize the information. Read each answer choice and compare it with the graph to see if the information in the answer choice is correct. Eliminate any wrong answer choices.

3. The low temperature overnight was -1°F . Each night for the next four nights, the low temperature was 7° lower than the previous night. What was the low temperature during the last night? (Lesson 2-4)

- (A) -29° (B) -27° (C) -22° (D) -8°

4. Write $\frac{6t^4}{18ts}$ in simplest form. (Lesson 4-5)

- (A) $\frac{1}{3}t^3s$ (B) $3t^5s$
(C) $\frac{t^3}{3s}$ (D) $\frac{t^5}{3s}$

5. What is 8×10^{-2} in standard notation? (Lesson 4-8)

- (A) 0.008 (B) 0.08
(C) 0.8 (D) 800

6. Add $\frac{2}{3} + \frac{1}{4} + \frac{5}{6}$. (Lesson 5-7)

- (A) $\frac{2}{3}$ (B) $\frac{5}{4}$
(C) $\frac{7}{4}$ (D) $\frac{11}{6}$

7. Evaluate $\frac{1}{4}(2 - x) - x$ if $x = \frac{1}{2}$. (Lesson 5-7)

- (A) $-\frac{1}{2}$ (B) $-\frac{1}{8}$ (C) $\frac{1}{8}$ (D) $\frac{1}{2}$

8. Antonia read four books that had the following number of pages: 324, 375, 420, 397. What is the mean number of pages in these books? (Lesson 5-8)

- (A) 375 (B) 379
(C) 380 (D) 386

9. Which sequence is a geometric sequence with a common ratio of 2? (Lesson 5-10)

- (A) 2, -4 , 8, -16 , ... (B) 2, 4, 6, 8, 10, ...
(C) 3, 5, 7, 9, 11, ... (D) 3, 6, 12, 24, ...

10. State the next three terms in the sequence 128, 32, 8, 2, (Lesson 5-10)

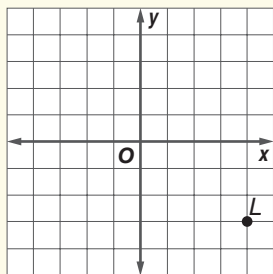
- (A) $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{32}$ (B) $\frac{1}{2}$, $\frac{1}{16}$, $\frac{1}{32}$
(C) $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$ (D) $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

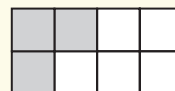
11. Nate is cutting shelves from a board that is 15 feet long. Each shelf is 3 feet 4 inches long. What is the greatest number of shelves he can make from the board?
(Prerequisite Skill, pp. 720–721)

12. Write the ordered pair that names point L .
(Lesson 2-6)



13. Find x if $3x + 4 = 28$. (Lesson 3-5)
14. Write an equation to represent the total number of Calories t in one box of snack crackers. The box of crackers contains 8 servings. Each serving has 125 Calories.
(Lesson 3-6)
15. On Saturday, Juan plans to drive 275 miles at a rate of 55 miles per hour. How many hours will his trip take? Use the formula $d = rt$, where d represents the distance, r represents rate, and t represents the time.
(Lesson 3-7)
16. Write $4^2 \times 5^3$ as a product of prime factors without using exponents. (Lesson 4-3)
17. The average American worker spends 44 minutes traveling to and from work each day. What fraction of the day is this?
(Lesson 4-5)
18. The Saturn V rocket that took the Apollo astronauts to the moon weighed 6.526×10^6 pounds at lift-off. Write its weight in standard notation. (Lesson 4-8)

19. Write a decimal to represent the shaded portion of the figure below. (Lesson 5-1)



20. One elevator in a 40-story building is programmed to stop at every third floor. Another is programmed to stop at every fourth floor. Which floors in the building are served by both elevators?
(Lesson 5-6)
21. Beth has $\frac{3}{4}$ cup of grated cheese. She needs $2\frac{1}{2}$ cups of grated cheese for making pizzas. How many more cups does she need?
(Lesson 5-7)
22. The number of tickets sold for each performance of the Spring Music Fest are 352, 417, 307, 367, 433, and 419. What is the median number of tickets sold per performance?
(Lesson 5-8)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

23. During seven regular season games, the Hawks basketball team scored the points shown in the table below. (Lesson 5-8)

Game	1	2	3	4	5	6	7
Points	68	60	73	74	64	78	73

- Find the mean, median, and mode of these seven scores.
- During the first playoff game after the regular season, the Hawks scored only 40 points. Find the mean, median, and mode of all eight scores.
- Which of these three measures of central tendency—mean, median, or mode—changed the most as a result of the playoff score? Explain your answer.
- Does the mean score or the median score best represent the team’s scores for all eight games? Explain your answer.

