UNIT 2

Algebra and Rational Numbers

Most of the numbers you encounter in the real world are *rational numbers*—fractions, decimals, and percents. In this unit, you will build on your foundation of algebra so that it includes rational numbers.

CONTENTS

WebQuest Internet Project

Kids Gobbling Empty Calories

"Teens are eating 150 more calories a day in snacks than they did two decades ago. And kids of all ages are munching on more of the richer goodies between meals than children did in the past."

Source: *USA TODAY*, April 30, 2001

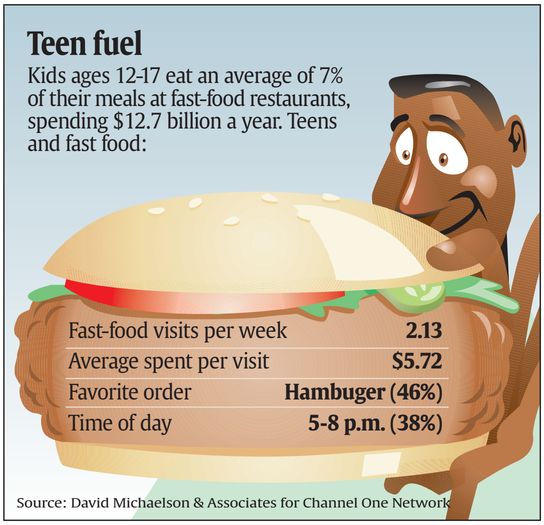
In this project, you will be exploring how rational numbers are related to nutrition.

Log on to www.pre-alg.com/webquest. Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 2.

Lesson	4-5	5-8	6-7
Page	173	242	301

USA TODAY Snapshots®



Teen fuel

Kids ages 12-17 eat an average of 7% of their meals at fast-food restaurants, spending $12.7 billion a year. Teens and fast food:

Fast-food visits per week	**2.13**
Average spent per visit	**$5.72**
Favorite order	**Hamburger (46%)**
Time of day	**5-8 p.m. (38%)**

Source: David Michaelson & Associates for Channel One Network

By Anne R. Carey and Jerry Mosemak, USA TODAY

CONTENTS

Chapter

4

Factors and Fractions

What You'll Learn

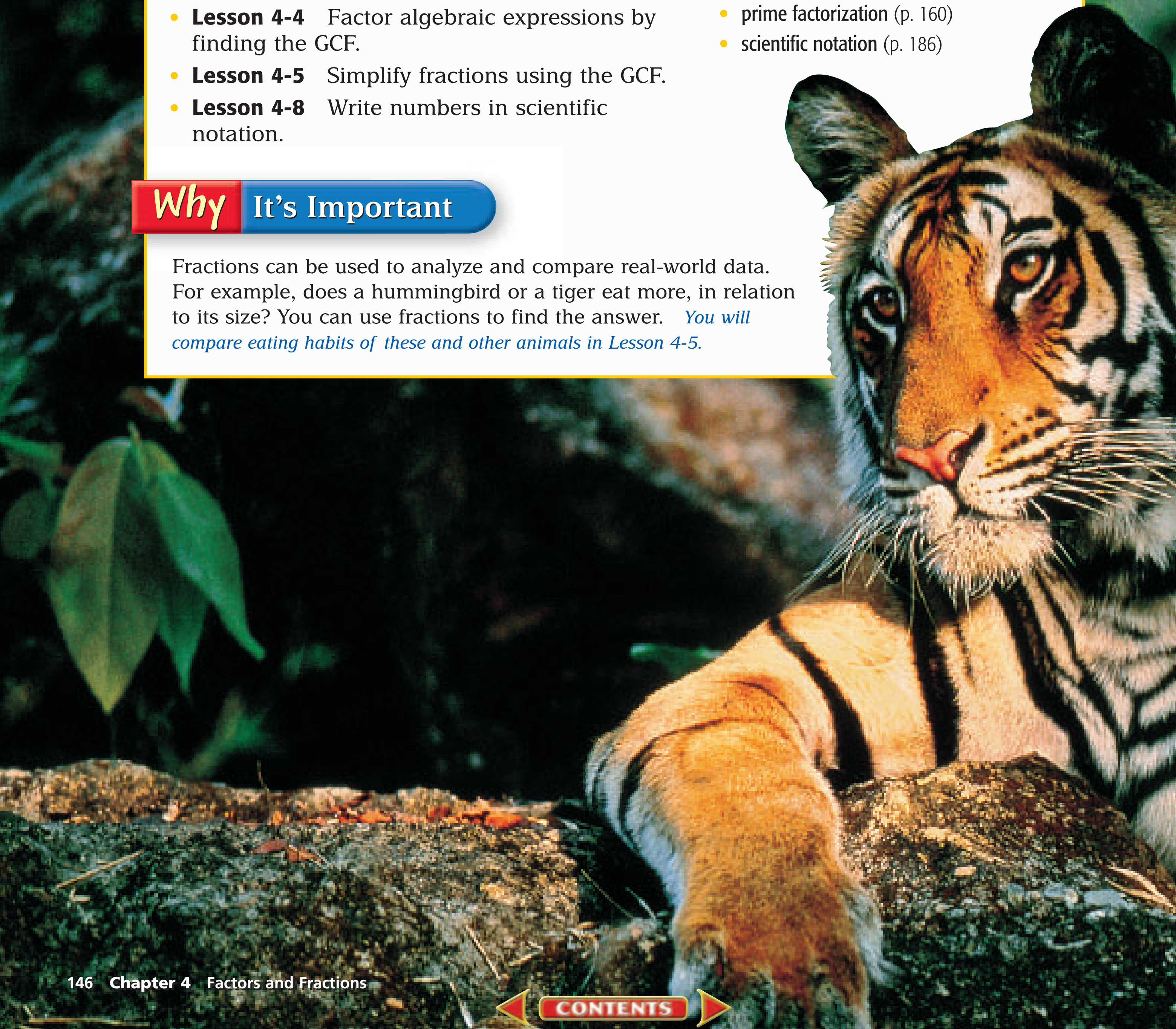
- **Lessons 4-1, 4-3, and 4-6** Identify, factor, multiply, and divide monomials.
- **Lessons 4-2 and 4-7** Evaluate expressions containing exponents.
- **Lesson 4-4** Factor algebraic expressions by finding the GCF.
- **Lesson 4-5** Simplify fractions using the GCF.
- **Lesson 4-8** Write numbers in scientific notation.

Key Vocabulary

- **factors** (p. 148)
- **monomial** (p. 150)
- **power** (p. 153)
- **prime factorization** (p. 160)
- **scientific notation** (p. 186)

Why It's Important

Fractions can be used to analyze and compare real-world data. For example, does a hummingbird or a tiger eat more, in relation to its size? You can use fractions to find the answer. *You will compare eating habits of these and other animals in Lesson 4-5.*



CONTENTS

Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 4.

For Lesson 4-1 — Distributive Property

Simplify. *(For review, see Lesson 3-1.)*

1. $2(x + 1)$
2. $3(n - 1)$
3. $-2(k + 8)$
4. $-4(x - 5)$
5. $6(2c + 4)$
6. $5(-3s + t)$
7. $7(a + b)$
8. $9(b - 2c)$

For Lesson 4-2 — Order of Operations

Evaluate each expression if $x = 2$, $y = 5$, and $z = -1$. *(For review, see Lesson 1-3.)*

9. $x + 12$
10. $z + (-5)$
11. $4y + 8$
12. $10 + 3z$
13. $(2 + y)9$
14. $6(x - 4)$
15. $3xy$
16. $2z + y$

For Lesson 4-8 — Product of Decimals

Find each product. *(For review, see page 715.)*

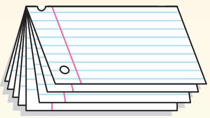
17. $4.5 \cdot 10$
18. $3.26 \cdot 100$
19. $0.1 \cdot 780$
20. $15 \cdot 0.01$
21. $3.9 \cdot 0.1$
22. $63.2 \cdot 0.1$
23. $0.01 \cdot 0.5$
24. $301.8 \cdot 0.001$

FOLDABLES™ Study Organizer

Factors and Monomials Make this Foldable to help you organize your notes. Begin with four sheets of notebook paper.

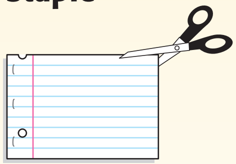
Step 1 Fold

Fold four sheets of notebook paper in half from top to bottom.



Step 2 Cut and Staple

Cut along the fold. Staple the eight half-sheets together to form a booklet.

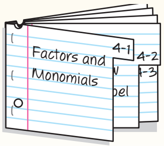


Step 3 Cut Tabs into Margin

Make the top tab 2 lines wide, the next tab 4 lines wide, and so on.

Step 4 Label

Label each of the tabs with the lesson number and title.



Reading and Writing As you read and study the chapter, write notes and examples on each page.

CONTENTS

4-1 Factors and Monomials

What You'll Learn

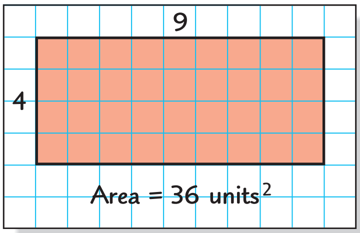
- Determine whether one number is a factor of another.
- Determine whether an expression is a monomial.

Vocabulary

- factors
- divisible
- monomial

How are side lengths of rectangles related to factors?

The rectangle at the right has an area of $9 \cdot 4$ or 36 square units.



a. Use grid paper to draw as many other rectangles as possible with an area of 36 square units. Label the length and width of each rectangle.

b. Did you draw a rectangle with a length of 5 units? Why or why not?

c. List all of the pairs of whole numbers whose product is 36. Compare this list to the lengths and widths of all the rectangles that have an area of 36 square units. What do you observe?

d. Predict the number of rectangles that can be drawn with an area of 64 square units. Explain how you can predict without drawing.

FIND FACTORS Two or more numbers that are multiplied to form a product are called **factors**.

$$\underbrace{4 \times 9}_{\text{factors}} = 36 \leftarrow \text{product}$$

So, 4 and 9 are factors of 36 because they each divide 36 with a remainder of 0. We can say that 36 is **divisible** by 4 and 9. However, 5 is not a factor of 36 because $36 \div 5 = 7$ with a remainder of 1.

Sometimes you can test for divisibility mentally. The following rules can help you determine whether a number is divisible by 2, 3, 5, 6, or 10.

Concept Summary — Divisibility Rules

A number is divisible by:	Examples		Reasons
• 2 if the ones digit is divisible by 2.	54	→	4 is divisible by 2.
• 3 if the sum of its digits is divisible by 3.	72	→	$7 + 2 = 9$, and 9 is divisible by 3.
• 5 if the ones digit is 0 or 5.	65	→	The ones digit is 5.
• 6 if the number is divisible by 2 and 3.	48	→	48 is divisible by 2 and 3.
• 10 if the ones digit is 0.	120	→	The ones digit is 0.

Reading Math

Even and Odd Numbers

A number that is divisible by 2 is called an *even number*. A number that is not divisible by 2 is called an *odd number*.

Concept Check Is 51 divisible by 3? Why or why not?

CONTENTS

Example 1 Use Divisibility Rules

Determine whether 138 is divisible by 2, 3, 5, 6, or 10.

Number	Divisible?	Reason
2	yes	The ones digit is 8, and 8 is divisible by 2.
3	yes	The sum of the digits is 1 + 3 + 8 or 12, and 12 is divisible by 3.
5	no	The ones digit is 8, not 0 or 5.
6	yes	138 is divisible by 2 and 3.
10	no	The ones digit is not 0.

So, 138 is divisible by 2, 3, and 6.

More About. . .

Weddings

On average, it costs four times more to book reception sites in Los Angeles than in the Midwest.

Source: newschannel5.webpoint.com/wedding

Example 2 Use Divisibility Rules to Solve a Problem

WEDDINGS **A bride must choose whether to seat 5, 6, or 10 people per table at her reception. If there are 192 guests and she wants all the tables to be full, which should she choose?**

Seats Per Table	Yes/No	Reason
5	no	The ones digit of 192 does not end in 0 or 5, so 192 is not divisible by 5. There would be empty seats.
6	yes	192 is divisible by 2 and 3, so it is also divisible by 6. Therefore, all the tables would be full.
10	no	The ones digit of 192 does not end in 0, so 192 is not divisible by 10. There would be empty seats.

The bride should choose tables that seat 6 people.

You can also use the rules for divisibility to find the factors of a number.

Example 3 Find Factors of a Number

List all the factors of 72.

Use the divisibility rules to determine whether 72 is divisible by 2, 3, 5, and so on. Then use division to find other factors of 72.

Number	72 Divisible by Number?	Factor Pairs
1	yes	$1 \cdot 72$
2	yes	$2 \cdot 36$
3	yes	$3 \cdot 24$
4	yes	$4 \cdot 18$
5	no	—
6	yes	$6 \cdot 12$
7	no	—
8	yes	$8 \cdot 9$
9	yes	$9 \cdot 8$

Use division to find the other factor in each factor pair. $72 \div 2 = 36$

You can stop finding factors when the numbers start repeating.

So, the factors of 72 are 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, and 72.

Reading Math

Divisible/Factor

The following statements mean the same thing.

- 72 is divisible by 2.
- 2 is a factor of 72.

ALGEBRA CONNECTION

MONOMIALS A number such as 80 or an expression such as $8x$ is called a monomial. A **monomial** is a number, a variable, or a product of numbers and/or variables.

$$8 \cdot 10 = 80$$

8 and 10 are factors of 80.

$$8 \cdot x = 8x$$

8 and x are factors of $8x$.

Reading Math

Monomial

The prefix *mono* means one. A monomial is an expression with one term.

Monomials		Not Monomials	
4	a number	$2 + x$	two terms are added
y	a variable	$5c - 6$	one term is subtracted from another term
$-2rs$	the product of a number and variables	$3(a + b)$	two terms are added $3(a + b) = 3a + 3b$

Concept Check Explain why $7q \cdot n$ is a monomial, but $7q + n$ is not.

Before you determine whether an expression is a monomial, be sure the expression is in simplest form.

Example 4 Identify Monomials

Determine whether each expression is a monomial.

a. $\mathbf{2(x - 3)}$

$2(x - 3) = 2x + 2(-3)$ Distributive Property

$= 2x - 6$ Simplify.

This expression is not a monomial because it has two terms involving subtraction.

b. $\mathbf{-48xyz}$

This expression is a monomial because it is the product of integers and variables.

Check for Understanding

Concept Check

1. **Explain** how you can mentally determine whether there is a remainder when 18,450 is divided by 6.
2. **Determine** whether 3 is a common factor of 125 and 132. Explain.
3. **OPEN ENDED** Use mental math, paper and pencil, or a calculator to find at least one number that satisfies each condition.
   - **a.** a 3-digit number that is divisible by 2, 3, and 6
   - **b.** a 4-digit number that is divisible by 3 and 5, but is not divisible by 10.
   - **c.** a 3-digit number that is not divisible by 2, 3, 5, or 10

Guided Practice

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

4. 51 **5.** 146 **6.** 876 **7.** 3050

List all the factors of each number.

8. 203
9. 80
10. 115

ALGEBRA **Determine whether each expression is a monomial. Explain why or why not.**

11. 38
12. $2n - 2$
13. $5(x + y)$
14. $17(4)k$

Application

15. **CALENDARS** Years that are divisible by 4, called *leap years*, are 366 days long. Also, years ending in "00" that are divisible by 400 are leap years. Use the rule given below to determine whether 2000, 2004, 2015, 2018, 2022, and 2032 are leap years.

> If the last two digits form a number that is divisible by 4, then the number is divisible by 4.

Practice and Apply

Homework Help

For Exercises	See Examples
16–27	1
28–35	3
36–47	4
48–51	2

Extra Practice
See page 730.

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.

16. 39
17. 135
18. 82
19. 120
20. 250
21. 118
22. 378
23. 955
24. 5010
25. 684
26. 10,523
27. 24,640

List all the factors of each number.

28. 75
29. 114
30. 57
31. 65
32. 90
33. 124
34. 102
35. 135

ALGEBRA **Determine whether each expression is a monomial. Explain why or why not.**

36. m
37. 110
38. $s + t$
39. $g - h$
40. $-12 + 12x$
41. $3c + 6$
42. $7(a + 1)$
43. $4(2t - 1)$
44. $4b$
45. $10(-t)$
46. $-25abc$
47. $8j(4k)$

MUSIC **For Exercises 48 and 49, use the following information.**
The band has 72 students who will march during halftime of the football game. For one drill, they need to march in rows with the same number of students in each row.

48. Can the whole band be arranged in rows of 7? Explain.

49. How many different ways could students be arranged? Describe the arrangements.

More About. . .

History

In 1912, when there were 48 states, the stars of the flag were arranged in equal rows. In 1959, after Alaska joined the Union, a new arrangement was proposed.

Source: www.usflag.org

HISTORY **For Exercises 50 and 51, use the following information.**
Each star on the U.S. flag represents a state. As states joined the Union, the rectangular arrangement of the stars changed.

50. Use the information at the left to make a conjecture about how you think the stars of the flag were arranged in 1912 and in 1959.

51. **Research** What is the correct arrangement of stars on the U.S. flag? Explain why the arrangement is not rectangular.

Determine whether each statement is *sometimes, always,* or *never* true. Explain your reasoning.

52. A number that is divisible by 3 is also divisible by 6.

53. A number that has 10 as a factor is not divisible by 5.

54. A number that has a factor of 10 is an even number.

55. MONEY The homecoming committee can spend $144 on refreshments for the dance. Soft drinks cost $6 per case, and cookies cost $4 per bag.

a. How many cases of soft drinks can they buy with $144?

b. How many bags of cookies can they buy with $144?

c. Suppose they want to buy approximately the same amounts of soft drinks and cookies. How many of each could they buy with $144?

56. CRITICAL THINKING Write a number that satisfies each set of conditions.

a. the greatest three-digit number that is not divisible by 2, 3, or 10

b. the least three-digit number that is not divisible by 2, 3, 5, or 10

57. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are side lengths of rectangles related to factors?

Include the following in your answer:

- a drawing of a rectangle with its dimensions and area labeled, and
- a definition of *factors* and a description of the relationship between rectangle dimensions and factor pairs of a number.

Standardized Test Practice

58. Which number is divisible by 3?

(A) 133 (B) 444 (C) 53 (D) 250

59. Determine which expression is *not* a monomial.

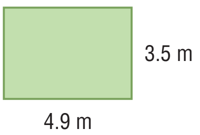
(A) $6d$ (B) $6d \cdot 5$ (C) $6d - 5$ (D) 5

Maintain Your Skills

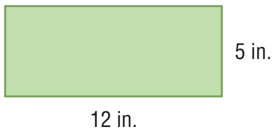
Mixed Review

GEOMETRY **Find the perimeter and area of each rectangle.** *(Lesson 3-7)*

60.



61.



ALGEBRA **Translate each sentence into an equation. Then find each number.** *(Lesson 3-6)*

62. Eight more than twice a number is −16.

63. Two less than 5 times a number equals 3.

ALGEBRA **Solve each equation. Check your solution.** *(Lesson 3-5)*

64. $2x - 1 = 9$ **65.** $14 = 8 + 3n$ **66.** $7 + \frac{k}{5} = -1$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each product.**
*(To review **multiplying integers**, see Lesson 2-4.)*

67. $4 \cdot 4 \cdot 4$ **68.** $10 \cdot 10 \cdot 10 \cdot 10$ **69.** $(-3)(-3)(-3)$

70. $(-2)(-2)(-2)(-2)$ **71.** $8 \cdot 8 \cdot 6 \cdot 6$ **72.** $(2)(2)(-5)(-5)(-5)$

CONTENTS

4-2 Powers and Exponents

What You'll Learn

- Write expressions using exponents.
- Evaluate expressions containing exponents.

Vocabulary

- base
- exponent
- power
- standard form
- expanded form

Why are exponents important in comparing computer data?

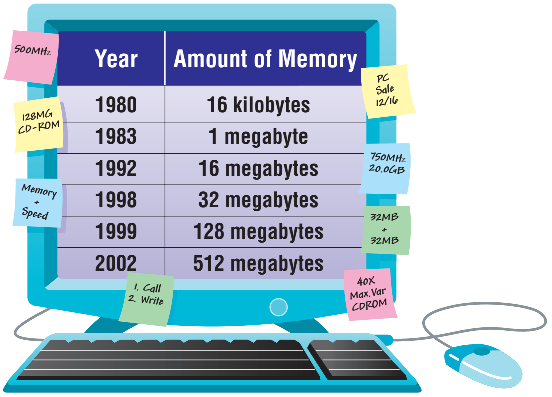
Computer data are measured in small units called *bytes*. These units are based on factors of 2.

a. Write 16 as a product of factors of 2. How many factors are there?

b. How many factors of 2 form the product 128?

c. One megabyte is 1024 kilobytes. How many factors of 2 form the product 1024?

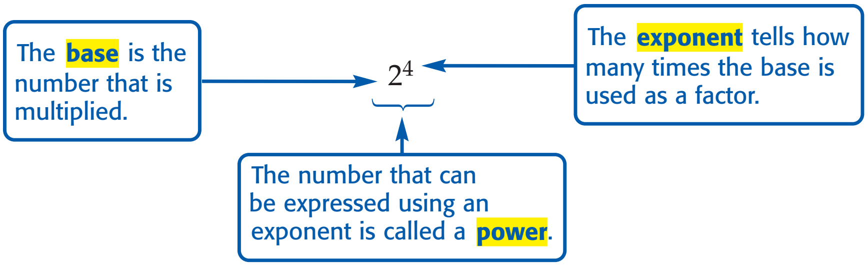
Personal Computers



Year	Amount of Memory
1980	16 kilobytes
1983	1 megabyte
1992	16 megabytes
1998	32 megabytes
1999	128 megabytes
2002	512 megabytes

Source: www.islandnet.com

EXPONENTS An expression like $2 \times 2 \times 2 \times 2$ can be written as a power. A power has two parts, a base and an exponent. The expression $2 \times 2 \times 2 \times 2$ can be written as 2^4.



The **base** is the number that is multiplied.

The **exponent** tells how many times the base is used as a factor.

The number that can be expressed using an exponent is called a **power**.

The table below shows how to write and read powers with positive exponents.

Powers	Words	Repeated Factors
2^1	2 to the first power	2
2^2	2 to the second power or 2 squared	$2 \cdot 2$
2^3	2 to the third power or 2 cubed	$2 \cdot 2 \cdot 2$
2^4	2 to the fourth power or 2 to the fourth	$2 \cdot 2 \cdot 2 \cdot 2$
⋮	⋮	⋮
2^n	2 to the *n*th power or 2 to the *n*th	$\underbrace{2 \cdot 2 \cdot 2 \cdot \ldots \cdot 2}_{n \text{ factors}}$

Study Tip

First Power When a number is raised to the first power, the exponent is usually omitted. So 2^1 is written as 2.

Any number, except 0, raised to the zero power is defined to be 1.

$1^0 = 1$ $2^0 = 1$ $3^0 = 1$ $4^0 = 1$ $5^0 = 1$ $x^0 = 1, x \neq 0$

Example 1 Write Expressions Using Exponents

Write each expression using exponents.

a. $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

The base is 3. It is a factor 5 times, so the exponent is 5.
$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5$

b. $t \cdot t \cdot t \cdot t$

The base is t. It is a factor 4 times, so the exponent is 4.
$t \cdot t \cdot t \cdot t = t^4$

c. $(-9)(-9)$

The base is -9. It is a factor 2 times, so the exponent is 2.
$(-9)(-9) = (-9)^2$

d. $(x + 1)(x + 1)(x + 1)$

The base is $x + 1$. It is a factor 3 times, so the exponent is 3.
$(x + 1)(x + 1)(x + 1) = (x + 1)^3$

e. $7 \cdot a \cdot a \cdot a \cdot b \cdot b$

First, group the factors with like bases. Then, write using exponents.

$$7 \cdot a \cdot a \cdot a \cdot b \cdot b = 7 \cdot (a \cdot a \cdot a) \cdot (b \cdot b)$$
$$= 7a^3b^2$$

$a \cdot a \cdot a = a^3$ and $b \cdot b = b^2$

Study Tip

Common Misconception

$(-9)^2$ is not the same as -9^2.
$-9^2 = -1 \cdot 9^2$

Concept Check How would you write *ten to the fourth* using an exponent?

The number 13,548 is in **standard form** because it does not contain exponents. You can use place value and exponents to express a number in **expanded form**.

Example 2 Use Exponents in Expanded Form

Express 13,048 in expanded form.

Step 1 Use place value to write the value of each digit in the number.

$$13{,}048 = 10{,}000 + 3000 + 0 + 40 + 8$$
$$= (1 \times 10{,}000) + (3 \times 1000) + (0 \times 100) + (4 \times 10) + (8 \times 1)$$

Step 2 Write each place value as a power of 10 using exponents.

$$13{,}048 = (1 \times 10^4) + (3 \times 10^3) + (0 \times 10^2) + (4 \times 10^1) + (8 \times 10^0)$$

Recall that $10^0 = 1$.

EVALUATE EXPRESSIONS Since powers are forms of multiplication, they need to be included in the rules for order of operations.

Concept Summary Order of Operations

	Words	Example
Step 1	Simplify the expressions inside grouping symbols. Start with the innermost grouping symbols.	$(3 + 4)^2 + 5 \cdot 2 = 7^2 + 5 \cdot 2$
Step 2	Evaluate all powers.	$= 49 + 5 \cdot 2$
Step 3	Do all multiplications or divisions in order from left to right.	$= 49 + 10$
Step 4	Do all additions or subtractions in order from left to right.	$= 59$

CONTENTS

Follow the order of operations to evaluate algebraic expressions.

Example 3 Evaluate Expressions

Evaluate each expression.

a. 2^3

$2^3 = 2 \cdot 2 \cdot 2$ 2 is a factor 3 times.

$= 8$ Multiply.

b. $y^2 + 5$ **if** $y = -3$

$y^2 + 5 = (-3)^2 + 5$ Replace y with -3.

$= (-3)(-3) + 5$ -3 is a factor two times.

$= 9 + 5$ Multiply.

$= 14$ Add.

c. $3(x + y)^4$ **if** $x = -2$ **and** $y = 1$

$3(x + y)^4 = 3(-2 + 1)^4$ Replace x with -2 and y with 1.

$= 3(-1)^4$ Simplify the expression inside the parentheses.

$= 3(1)$ Evaluate $(-1)^4$.

$= 3$ Simplify.

Study Tip

Exponents

An exponent goes with the number, variable, or quantity in parentheses immediately preceding it.

- In $5 \cdot 3^2$, 3 is squared. $5 \cdot 3^2 = 5 \cdot 3 \cdot 3$
- In $(5 \cdot 3)^2$, $(5 \cdot 3)$ is squared. $(5 \cdot 3)^2 = (5 \cdot 3)(5 \cdot 3)$

Check for Understanding

Concept Check

1. **OPEN ENDED** Use exponents to write a numerical expression and an algebraic expression in which the base is a factor 5 times.
2. **Explain** how the expression *6 cubed* is repeated multiplication.
3. **Make a conjecture** about the value of 1^n and the value of $(-1)^n$ for any value of n. Explain.

Guided Practice

Write each expression using exponents.

4. $n \cdot n \cdot n$
5. $7 \cdot 7$
6. $3 \cdot 3 \cdot x \cdot x \cdot x \cdot x$

7. Express 2695 in expanded form.

ALGEBRA **Evaluate each expression.**

8. 2^4
9. $x^3 - 3$ if $x = -2$
10. $5(y - 1)^2$ if $y = 4$

Application

11. **SOUND** Fireworks can easily reach a sound of 169 decibels, which can be dangerous if prolonged. Write this number using exponents and a smaller base.

Practice and Apply

Write each expression using exponents.

12. $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$
13. 6
14. $(-5)(-5)(-5)$
15. $(-8)(-8)(-8)(-8)$
16. $k \cdot k$
17. $(-t)(-t)(-t)$
18. $(r \cdot r)(r \cdot r)$
19. $m \cdot m \cdot m \cdot m$
20. $a \cdot a \cdot b \cdot b \cdot b \cdot b$
21. $2 \cdot x \cdot x \cdot y \cdot y$
22. $7 \cdot 7 \cdot 7 \cdot n \cdot n \cdot n \cdot n$
23. $9 \cdot (p + 1) \cdot (p + 1)$

www.pre-alg.com/extra_examples

CONTENTS

Homework Help

For Exercises	See Examples
12–23, 43, 47	1
24–27	2
28–42	3

Extra Practice
See page 731.

Express each number in expanded form.

24. 452 **25.** 803 **26.** 6994 **27.** 23,781

ALGEBRA **Evaluate each expression if $a = 2$, $b = 4$, and $c = -3$.**

28. 7^2 **29.** 10^3 **30.** $(-9)^3$

31. $(-2)^5$ **32.** b^4 **33.** c^4

34. $5a^4$ **35.** ac^3 **36.** $b^0 - 10$

37. $c^2 + a^2$ **38.** $3a + b^3$ **39.** $a^2 + 3a - 1$

40. $b^2 - 2b + 6$ **41.** $3(b - 1)^4$ **42.** $2(3c + 7)^2$

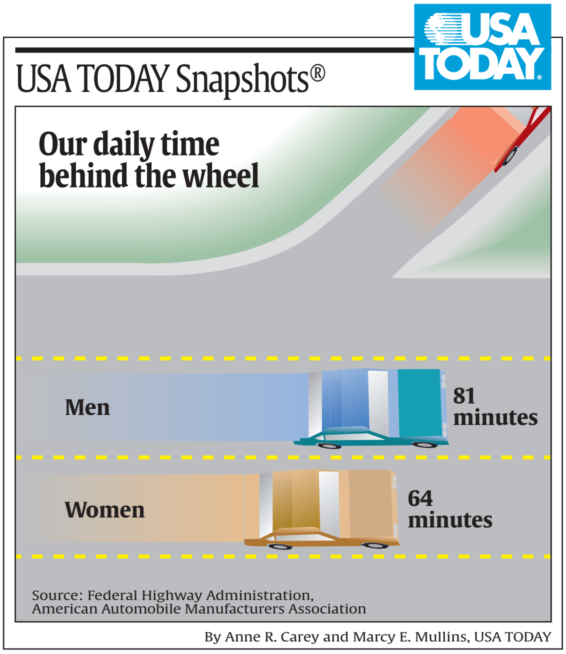
43. **TRAVEL** Write each number in the graphic as a power.

44. Write *7 cubed times x squared* as repeated multiplication.

45. Write *negative eight, cubed* using exponents, as a product of repeated factors, and in standard form.

46. Without using a calculator, order 96, 96^2, 96^{10}, 96^5, and 96^0 from least to greatest. Explain your reasoning.

47. **NUMBER THEORY** Explain whether the square of any nonzero number is *sometimes*, *always*, or *never* a positive number.



Source: Federal Highway Administration, American Automobile Manufacturers Association

By Anne R. Carey and Marcy E. Mullins, USA TODAY

48. **BIOLOGY** A man burns approximately 121 Calories by standing for an hour. A woman burns approximately 100 Calories per hour when standing. Write each of these numbers as a power with an exponent other than 1.

More About. . .

History

The *so* noodles are about a yard long and as thin as a piece of yarn. Very few chefs still know how to make these noodles.

Source: *The Mathematics Teacher*

HISTORY **For Exercises 49–51, use the following information.**

In an ancient Chinese tradition, a chef stretches and folds dough to make long, thin noodles called *so*. After the first fold, he makes 2 noodles. He stretches and folds it a second time to make 4 noodles. Each time he repeats this process, the number of noodles doubles.

49. Use exponents to express the number of noodles after each of the first five folds.

50. Legendary chefs have completed as many as thirteen folds. How many noodles is this?

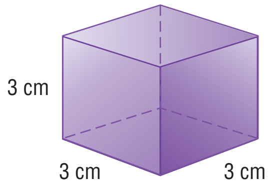
51. If the noodles are laid end to end and each noodle is 5 feet long, after how many of these folds will the length be more than a mile?

Replace each ● with <, >, or = to make a true statement.

52. 3^7 ● 7^3 **53.** 2^4 ● 4^2 **54.** 6^3 ● 4^4

CONTENTS

GEOMETRY **For Exercises 55–57, use the cube below.**



55. The *surface area* of a cube is the sum of the areas of the faces. Use exponents to write an expression for the surface area of the cube.

56. The *volume* of a cube, or the amount of space that it occupies, is the product of the length, width, and height. Use exponents to write an expression for the volume of the cube.

57. If you double the length of each edge of the cube, are the surface area and volume also doubled? Explain.

58. **CRITICAL THINKING** Suppose the length of a side of a square is n units and the length of an edge of a cube is n units.

- **a.** If all the side lengths of a square are doubled, are the perimeter and the area of the square doubled? Explain.
- **b.** If all the side lengths of a square are tripled, show that the area of the new square is 9 times the area of the original square.
- **c.** If all the edge lengths of a cube are tripled, show that the volume of the new cube is 27 times the volume of the original cube.

59. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why are exponents important in comparing computer data?

Include the following in your answer:

- an explanation of how factors of 2 describe computer memory, and
- a sentence explaining the advantage of using exponents.

Online Research **Data Update** How many megabytes of memory are common today? Visit www.pre-alg.com/data_update to learn more.

Standardized Test Practice

60. Write *ten million* as a power of ten.

Ⓐ 10^5 Ⓑ 10^6 Ⓒ 10^7 Ⓓ 10^8

61. What value of x will make $256 = 2^x$ true?

Ⓐ 7 Ⓑ 8 Ⓒ 9 Ⓓ 128

Maintain Your Skills

Mixed Review

State whether each number is divisible by 2, 3, 5, 6, or 10. *(Lesson 4-1)*

62. 128 **63.** 370 **64.** 945

65. **METEOROLOGY** A tornado travels 300 miles in 2 hours. Use the formula $d = rt$ to find the tornado's speed in miles per hour. *(Lesson 3-7)*

ALGEBRA **Solve each equation. Check your solution.** *(Lesson 3-5)*

66. $2x + 1 = 7$ **67.** $16 = 5k - 4$ **68.** $\frac{n}{3} + 8 = 6$

69. **ALGEBRA** Simplify $4(y + 2) - y$. *(Lesson 3-2)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **List all the factors for each number.**
*(To review **factoring**, see Lesson 4-1.)*

70. 11 **71.** 5 **72.** 9

73. 16 **74.** 19 **75.** 35

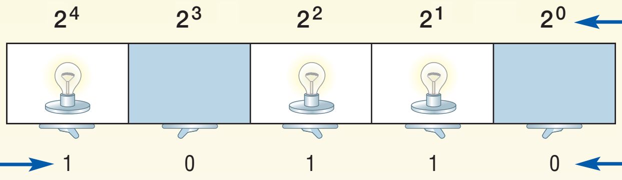
Algebra Activity

A Follow-Up of Lesson 4-2

Base 2

Activity

A computer contains a large number of tiny electronic switches that can be turned ON or OFF. The digits 0 and 1, also called *bits*, are the alphabet of computer language. This **binary** language uses a **base two** system of numbers.



Place values are factors of 2.

The digit 1 represents the ON switch.

The digit 0 represents the OFF switch.

$$10110_2 = \underbrace{(1 \times 2^4)}_{16} + \underbrace{(0 \times 2^3)}_{0} + \underbrace{(1 \times 2^2)}_{4} + \underbrace{(1 \times 2^1)}_{2} + \underbrace{(0 \times 2^0)}_{0}$$

$$= 16 + 0 + 4 + 2 + 0$$

$$= 22$$

So, $10110_2 = 22_{10}$ or 22.

You can also reverse the process and express base ten numbers as equivalent numbers in base two.

Express the decimal number 13 as a number in base two.

Step 1 Make a base 2 place-value chart. Find the greatest factor of 2 that is less than 13. Place a 1 in that place value.

Step 2 Subtract 13 – 8 = 5. Now find the greatest factor of 2 that is less than 5. Place a 1 in that place value.

Step 3 Subtract 5 – 4 = 1. Place a 1 in that place value.

Step 4 There are no factors of 2 left, so place a 0 in any unfilled spaces.

So, 13 in the base 10 system is equal to 1101 in the base 2 system. Or, $13 = 1101_2$.

	1			
16	**8**	**4**	**2**	**1**

	1	1		
16	**8**	**4**	**2**	**1**

	1	1	0	1
16	**8**	**4**	**2**	**1**

Exercises

1. Express 1011_2 as an equivalent number in base 10.

Express each base 10 number as an equivalent number in base 2.

2. 6 **3.** 9 **4.** 15 **5.** 21

Extend the Activity

6. The first five place values for base 5 are shown. Any digit from 0 to 4 can be used to write a base 5 number. Write 179 in base 5.

625	**125**	**25**	**5**	**1**

7. **OPEN ENDED** Write 314 as an equivalent number in a base other than 2, 5, or 10. Include a place-value chart.

8. **OPEN ENDED** Choose a base 10 number and write it as an equivalent number in base 8. Include a place-value chart.

CONTENTS

4-3 Prime Factorization

What You'll Learn

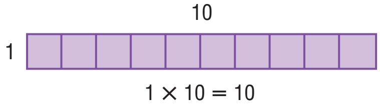
- Write the prime factorizations of composite numbers.
- Factor monomials.

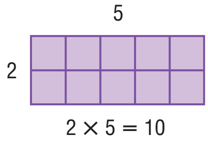
Vocabulary

- prime number
- composite number
- prime factorization
- factor tree
- factor

How can models be used to determine whether numbers are prime?

There are two ways that 10 can be expressed as the product of whole numbers. This can be shown by using 10 squares to form rectangles.





a. Use grid paper to draw as many different rectangular arrangements of 2, 3, 4, 5, 6, 7, 8, and 9 squares as possible.

b. Which numbers of squares can be arranged in more than one way?

c. Which numbers of squares can only be arranged one way?

d. What do the rectangles in part **c** have in common? Explain.

Reading Math

Composite

Everyday Meaning: materials that are made up of many substances

Math Meaning: numbers having many factors

PRIME NUMBERS AND COMPOSITE NUMBERS A **prime number** is a whole number that has exactly two factors, 1 and itself. A **composite number** is a whole number that has more than two factors. Zero and 1 are neither prime nor composite.

	Whole Numbers	Factors	Number of Factors
Prime Numbers	2	1, 2	2
	3	1, 3	2
	5	1, 5	2
	7	1, 7	2
Composite Numbers	4	1, 2, 4	3
	6	1, 2, 3, 6	4
	8	1, 2, 4, 8	4
	9	1, 3, 9	3
Neither Prime nor Composite	0	all numbers	infinite
	1	1	1

Example 1 Identify Numbers as Prime or Composite

a. Determine whether 19 is prime or composite.

Find factors of 19 by listing the whole number pairs whose product is 19.

$19 = 1 \times 19$

The number 19 has only two factors. Therefore, 19 is a prime number.

Study Tip

Mental Math
To determine whether a number is prime or composite, you can mentally use the rules for divisibility rather than listing factors.

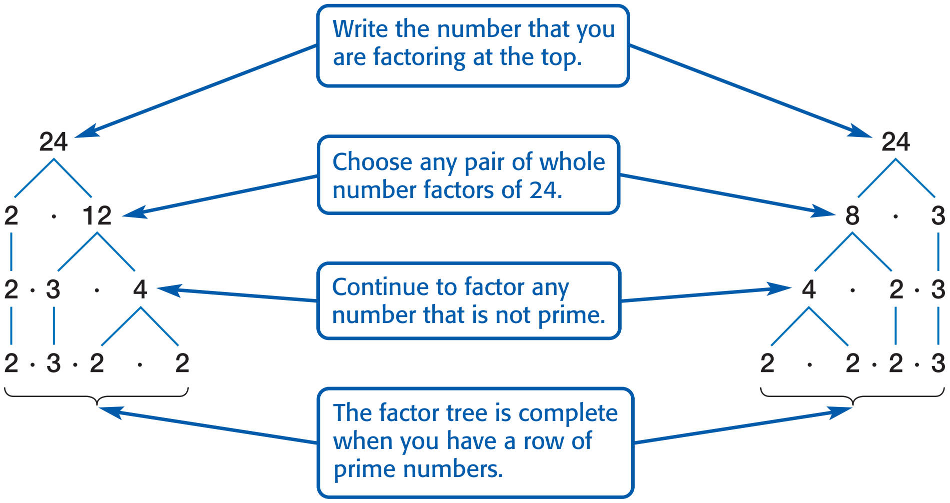
b. Determine whether 28 is prime or composite.

Find factors of 28 by listing the whole number pairs whose product is 28.

$28 = 1 \times 28$
$28 = 2 \times 14$
$28 = 4 \times 7$

The factors of 28 are 1, 2, 4, 7, 14, and 28. Since the number has more than two factors, it is composite.

When a composite number is expressed as the product of prime factors, it is called the **prime factorization** of the number. One way to find the prime factorization of a number is to use a **factor tree**.



Study Tip

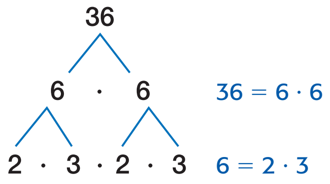
Commutative
The order of the factors does not matter because the operation of multiplication is commutative.

Both trees give the same prime factors, except in different orders. There is exactly one prime factorization of 24. The prime factorization of 24 is $2 \cdot 2 \cdot 2 \cdot 3$ or $2^3 \cdot 3$.

Concept Check Could a different factor tree have been used to write the prime factorization of 24? If so, would the result be the same?

Example 2 Write Prime Factorization

Write the prime factorization of 36.



The factorization is complete because 2 and 3 are prime numbers.

The prime factorization of 36 is $2 \cdot 2 \cdot 3 \cdot 3$ or $2^2 \cdot 3^2$.

You can also use a strategy involving division called the *cake method* to find a prime factorization. The prime factorization of 210 is shown below using the cake method.

Study Tip

Remainders
If you get a remainder when using the cake method, choose a different prime number to divide the quotient.

Step 1
Begin with the smallest prime that is a factor of 210, in this case, 2. Divide 210 by 2.

Step 2
Divide the quotient 105 by the smallest possible prime factor, 3.

Step 3
Repeat until the quotient is prime.

$$\begin{array}{r} 105 \\ 2\overline{)210} \end{array} \rightarrow \begin{array}{r} 35 \\ 3\overline{)105} \\ 2\overline{)210} \end{array} \rightarrow \begin{array}{r} 7 \\ 5\overline{)35} \\ 3\overline{)105} \\ 2\overline{)210} \end{array}$$

The prime factorization of 210 is $2 \cdot 3 \cdot 5 \cdot 7$. Multiply to check the result.

ALGEBRA CONNECTION

FACTOR MONOMIALS To **factor** a number means to write it as a product of its factors. A monomial can also be factored as a product of prime numbers and variables with no exponent greater than 1. Negative coefficients can be factored using -1 as a factor.

Example 3 Factor Monomials

Factor each monomial.

a. $8ab^2$

$$\begin{aligned} 8ab^2 &= 2 \cdot 2 \cdot 2 \cdot a \cdot b^2 && 8 = 2 \cdot 2 \cdot 2 \\ &= 2 \cdot 2 \cdot 2 \cdot a \cdot b \cdot b && a \cdot b^2 = a \cdot b \cdot b \end{aligned}$$

b. $-30x^3y$

$$\begin{aligned} -30x^3y &= -1 \cdot 2 \cdot 3 \cdot 5 \cdot x^3 \cdot y && -30 = -1 \cdot 2 \cdot 3 \cdot 5 \\ &= -1 \cdot 2 \cdot 3 \cdot 5 \cdot x \cdot x \cdot x \cdot y && x^3 \cdot y = x \cdot x \cdot x \cdot y \end{aligned}$$

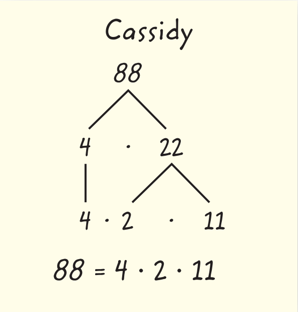
Check for Understanding

Concept Check

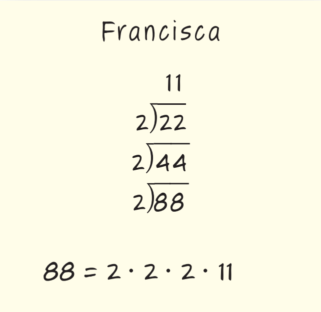
1. **Explain** the difference between a prime and composite number.

2. **OPEN ENDED** Write a 2-digit number with prime factors that include 2 and 3.

3. **FIND THE ERROR** Cassidy and Francisca each factored 88.



$88 = 4 \cdot 2 \cdot 11$


Francisca

$$\begin{array}{r} 11 \\ 2\overline{)22} \\ 2\overline{)44} \\ 2\overline{)88} \end{array}$$

$88 = 2 \cdot 2 \cdot 2 \cdot 11$

Who is correct? Explain your reasoning.

www.pre-alg.com/extra_examples

CONTENTS

Guided Practice

Determine whether each number is *prime* or *composite*.

4. 7
5. 23
6. 15

Write the prime factorization of each number. Use exponents for repeated factors.

7. 18
8. 39
9. 50

ALGEBRA **Factor each monomial.**

10. $4c^2$
11. $5a^2b$
12. $-70xyz$

Application

13. **NUMBER THEORY** One mathematical conjecture that is unproved states that there are infinitely many *twin primes*. Twin primes are prime numbers that differ by 2, such as 3 and 5. List all the twin primes that are less than 50.

Practice and Apply

Homework Help

For Exercises	See Examples
14–21	1
22–29	2
30–41	3

Extra Practice
See page 731.

Determine whether each number is *prime* or *composite*.

14. 21
15. 33
16. 23
17. 70
18. 17
19. 51
20. 43
21. 31

Write the prime factorization of each number. Use exponents for repeated factors.

22. 26
23. 81
24. 66
25. 63
26. 56
27. 100
28. 392
29. 110

ALGEBRA **Factor each monomial.**

30. $14w$
31. $9t^2$
32. $-7c^2$
33. $-25z^3$
34. $20st$
35. $-38mnp$
36. $28x^2y$
37. $21gh^3$
38. $13q^2r^2$
39. $64n^3$
40. $-75ab^2$
41. $-120r^2st^3$

42. Is the value of $n^2 - n + 41$ prime or composite if $n = 3$?

43. **OPEN ENDED** Write a monomial whose factors include -1, 5, and x.

44. **TECHNOLOGY** *Mersenne primes* are prime numbers in the form $2^n - 1$. In 2001, a computer programmer used special software to discover the largest prime number so far, $2^{13,466,917} - 1$. Write the prime factorization of each number, or write *prime* if the number is a Mersenne prime.

a. $2^5 - 1$
b. $2^6 - 1$
c. $2^7 - 1$
d. $2^8 - 1$

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can models be used to determine whether numbers are prime?

Include the following in your answer:

- the number of rectangles that can be drawn to represent prime and composite numbers, and
- an explanation of how one model can show that a number is *not* prime.

CONTENTS

46. **CRITICAL THINKING** Find the prime factors of these numbers that are divisible by 12: 12, 60, 84, 132, and 180. Then, write a rule to determine when a number is divisible by 12.

Standardized Test Practice

Ⓐ Ⓑ Ⓒ Ⓓ

47. Which table of values represents the following rule?
Add the input number to the square of the input number.

Ⓐ

Input (x)	Output (y)
0	1
2	3
4	5

Ⓑ

Input (x)	Output (y)
1	1
2	6
4	8

Ⓒ

Input (x)	Output (y)
1	2
2	6
4	20

Ⓓ

Input (x)	Output (y)
1	2
2	4
4	8

48. Determine which number is *not* a prime factor of 70.

Ⓐ 2 Ⓑ 5 Ⓒ 7 Ⓓ 10

Maintain Your Skills

Mixed Review

49. Write $(-5) \cdot (-5) \cdot (-5) \cdot h \cdot h \cdot k$ using exponents. *(Lesson 4-2)*

Determine whether each expression is a monomial. *(Lesson 4-1)*

50. $14cd$ **51.** -5 **52.** $x - y$ **53.** $3(1 + 3r)$

ALGEBRA **Solve each equation. Check your solution.** *(Lesson 3-4)*

54. $\frac{n}{8} = -4$ **55.** $2x = -18$ **56.** $30 = 6n$ **57.** $-7 = \frac{y}{4}$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Use the Distributive Property to rewrite each expression.** *(To review the **Distributive Property**, see Lesson 3-1.)*

58. $2(n + 4)$ **59.** $5(x - 7)$ **60.** $-3(t + 4)$

61. $(a + 6)10$ **62.** $(b - 3)(-2)$ **63.** $8(9 - y)$

Practice Quiz 1

Lessons 4-1 through 4-3

Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10. *(Lesson 4-1)*

1. 105 **2.** 270 **3.** 511 **4.** 1368

5. **ALGEBRA** Evaluate $b^2 - 4ac$ if $a = -1$, $b = 5$, and $c = 3$. *(Lesson 4-2)*

6. **LITERATURE** In a story, a knight received a reward for slaying a dragon. He received 1 cent on the first day, 2 cents on the second day, 4 cents on the third day, and so on, continuing to double the amount for 30 days. *(Lesson 4-2)*

a. Express his reward on each of the first three days as a power of 2.

b. Express his reward on the 8th day as a power of 2. Then evaluate.

Factor each monomial. *(Lesson 4-3)*

7. $77x$ **8.** $18st$ **9.** $-23n^3$ **10.** $30cd^2$

www.pre-alg.com/self_check_quiz

CONTENTS

4-4

Greatest Common Factor (GCF)

What You'll Learn

- Find the greatest common factor of two or more numbers or monomials.
- Use the Distributive Property to factor algebraic expressions.

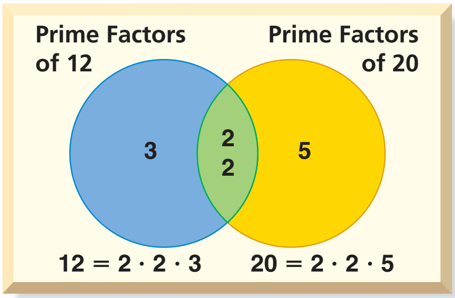
Vocabulary

- Venn diagram
- greatest common factor

How can a diagram be used to find the greatest common factor?

A **Venn diagram** shows the relationships among sets of numbers or objects by using overlapping circles in a rectangle.

The Venn diagram at the right shows the prime factors of 12 and 20. The common prime factors are in both circles.



a. Which numbers are in both circles?

b. Find the product of the numbers that are in both circles.

c. Is the product also a factor of 12 and 20?

d. Make a Venn diagram showing the prime factors of 16 and 28. Then use it to find the common factors of the numbers.

GREATEST COMMON FACTOR Often, numbers have some of the same factors. The greatest number that is a factor of two or more numbers is called the **greatest common factor (GCF)**. Below are two ways to find the GCF of 12 and 20.

Example 1 Find the GCF

Find the GCF of 12 and 20.

Method 1 List the factors.

factors of 12: 1, 2, 3, ④ 6, 12
factors of 20: 1, 2, ④ 5, 10, 20

Common factors of 12 and 20: 1, 2, 4

The greatest common factor of 12 and 20 is 4.

Method 2 Use prime factorization.

$12 = 2 \cdot 2 \cdot 3$
$20 = 2 \cdot 2 \cdot 5$

Common prime factors of 12 and 20: 2, 2

The GCF is the product of the common prime factors.
$2 \cdot 2 = 4$

Again, the GCF of 12 and 20 is 4.

Study Tip

Choosing a Method

To find the GCF of two or more numbers, it is easier to

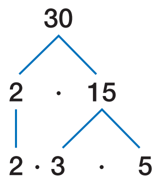
- list the factors if the numbers are small, or
- use prime factorization if the numbers are large.

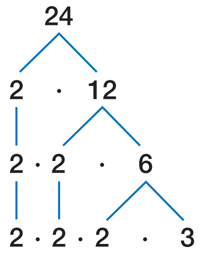
Example 2 Find the GCF

Find the GCF of each set of numbers.

a. 30, 24

First, factor each number completely. Then circle the common factors.





$30 = \boxed{2} \cdot \boxed{3} \cdot 5$ The common prime

$24 = \boxed{2} \cdot 2 \cdot 2 \cdot \boxed{3}$ factors are 2 and 3.

The GCF of 30 and 24 is 2 · 3 or 6.

Study Tip

Writing Prime Factors

Try to line up the common prime factors so that it is easier to circle them.

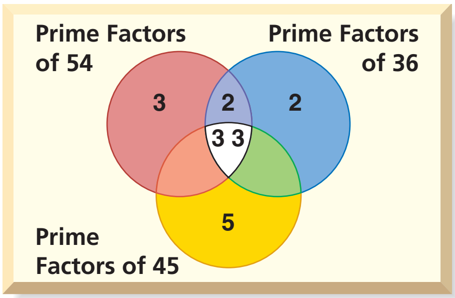
b. 54, 36, 45

$54 = 2 \cdot \boxed{3} \cdot \boxed{3} \cdot 3$ The common

$36 = 2 \cdot 2 \cdot \boxed{3} \cdot \boxed{3}$ prime factors

$45 = \boxed{3} \cdot \boxed{3} \cdot 5$ are 3 and 3.

The GCF is 3 · 3 or 9.



Example 3 Use the GCF to Solve Problems

TRACK AND FIELD **There are 208 boys and 240 girls participating in a field day competition.**

a. What is the greatest number of teams that can be formed if each team has the same number of girls and each team has the same number of boys?

Find the GCF of 208 and 240.

$208 = \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot 13$ The common prime factors

$240 = \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot \boxed{2} \cdot 3 \cdot 5$ are 2, 2, 2, and 2.

The greatest common factor of 208 and 240 is 2 · 2 · 2 · 2 or 16.
So, 16 teams can be formed.

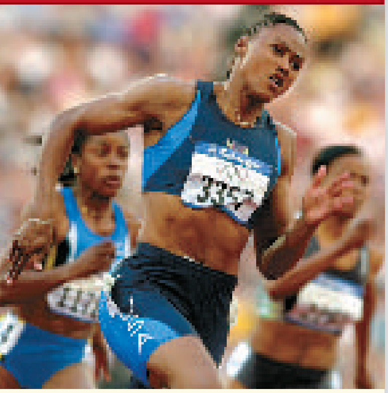
b. How many boys and girls will be on each team?

$208 \div 16 = 13$

$240 \div 16 = 15$

So, each team will have 13 boys and 15 girls.

More About. . .



Track and Field

In some events such as sprints and the long jump, if the wind speed is greater than 2 meters per second, then the time or mark cannot be considered for record purposes.

Source: www.encarta.msn.com

www.pre-alg.com/extra_examples

ALGEBRA CONNECTION

FACTOR ALGEBRAIC EXPRESSIONS You can also find the GCF of two or more monomials by finding the product of their common prime factors.

Example 4 Find the GCF of Monomials

Find the GCF of $16xy^2$ and $30xy$.

Completely factor each expression.

$$\begin{aligned} 16xy^2 &= \boxed{2} \cdot 2 \cdot 2 \cdot 2 \cdot \boxed{x} \cdot \boxed{y} \cdot y \\ 30xy &= \boxed{2} \cdot 3 \cdot 5 \cdot \boxed{x} \cdot \boxed{y} \end{aligned}$$

Circle the common factors.

The GCF of $16xy^2$ and $30xy$ is $2 \cdot x \cdot y$ or $2xy$.

Study Tip

Look Back

To review the **Distributive Property**, see Lesson 3-1.

In Lesson 3-1, you used the Distributive Property to rewrite $2(x + 3)$ as $2x + 6$. You can also use this property to factor an algebraic expression such as $2x + 6$.

Example 5 Factor Expressions

Factor $2x + 6$.

First, find the GCF of $2x$ and 6.

$$\begin{aligned} 2x &= \boxed{2} \cdot x \\ 6 &= \boxed{2} \cdot 3 \end{aligned}$$

The GCF is 2.

Now write each term as a product of the GCF and its remaining factors.

$$\begin{aligned} 2x + 6 &= 2(x) + 2(3) \\ &= 2(x + 3) \end{aligned}$$

Distributive Property

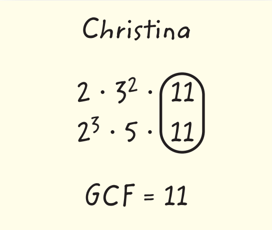
So, $2x + 6 = 2(x + 3)$.

Concept Check Which property allows you to factor $3x + 9$?

Check for Understanding

Concept Check

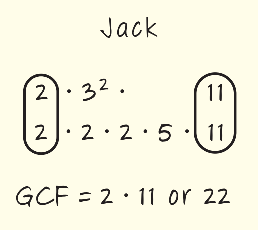
1. **Explain** how to find the greatest common factor of two or more numbers.
2. **OPEN ENDED** Name two different numbers whose GCF is 12.
3. **FIND THE ERROR** Christina and Jack both found the GCF of $2 \cdot 3^2 \cdot 11$ and $2^3 \cdot 5 \cdot 11$.



Christina

$$\begin{aligned} 2 \cdot 3^2 \cdot \boxed{11} \\ 2^3 \cdot 5 \cdot \boxed{11} \end{aligned}$$

GCF = 11



Jack

$$\begin{aligned} \boxed{2} \cdot 3^2 \cdot \boxed{11} \\ \boxed{2} \cdot 2 \cdot 2 \cdot 5 \cdot \boxed{11} \end{aligned}$$

GCF = 2 · 11 or 22

Who is correct? Explain your reasoning.

Guided Practice

Find the GCF of each set of numbers or monomials.

4. 6, 8
5. 21, 45
6. 16, 56
7. 28, 42
8. 7, 30
9. 108, 144
10. 12, 24, 36
11. $14n$, $42n^2$
12. $36a^3b$, $56ab^2$

Factor each expression.

13. $3n + 9$
14. $t^2 + 4t$
15. $15 + 20x$

Application

16. **PARADES** In the parade, 36 members of the color guard are to march in front of 120 members of the high school marching band. Both groups are to have the same number of students in each row. Find the greatest number of students in each row.

Practice and Apply

Homework Help

For Exercises	See Examples
17–34	1, 2
35–42	4
44–52	5
53, 54	3

Extra Practice
See page 731.

Find the GCF of each set of numbers or monomials.

17. 12, 8
18. 3, 9
19. 24, 40
20. 21, 14
21. 20, 30
22. 12, 18
23. 18, 45
24. 22, 21
25. 16, 40
26. 42, 56
27. 30, 35
28. 12, 60
29. 116, 100
30. 135, 315
31. 9, 15, 24
32. 20, 21, 25
33. 20, 28, 36
34. 66, 90, 150
35. $12x$, $40x^2$
36. 18, $45mn$
37. $4st$, $10s$
38. $5ab$, $6b^2$
39. $14b$, $56b^2$
40. $30a^3b^2$, $24a^2b$

41. What is the greatest common factor of $32mn^2$, $16n$, and $12n^3$?

42. Name the GCF of $15v^2$, $70vw$, and $36w^2$.

43. Name two monomials whose GCF is $2x$.

Factor each expression.

44. $2x + 8$
45. $3r + 12$
46. $8 + 32a$
47. $6 + 3y$
48. $9 + 3t$
49. $14 + 21c$
50. $k^2 + 5k$
51. $4y - 16$
52. $5n - 10$

53. **PATTERNS** Consider the pattern 7, 14, 21, 28, 35, … .

a. Find the GCF of the terms in the pattern. Explain how you know.

b. Write the next two terms in the pattern.

54. **CARPENTRY** Tamika is helping her father make shelves to store her sports equipment in the garage. How many shelves measuring 12 inches by 16 inches can be cut from a 48-inch by 72-inch piece of plywood so that there is no waste?

55. **DESIGN** Lauren is covering the surface of an end table with equal-sized ceramic tiles. The table is 30 inches long and 24 inches wide.

a. What is the largest square tile that Lauren can use and not have to cut any tiles?

b. How many tiles will Lauren need?

www.pre-alg.com/self_check_quiz

CONTENTS

56. **HISTORY** *The Nine Chapters on the Mathematical Art* is a Chinese math book written during the first century. It describes a procedure for finding the greatest common factors. Follow each step below to find the GCF of 86 and 110.

a. Subtract the lesser number, a, from the greater number, b.

b. If the result in part **a** is a factor of both numbers, it is the GCF. If the result is not a factor of both numbers, subtract the result from a or subtract a from the result so that the difference is a positive number.

c. Continue subtracting and checking the results until you find a number that is a factor of both numbers.

57. **CRITICAL THINKING** Can the GCF of a set of numbers be equal to one of the numbers? Give an example or a counterexample to support your answer.

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can a diagram be used to find the greatest common factor?

Include the following in your answer:

- a description of how a Venn diagram can be used to display the prime factorization of two or more numbers, and
- the part of a Venn diagram that is used to find the greatest common factor.

Standardized Test Practice

59. Write $6y + 21$ in factored form.

Ⓐ $6(y + 3)$ Ⓑ $2(3y + 7)$ Ⓒ $3(y + 7)$ Ⓓ $3(2y + 7)$

60. Find the GCF of $42x^2y$ and $38xy^2$.

Ⓐ $2x^2y$ Ⓑ $3xy$ Ⓒ $2xy$ Ⓓ $6x^2y^2$

Extending the Lesson

Two numbers are **relatively prime** if their only common factor is 1. Determine whether the numbers in each pair are relatively prime. Write *yes* or *no*.

61. 7 and 8 **62.** 13 and 11 **63.** 27 and 18

64. 20 and 25 **65.** 22 and 23 **66.** 8 and 12

Maintain Your Skills

Mixed Review

ALGEBRA **Factor each monomial.** *(Lesson 4-3)*

67. $9n$ **68.** $15x^2$ **69.** $-5jk$ **70.** $22ab^3$

71. **ALGEBRA** Evaluate $7x^2 + y^3$ if $x = -2$ and $y = 4$. *(Lesson 4-2)*

Find each quotient. *(Lesson 2-5)*

72. $69 \div 23$ **73.** $48 \div (-8)$ **74.** $-24 \div (-12)$ **75.** $-50 \div 5$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each equivalent measure.**

(To review ***converting measurements****, see pages 718–721.)*

76. 1 ft = _?_ in. **77.** 1 yd = _?_ in. **78.** 1 lb = _?_ oz

79. 1 day = _?_ h **80.** 1 m = _?_ cm **81.** 1 kg = _?_ g

CONTENTS

4-5 Simplifying Algebraic Fractions

What You'll Learn

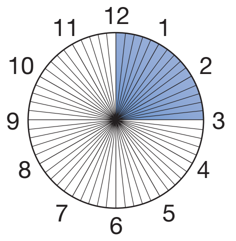
- Simplify fractions using the GCF.
- Simplify algebraic fractions.

Vocabulary

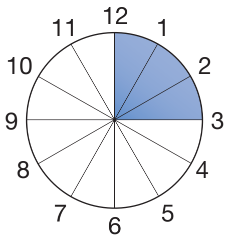
- simplest form
- algebraic fraction

How are simplified fractions useful in representing measurements?

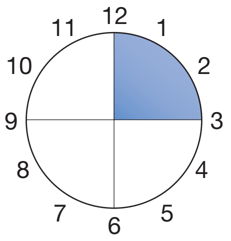
You can use a fraction to compare a *part* of something to a *whole*. The figures below show what part 15 minutes is of 1 hour.



$\frac{15}{60}$ is shaded.



$\frac{3}{12}$ is shaded.



$\frac{1}{4}$ is shaded.

a. Are the three fractions equivalent? Explain your reasoning.

b. Which figure is divided into the least number of parts?

c. Which fraction would you say is written in simplest form? Why?

SIMPLIFY NUMERICAL FRACTIONS A fraction is in **simplest form** when the GCF of the numerator and the denominator is 1.

Fractions in Simplest Form	Fractions *not* in Simplest Form
$\frac{1}{4}, \frac{1}{3}, \frac{3}{4}, \frac{17}{50}$	$\frac{3}{12}, \frac{15}{60}, \frac{6}{8}, \frac{5}{20}$

One way to write a fraction in simplest form is to write the prime factorization of the numerator and the denominator. Then divide the numerator and denominator by the GCF.

Example 1 Simplify Fractions

Write $\frac{9}{12}$ in simplest form.

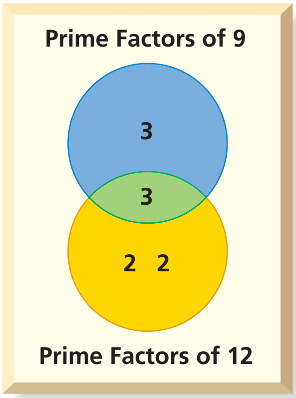
$9 = 3 \cdot \boxed{3}$ Factor the numerator.

$12 = 2 \cdot 2 \cdot \boxed{3}$ Factor the denominator.

The GCF of 9 and 12 is 3.

$\frac{9}{12} = \frac{9 \div 3}{12 \div 3}$ Divide the numerator and the denominator by the GCF.

$= \frac{3}{4}$ Simplest form



Study Tip

Use a Venn Diagram To simplify fractions, let one circle in a Venn diagram represent the numerator and let another circle represent the denominator. The number or product of numbers in the intersection is the GCF.

CONTENTS

The division in Example 1 can be represented in another way.

$$\frac{9}{12} = \frac{\overset{1}{\cancel{3}} \cdot 3}{2 \cdot 2 \cdot \underset{1}{\cancel{3}}}$$ The slashes mean that the numerator and the denominator are both divided by the GCF, 3.

$$= \frac{3}{2 \cdot 2} \text{ or } \frac{3}{4}$$ Simplify.

Example 2 Simplify Fractions

Write $\frac{15}{60}$ in simplest form.

$$\frac{15}{60} = \frac{\overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}}}{2 \cdot 2 \cdot \underset{1}{\cancel{3}} \cdot \underset{1}{\cancel{5}}}$$ Divide the numerator and denominator by the GCF, 3 · 5.

$$= \frac{1}{4}$$ Simplify.

Concept Check How do you know when a fraction is in simplest form?

Simplifying fractions is a useful tool in measurement.

Example 3 Simplify Fractions in Measurement

MEASUREMENT **Eighty-eight feet is what part of 1 mile?**

There are 5280 feet in 1 mile. Write the fraction $\frac{88}{5280}$ in simplest form.

$$\frac{88}{5280} = \frac{\overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{2}} \cdot \overset{1}{\cancel{11}}}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot \underset{1}{\cancel{11}}}$$ Divide the numerator and denominator by the GCF, 2 · 2 · 2 · 11.

$$= \frac{1}{60}$$ Simplify.

So, 88 feet is $\frac{1}{60}$ of a mile.

Study Tip

Alternative Method

You can also divide the numerator and denominator by common factors until the fraction is in simplest form.

$$\frac{88}{5280} = \frac{44}{2640}$$
$$= \frac{22}{1320}$$
$$= \frac{11}{660} \text{ or } \frac{1}{60}$$

ALGEBRA CONNECTION

SIMPLIFY ALGEBRAIC FRACTIONS A fraction with variables in the numerator or denominator is called an **algebraic fraction**. Algebraic fractions can also be written in simplest form.

Example 4 Simplify Algebraic Fractions

Simplify $\frac{21x^2y}{35xy}$.

$$\frac{21x^2y}{35xy} = \frac{3 \cdot \overset{1}{\cancel{7}} \cdot \overset{1}{\cancel{x}} \cdot x \cdot \overset{1}{\cancel{y}}}{5 \cdot \underset{1}{\cancel{7}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{y}}}$$ Divide the numerator and denominator by the GCF, 7 · x · y.

$$= \frac{3x}{5}$$ Simplify.

Standardized Test Practice

Example 5 Simplify Algebraic Fractions

Multiple-Choice Test Item

Which fraction is $\frac{abc^3}{a^2b}$ written in simplest form?

Ⓐ $\frac{bc^3}{a}$ Ⓑ $\frac{bc^2}{a}$ Ⓒ $\frac{c^3}{a}$ Ⓓ $\frac{c^2}{a}$

Read the Test Item *In simplest form* means that the GCF of the numerator and the denominator is 1.

Solve the Test Item

$$\frac{abc^3}{a^2b} = \frac{a\cancel{b}c^3}{a^2\cancel{b}}$$

Without factoring, you can see that the variable b will not appear in the simplified fraction. That eliminates choices A and B.

$$\frac{abc^3}{a^2b} = \frac{\overset{1}{\cancel{a}} \cdot \overset{1}{\cancel{b}} \cdot c \cdot c \cdot c}{\underset{1}{\cancel{a}} \cdot a \cdot \underset{1}{\cancel{b}}} \quad \text{Factor.}$$

$$= \frac{c^3}{a} \quad \text{Multiply.}$$

The answer is C.

Test-Taking Tip

Shortcuts You can solve some problems without much calculating if you understand the basic mathematical concepts. Look carefully at what is asked, and think of possible shortcuts for solving the problem.

Check for Understanding

Concept Check

1. **Explain** what it means to express a fraction in simplest form.

2. **OPEN ENDED** Write examples of a numerical fraction and an algebraic fraction in simplest form and examples of a numerical fraction and an algebraic fraction not in simplest form.

Guided Practice

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

3. $\frac{2}{14}$ 4. $\frac{9}{15}$ 5. $\frac{5}{11}$ 6. $\frac{25}{40}$ 7. $\frac{64}{68}$

ALGEBRA **Simplify each fraction. If the fraction is already in simplest form, write *simplified*.**

8. $\frac{x}{x^3}$ 9. $\frac{8a^2}{16a}$ 10. $\frac{12c}{15d}$ 11. $\frac{24}{5k}$

12. **MEASUREMENT** Nine inches is what part of 1 yard?

Standardized Test Practice

13. Which fraction is $\frac{25mn}{65n}$ written in simplest form?

Ⓐ $\frac{2m}{6}$ Ⓑ $\frac{5m}{13}$ Ⓒ $\frac{5m}{13n}$ Ⓓ $\frac{25mn}{65}$

www.pre-alg.com/extra_examples

Practice and Apply

Homework Help

For Exercises	See Examples
14–28	1, 2
30–41	4
42–43	3

Extra Practice
See page 732.

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

14. $\frac{3}{18}$ **15.** $\frac{10}{12}$ **16.** $\frac{15}{21}$ **17.** $\frac{8}{36}$ **18.** $\frac{17}{20}$

19. $\frac{18}{44}$ **20.** $\frac{16}{64}$ **21.** $\frac{30}{37}$ **22.** $\frac{34}{38}$ **23.** $\frac{17}{51}$

24. $\frac{51}{60}$ **25.** $\frac{25}{60}$ **26.** $\frac{36}{96}$ **27.** $\frac{133}{140}$ **28.** $\frac{765}{2023}$

29. **AIRCRAFT** A model of Lindbergh's *Spirit of St. Louis* has a wingspan of 18 inches. The wingspan of the actual airplane is 46 feet. Write a fraction in simplest form comparing the wingspan of the model and the wingspan of the actual airplane. (*Hint:* convert 46 feet to inches.)

ALGEBRA **Simplify each fraction. If the fraction is already in simplest form, write *simplified*.**

30. $\frac{a}{a^4}$ **31.** $\frac{y^3}{y}$ **32.** $\frac{12m}{15m}$ **33.** $\frac{40d}{42d}$

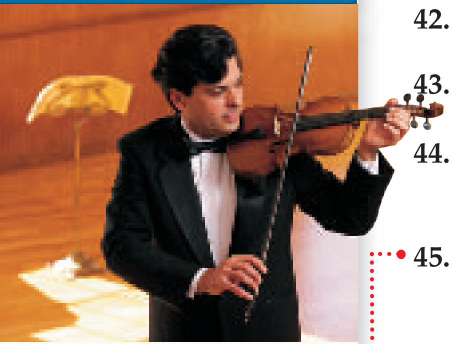
34. $\frac{4k}{19m}$ **35.** $\frac{8t}{64t^2}$ **36.** $\frac{16n}{18n^2p}$ **37.** $\frac{28z^3}{16z}$

38. $\frac{6r}{15rs}$ **39.** $\frac{12cd}{19e}$ **40.** $\frac{30x^2}{51xy}$ **41.** $\frac{17g^2h}{51g}$

42. **MEASUREMENT** Fifteen hours is what part of one day?

43. **MEASUREMENT** Ninety-six centimeters is what part of a meter?

44. **MEASUREMENT** Twelve ounces is what part of a pound? (*Hint*: 1 lb = 16 oz)

Career Choices

Musician

Pitch is the frequency at which an instrument's string vibrates when it is struck. To correct the pitch, a musician must increase or decrease tension in the strings.

Online Research
For information about a career as a musician, visit: www.pre-alg.com/careers

45. **MUSIC** Musical notes C and A sound harmonious together because of their *frequencies*, or vibrations. The fraction that is formed by the two frequencies can be simplified, as shown below.

$$\frac{\text{C}}{\text{A}} = \frac{264}{440} \text{ or } \frac{3}{5}$$

When a fraction formed by two frequencies *cannot* be simplified, the notes sound like noise. Determine whether each pair of notes would sound harmonious together. Explain why or why not.

Note	Frequency (hz)
C	264
D	294
E	330
F	349
G	392
A	440
B	494
C	528

a. E and A **b.** D and F **c.** first C and last C

46. **ANIMALS** The table shows the average amount of food each animal can eat in a day and its average weight. What fraction of its weight can each animal eat per day?

Animal	Daily Amount of Food	Weight of Animal
elephant	450 lb	9000 lb
hummingbird	2 g	3 g
polar bear	25 lb	1500 lb
tiger	20 lb	500 lb

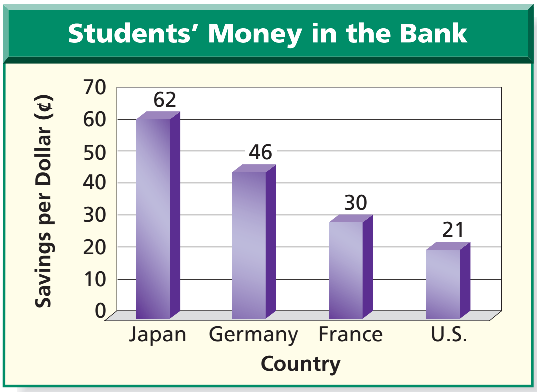
Source: *Animals as Our Companions, Wildlife Fact File*

CONTENTS

WebQuest

Simplifying fractions will help you determine the portion of your daily allowance of fat grams. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

MONEY **For Exercises 47–49, use the graph to write each fraction in simplest form.**


Students' Money in the Bank

47. the fraction of a dollar that students in Japan save

48. the fraction of a dollar that students in the U.S. save

49. a fraction showing the amount of a dollar that students in the U.S. save compared to students in France

50. **CRITICAL THINKING** Is it true that $\frac{23}{53} = \frac{\overset{1}{\cancel{23}}}{\underset{1}{\cancel{53}}}$ or $\frac{2}{5}$? Explain.

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

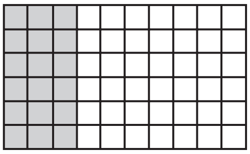
How are simplified fractions useful in representing measurements?

Include the following in your answer:

- an explanation of how measurements represent parts of a whole, and
- examples of fractions that represent measurements.

Standardized Test Practice

52. Which fraction represents the shaded area written in simplest form?



Ⓐ $\frac{9}{30}$ Ⓑ $\frac{3}{10}$ Ⓒ $\frac{6}{20}$ Ⓓ $\frac{30}{100}$

53. Write $\frac{15ab}{25b^2}$ in simplest form.

Ⓐ $\frac{3a}{5b}$ Ⓑ $\frac{15a}{25b}$ Ⓒ $\frac{3ab}{5}$ Ⓓ $\frac{15a}{25b^2}$

Maintain Your Skills

Mixed Review

Find the greatest common factor of each set of numbers or monomials. *(Lesson 4-4)*

54. 9, 15 **55.** 4, 12, 10 **56.** $40x^2, 16x$ **57.** $25a, 30b$

Determine whether each number is *prime* or *composite*. *(Lesson 4-3)*

58. 13 **59.** 34 **60.** 99 **61.** 79

ALGEBRA **Solve each equation. Check your solution.** *(Lesson 3-3)*

62. $t - 18 = 24$ **63.** $30 = 3 + y$ **64.** $-7 = x + 11$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **For each expression, use parentheses to group the numbers together and to group the powers with like bases together.**
*(To review **properties of multiplication**, see Lesson 1-4.)*

Example: $a \cdot 4 \cdot a^3 \cdot 2 = (4 \cdot 2)(a \cdot a^3)$

65. $6 \cdot 7 \cdot k^3$ **66.** $s \cdot t^2 \cdot s \cdot t$

67. $3 \cdot x^4 \cdot (-5) \cdot x^2$ **68.** $5 \cdot n^3 \cdot p \cdot 2 \cdot n \cdot p$

Reading Mathematics

Powers

The phrase *the quantity* is used to indicate parentheses when reading expressions. Recall that an exponent indicates the number of times that the base is used as a factor. Suppose you are to write each of the following in symbols.

Words	Symbols	Examples (Let $x = 2$.)
three times x squared	$3x^2$	$3x^2 = 3 \cdot 2^2$ $= 3 \cdot 4$ Evaluate 2^2. $= 12$ Multiply $3 \cdot 4$.
three times x the quantity squared	$(3x)^2$	$(3x)^2 = (3 \cdot 2)^2$ $= 6^2$ Evaluate $3 \cdot 2$. $= 36$ Square 6.

In the expression $(3x)^2$, parentheses are used to show that $3x$ is used as a factor twice.

$$(3x)^2 = (3x)(3x)$$

The quantity can also be used to describe division of monomials.

Words	Symbols	Examples (Let $x = 2$.)
eight divided by x squared	$\frac{8}{x^2}$	$\frac{8}{x^2} = \frac{8}{2^2}$ $= \frac{8}{4}$ Evaluate 2^2. $= 2$ Divide $8 \div 4$.
eight divided by x the quantity squared	$\left(\frac{8}{x}\right)^2$	$\left(\frac{8}{x}\right)^2 = \left(\frac{8}{2}\right)^2$ $= 4^2$ Evaluate $8 \div 2$. $= 16$ Square 4.

Reading to Learn

State how you would read each expression.

1. $4a^2$

2. $(10x)^5$

3. $\frac{5}{n^3}$

4. $\left(\frac{4}{r}\right)^2$

5. $(m + n)^3$

6. $(a - b)^4$

7. $a - b^4$

8. $\frac{a}{b^4}$

9. $(4c^2)^3$

10. $\left(\frac{8}{c^2}\right)^3$

Determine whether each pair of expressions is equivalent. Write *yes* or *no*.

11. $4ab^5$ and $4(ab)^5$

12. $(2x)^3$ and $8x^3$

13. $(mn)^4$ and $m^4 \cdot n^4$

14. c^3d^3 and cd^3

15. $\frac{x}{y^2}$ and $\left(\frac{x}{y}\right)^2$

16. $\frac{n^2}{r^2}$ and $\left(\frac{n}{r}\right)^2$

CONTENTS

4-6 Multiplying and Dividing Monomials

What You'll Learn

- Multiply monomials.
- Divide monomials.

How are powers of monomials useful in comparing earthquake magnitudes?

For each increase on the Richter scale, an earthquake's vibrations, or *seismic waves*, are 10 times greater. So, an earthquake of magnitude 4 has seismic waves that are 10 times greater than that of a magnitude 3.

Richter Scale	Times Greater than Magnitude 3 Earthquake	Written Using Powers
4	10	10^1
5	10 × 10 = 100	$10^1 \times 10^1 = 10^2$
6	10 × 100 = 1000	$10^1 \times 10^2 = 10^3$
7	10 × 1000 = 10,000	$10^1 \times 10^3 = 10^4$
8	10 × 10,000 = 100,000	$10^1 \times 10^4 = 10^5$

a. Examine the exponents of the factors and the exponents of the products in the last column. What do you observe?

b. **Make a conjecture** about a rule for determining the exponent of the product when you multiply powers with the same base. Test your rule by multiplying $2^2 \cdot 2^4$ using a calculator.

MULTIPLY MONOMIALS Recall that exponents are used to show repeated multiplication. You can use the definition of exponent to help find a rule for multiplying powers with the same base.

$$2^3 \cdot 2^4 = \underbrace{\overbrace{(2 \cdot 2 \cdot 2)}^{\text{3 factors}} \cdot \overbrace{(2 \cdot 2 \cdot 2 \cdot 2)}^{\text{4 factors}}}_{\text{7 factors}}$$

$$= 2^7$$

Notice the sum of the original exponents and the exponent in the final product. This relationship is stated in the following rule.

Key Concept — Product of Powers

- **Words** You can multiply powers with the same base by adding their exponents.
- **Symbols** $a^m \cdot a^n = a^{m+n}$
- **Example** $3^2 \cdot 3^4 = 3^{2+4}$ or 3^6

Study Tip

Common Misconception

When multiplying powers, do not multiply the bases. $3^2 \cdot 3^4 = 3^6$, not 9^6

CONTENTS

Example 1 Multiply Powers

Find $7^3 \cdot 7$.

$7^3 \cdot 7 = 7^3 \cdot 7^1$ $\quad 7 = 7^1$

$= 7^{3+1}$ $\quad$ The common base is 7.

$= 7^4$ $\quad$ Add the exponents.

CHECK $7^3 \cdot 7 = (7 \cdot 7 \cdot 7)(7)$

$= 7 \cdot 7 \cdot 7 \cdot 7$ or 7^4 ✓

Concept Check Can you simplify $2^3 \cdot 3^3$ using the Product of Powers rule? Explain.

ALGEBRA CONNECTION

Monomials can also be multiplied using the rule for the product of powers.

Example 2 Multiply Monomials

Find each product.

a. $x^5 \cdot x^2$

$x^5 \cdot x^2 = x^{5+2}$ $\quad$ The common base is x.

$= x^7$ $\quad$ Add the exponents.

b. $(-4n^3)(2n^6)$

$(-4n^3)(2n^6) = (-4 \cdot 2)(n^3 \cdot n^6)$ $\quad$ Use the Commutative and Associative Properties.

$= (-8)(n^{3+6})$ $\quad$ The common base is n.

$= -8n^9$ $\quad$ Add the exponents.

Study Tip

Look Back
To review the **Commutative and Associative Properties of Multiplication**, see Lesson 1-4.

DIVIDE MONOMIALS You can also write a rule for finding quotients of powers.

$$\frac{2^6}{2^1} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}{2} \begin{array}{l} \leftarrow \boxed{\text{6 factors}} \\ \leftarrow \boxed{\text{1 factor}} \end{array}$$

$$= \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot \cancelto{1}{2}}{\underset{1}{\cancel{2}}}$$

Divide the numerator and the denominator by the GCF, 2.

$$= 2^5 \leftarrow \boxed{\text{5 factors}}$$

Simplify.

Compare the difference between the original exponents and the exponent in the final quotient. This relationship is stated in the following rule.

Key Concept — Quotient of Powers

- **Words** You can divide powers with the same base by subtracting their exponents.
- **Symbols** $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$
- **Example** $\frac{4^5}{4^2} = 4^{5-2}$ or 4^3

CONTENTS

Example 3 Divide Powers

Find each quotient.

a. $\frac{5^7}{5^4}$

$\frac{5^7}{5^4} = 5^{7-4}$ The common base is 5.

$= 5^3$ Subtract the exponents.

b. $\frac{y^5}{y^3}$

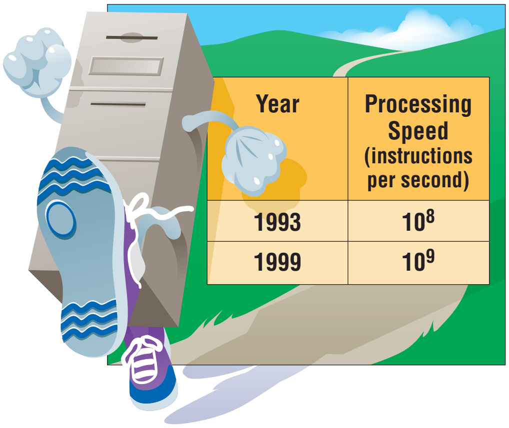
$\frac{y^5}{y^3} = y^{5-3}$ The common base is y.

$= y^2$ Subtract the exponents.

Concept Check Can you simplify $\frac{x^7}{y^2}$ using the Quotient of Powers rule? Why or why not?

Example 4 Divide Powers to Solve a Problem

COMPUTERS **The table compares the processing speeds of a specific type of computer in 1993 and in 1999. Find how many times faster the computer was in 1999 than in 1993.**



Year	Processing Speed (instructions per second)
1993	10^8
1999	10^9

Source: *The Intel Microprocessor Quick Reference Guide*

Write a division expression to compare the speeds.

$\frac{10^9}{10^8} = 10^{9-8}$ Subtract the exponents.

$= 10^1$ or 10 Simplify.

So, the computer was 10 times faster in 1999 than in 1993.

Reading Math

How Many/How Much

How many times faster indicates that division is to be used to solve the problem. If the question had said *how much faster*, then subtraction ($10^9 - 10^8$) would have been used to solve the problem.

Check for Understanding

Concept Check

1. **State** whether you could use the Product of Powers rule, Quotient of Powers rule, or neither to find $m^5 \cdot n^4$. Explain.

2. **Explain** whether $4^8 \cdot 4^6$ and $4^4 \cdot 4^{10}$ are equivalent expressions.

3. **OPEN ENDED** Write a multiplication expression whose product is 5^3.

Guided Practice

Find each product or quotient. Express using exponents.

4. $9^3 \cdot 9^2$
5. $a \cdot a^5$
6. $(n^4)(n^4)$
7. $-3x^2(4x^3)$
8. $\frac{3^8}{3^5}$
9. $\frac{10^5}{10^3}$
10. $\frac{x^3}{x}$
11. $\frac{a^{10}}{a^6}$

Application

12. **EARTHQUAKES** In 2000, an earthquake measuring 8 on the Richter scale struck Indonesia. Two months later, an earthquake of magnitude 5 struck northern California. How many times greater were the seismic waves in Indonesia than in California? (*Hint:* Let 10^8 and 10^5 represent the earthquakes, respectively.)

Practice and Apply

Homework Help

For Exercises	See Examples
13–24	1, 2
25–36	3
41–44	4

Extra Practice
See page 732.

Find each product or quotient. Express using exponents.

13. $3^3 \cdot 3^2$

14. $6 \cdot 6^7$

15. $d^4 \cdot d^6$

16. $10^4 \cdot 10^3$

17. $n^8 \cdot n$

18. $t^2 \cdot t^4$

19. $9^4 \cdot 9^5$

20. $a^6 \cdot a^6$

21. $2y \cdot 9y^4$

22. $(5r^3)(4r^4)$

23. $ab^5 \cdot 8a^2b^5$

24. $10x^3y \cdot (-2xy^2)$

25. $\frac{5^5}{5^2}$

26. $\frac{8^4}{8^3}$

27. $b^6 \div b^3$

28. $10^{10} \div 10^2$

29. $\frac{m^{20}}{m^8}$

30. $\frac{a^8}{a^8}$

31. $\frac{(-2)^6}{(-2)^5}$

32. $\frac{(-x)^5}{(-x)}$

33. $\frac{n^3(n^5)}{n^2}$

34. $\frac{s^7}{s \cdot s^2}$

35. $\left(\frac{k^3}{k}\right)\left(\frac{m^2}{m}\right)$

36. $\left(\frac{15}{5}\right)\left(\frac{n^9}{n}\right)$

37. the product of nine to the fourth power and nine cubed

38. the quotient of k to the fifth power and k squared

39. What is the product of 7^3, 7^5, and 7?

40. Find $a^4 \cdot a^6 \div a^2$.

CHEMISTRY **For Exercises 41–43, use the following information.**
The pH of a solution describes its acidity. Neutral water has a pH of 7. Each one-unit *decrease* in the pH means that the solution is 10 times more acidic. For example, a pH of 4 is 10 times more acidic than a pH of 5.

41. Suppose the pH of a lake is 5 due to acid rain. How much more acidic is the lake than neutral water?

42. Use the information at the left to find how much more acidic vinegar is than baking soda.

43. Cola is 10^4 times more acidic than neutral water. What is the pH value of cola?

More About. . .

Chemistry

The pH values of different kitchen items are shown below.

Item	pH
lemon juice	2
vinegar	3
tomatoes	4
baking soda	9

Source: *Biology*, Raven

44. **LIFE SCIENCE** When bacteria reproduce, they split so that one cell becomes two. The number of cells after t cycles of reproduction is 2^t.

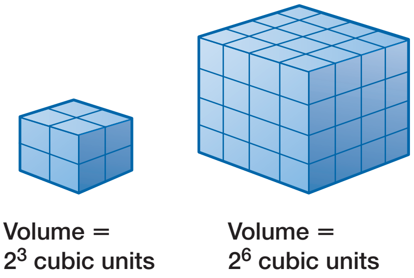
a. *E. coli* reproduce very quickly, about every 15 minutes. If there are 100 *E. coli* in a dish now, how many will there be in 30 minutes?

b. How many times more *E. coli* are there in a population after 3 hours than there were after 1 hour?

GEOMETRY **For Exercises 45 and 46, use the information in the figures.**

45. How many times greater is the length of the edge of the larger cube than the smaller one?

46. How many times greater is the volume of the larger cube than the smaller one?



Find each missing exponent.

47. $(4^{\bullet})(4^3) = 4^{11}$

48. $\frac{t^{\bullet}}{t^2} = t^{14}$

49. $\frac{13^5}{13^{\bullet}} = 1$

CONTENTS

50. **CRITICAL THINKING** Use the laws of exponents to show why the value of *any* nonzero number raised to the zero power equals 1.

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are powers of monomials useful in comparing earthquake magnitudes?

Include the following in your answer:

- a description of the Richter scale, and
- a comparison of two earthquakes of different magnitudes by using the Quotient of Powers rule.

Standardized Test Practice

52. Multiply $7xy$ and $x^{14}z$.

Ⓐ $7x^{15}yz$ Ⓑ $7x^{15}y$ Ⓒ $7x^{13}yz$ Ⓓ $x^{15}yz$

53. Find the quotient $a^5 \div a$.

Ⓐ a^5 Ⓑ a^4 Ⓒ a^6 Ⓓ a

Maintain Your Skills

Mixed Review

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*. *(Lesson 4-5)*

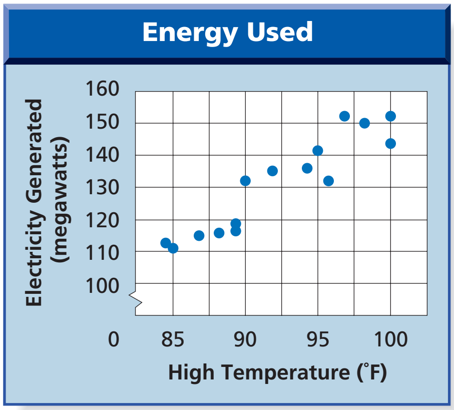
54. $\frac{12}{40}$ **55.** $\frac{20}{53}$ **56.** $\frac{8n^2}{32n}$ **57.** $\frac{6x^3}{4x^2y}$

Find the greatest common factor of each set of numbers or monomials. *(Lesson 4-4)*

58. 36, 4 **59.** 18, 28 **60.** 42, 54 **61.** $9a$, $10a^3$

62. Evaluate $|a| - |b| \cdot |c|$ if $a = -16$, $b = 2$, and $c = 3$. *(Lesson 2-1)*

63. **ENERGY** The graph shows the high temperature and the maximum amount of electricity that was used during each of fifteen summer days. Do the data show a *positive*, *negative*, or *no* relationship? Explain. *(Lesson 1-7)*



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate each expression if $x = 10$, $y = -5$, and $z = 4$. Write as a fraction in simplest form.**
*(To review **evaluating expressions**, see Lesson 1-3.)*

64. $\frac{1}{x}$ **65.** $\frac{y}{50}$ **66.** $\frac{z}{100}$

67. $\frac{1}{zy}$ **68.** $\frac{1}{x \cdot x}$ **69.** $\frac{1}{(z)(z)(z)}$

Algebra Activity

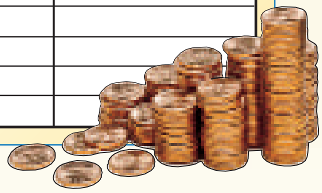
A Follow-Up of Lesson 4-6

A Half-Life Simulation

A radioactive material such as uranium decomposes or decays in a regular manner best described as a *half-life*. A half-life is the time it takes for half of the atoms in the sample to decay.

Collect the Data

Number of Half-Lives	Number of Pennies That Remain
1	
2	
3	
4	
5	



Step 1 Place 50 pennies heads up in a shoebox. Put the lid on the box and shake it up and down one time. This simulates one half-life.

Step 2 Open the lid of the box and remove all the pennies that are now tails up. In a table like the one at the right, record the number of pennies that remain.

Step 3 Put the lid back on the box and again shake the box up and down one time. This represents another half-life.

Step 4 Open the lid. Remove all the tails up pennies. Count the pennies that remain.

Step 5 Repeat the half-life of decay simulation until less than five pennies remain in the shoebox.

Analyze the Data

1. On grid paper, draw a coordinate grid in which the x-axis represents the number of half-lives and the y-axis represents the number of pennies that remain. Plot the points (number of half-lives, number of remaining pennies) from your table.

2. Describe the graph of the data.

After each half-life, you expect to remove about one-half of the pennies. So, you expect about one-half to remain. The expressions at the right represent the average number of pennies that remain if you start with 50, after one, two, and three half-lives.

one half-life:	$50\left(\frac{1}{2}\right) = 50\left(\frac{1}{2}\right)^1$
two half-lives:	$50\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 50\left(\frac{1}{2}\right)^2$
three half-lives:	$50\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 50\left(\frac{1}{2}\right)^3$

Make a Conjecture

3. Use the expressions to predict how many pennies remain after three half-lives. Compare this number to the number in the table above. Explain any differences.

4. Suppose you started with 1000 pennies. Predict how many pennies would remain after three half-lives.

CONTENTS

4-7 Negative Exponents

What You'll Learn

- Write expressions using negative exponents.
- Evaluate numerical expressions containing negative exponents.

How do negative exponents represent repeated division?

Copy the table at the right.

a. Describe the pattern of the powers in the first column. Continue the pattern by writing the next two powers in the table.

b. Describe the pattern of values in the second column. Then complete the second column.

c. Verify that the powers you wrote in part **a** are equal to the values that you found in part **b**.

d. Determine how 3^{-1} should be defined.

Power	Value
2^6	64
2^5	32
2^4	16
2^3	8
2^2	4
2^1	2
?	?
?	?

ALGEBRA CONNECTION

NEGATIVE EXPONENTS Extending the pattern at the right shows that 2^{-1} can be defined as $\frac{1}{2}$.

$$\begin{tikzcd}[row sep=small]
2^2 = 4 \arrow[d, bend left, "\div 2"] \\
2^1 = 2 \arrow[d, bend left, "\div 2"] \\
2^0 = 1 \arrow[d, bend left, "\div 2"] \\
2^{-1} = \frac{1}{2}
\end{tikzcd}$$

You can apply the Quotient of Powers rule and the definition of a power to $\frac{x^3}{x^5}$ and write a general rule about negative powers.

Method 1 Quotient of Powers

$$\frac{x^3}{x^5} = x^{3-5}$$
$$= x^{-2}$$

Method 2 Definition of Power

$$\frac{x^3}{x^5} = \frac{\overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}} \cdot \overset{1}{\cancel{x}}}{\underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot \underset{1}{\cancel{x}} \cdot x \cdot x}$$
$$= \frac{1}{x \cdot x}$$
$$= \frac{1}{x^2}$$

Since $\frac{x^3}{x^5}$ cannot have two different values, you can conclude that $x^{-2} = \frac{1}{x^2}$. This suggests the following definition.

Key Concept — *Negative Exponents*

- **Symbols** $a^{-n} = \frac{1}{a^n}$, for $a \neq 0$ and any integer n
- **Example** $5^{-4} = \frac{1}{5^4}$

www.pre-alg.com/extra_examples

CONTENTS

Example 1 Use Positive Exponents

Write each expression using a positive exponent.

a. 6^{-2}

$6^{-2} = \frac{1}{6^2}$ Definition of negative exponent

b. x^{-5}

$x^{-5} = \frac{1}{x^5}$ Definition of negative exponent

One way to write a fraction as an equivalent expression with negative exponents is to use prime factorization.

Example 2 Use Negative Exponents

Write $\frac{1}{9}$ as an expression using a negative exponent.

$\frac{1}{9} = \frac{1}{3 \cdot 3}$ Find the prime factorization of 9.

$= \frac{1}{3^2}$ Definition of exponent

$= 3^{-2}$ Definition of negative exponent

Concept Check How can $\frac{1}{9}$ be written as an expression with a negative exponent other than 3^{-2}?

Negative exponents are often used in science when dealing with very small numbers. Usually the number is a power of ten.

Example 3 Use Exponents to Solve a Problem

WATER **A molecule of water contains two hydrogen atoms and one oxygen atom. A hydrogen atom is only 0.00000001 centimeter in diameter. Write the decimal as a fraction and as a power of ten.**

The digit 1 is in the 100-millionths place.

$0.00000001 = \frac{1}{100{,}000{,}000}$ Write the decimal as a fraction.

$= \frac{1}{10^8}$ $100{,}000{,}000 = 10^8$

$= 10^{-8}$ Definition of negative exponent

More About. . .

Water

A single drop of water contains about 10^{20} molecules.

Source: www.composite.about.com

EVALUATE EXPRESSIONS Algebraic expressions containing negative exponents can be written using positive exponents and evaluated.

Example 4 Algebraic Expressions with Negative Exponents

Evaluate n^{-3} if $n = 2$.

$n^{-3} = 2^{-3}$ Replace n with 2.

$= \frac{1}{2^3}$ Definition of negative exponent

$= \frac{1}{8}$ Find 2^3.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a convincing argument that $3^0 = 1$ using the fact that $3^4 = 81$, $3^3 = 27$, $3^2 = 9$, and $3^1 = 3$.

2. Order 8^{-8}, 8^3 and 8^0 from greatest to least. Explain your reasoning.

Guided Practice

Write each expression using a positive exponent.

3. 5^{-2}
4. $(-7)^{-1}$
5. t^{-6}
6. n^{-2}

Write each fraction as an expression using a negative exponent other than −1.

7. $\frac{1}{3^4}$
8. $\frac{1}{9^2}$
9. $\frac{1}{49}$
10. $\frac{1}{8}$

ALGEBRA **Evaluate each expression if $a = 2$ and $b = -3$.**

11. a^{-5}
12. $(ab)^{-2}$

Application

13. **MEASUREMENT** A unit of measure called a *micron* equals 0.001 millimeter. Write this number using a negative exponent.

Practice and Apply

Homework Help

For Exercises	See Examples
14–27	1
28–35	2
36–39	3
40–43	4

Extra Practice
See page 732.

Write each expression using a positive exponent.

14. 4^{-1}
15. 5^{-3}
16. $(-6)^{-2}$
17. $(-3)^{-3}$
18. 3^{-5}
19. 10^{-4}
20. p^{-1}
21. a^{-10}
22. d^{-3}
23. q^{-4}
24. $2s^{-5}$
25. $\frac{1}{x^{-2}}$

For Exercises 26 and 27, write each expression using a positive exponent. Then write as a decimal.

26. A snowflake weighs 10^{-6} gram.

27. A small bird uses 5^{-4} Joules of energy to sing a song.

Write each fraction as an expression using a negative exponent other than −1.

28. $\frac{1}{9^4}$
29. $\frac{1}{5^5}$
30. $\frac{1}{8^3}$
31. $\frac{1}{13^2}$
32. $\frac{1}{100}$
33. $\frac{1}{81}$
34. $\frac{1}{27}$
35. $\frac{1}{16}$

Write each decimal using a negative exponent.

36. 0.1
37. 0.01
38. 0.0001
39. 0.00001

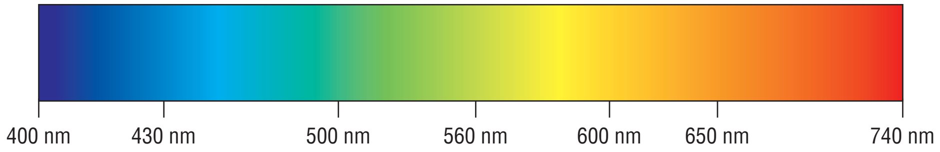
ALGEBRA **Evaluate each expression if $w = -2$, $x = 3$, and $y = -1$.**

40. x^{-4}
41. w^{-7}
42. 8^w
43. $(xy)^{-6}$

www.pre-alg.com/self_check_quiz

CONTENTS

44. **PHYSICAL SCIENCE** A nanometer is equal to a billionth of a meter. The visible range of light waves ranges from 400 nanometers (violet) to 740 nanometers (red).



a. Write one billionth of a meter as a fraction and with a negative exponent.

b. Use the information at the left to express the greatest wavelength of an X ray in meters. Write the expression using a negative exponent.

More About. . .

Physical Science

The wavelengths of X rays are between 1 and 10 nanometers.

Source: *Biology*, Raven

45. **ANIMALS** A common flea 2^{-4} inch long can jump about 2^3 inches high. How many times its body size can a flea jump?

46. **MEDICINE** Which type of molecule in the table has a greater mass? How many times greater is it than the other type?

Molecule	Mass (kg)
penicillin	10^{-18}
insulin	10^{-23}

Use the Product of Power and Quotient of Power rules to simplify each expression.

47. $x^{-2} \cdot x^{-3}$

48. $r^{-5} \cdot r^9$

49. $\frac{x^4}{x^7}$

50. $\frac{y^6}{y^{-10}}$

51. $\frac{a^4b^{-4}}{ab^{-2}}$

52. $\frac{36s^3t^5}{12s^6t^{-3}}$

53. **CRITICAL THINKING** Using what you learned about exponents, is $(x^3)^{-2} = (x^{-2})^3$? Why or why not?

54. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How do negative exponents represent repeated division?

Include the following in your answer:

- an example of a power containing a negative exponent written in fraction form, and
- a discussion about whether the value of a fraction such as $\frac{1}{2^n}$ increases or decreases as the value of n increases.

Standardized Test Practice

55. Which is 15^{-5} written as a fraction?

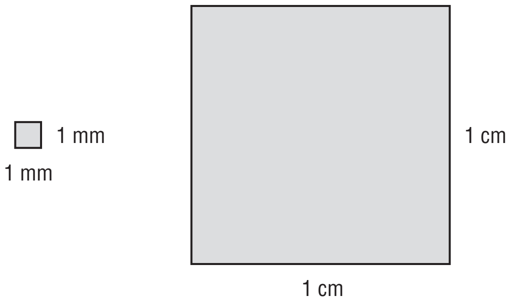
Ⓐ $\frac{1}{5^5}$ Ⓑ $\frac{1}{15}$

Ⓒ $\frac{1}{15^5}$ Ⓓ $-\frac{1}{15^5}$

56. One square millimeter equals __?__ square centimeter(s). (*Hint*: 1 cm = 10 mm)

Ⓐ 10^{-1} Ⓑ 10^{-2}

Ⓒ 10^{-3} Ⓓ 10^3



CONTENTS

Extending the Lesson Numbers less than 1 can also be expressed in expanded form.

Example: $0.568 = 0.5 + 0.06 + 0.008$
$= (5 \times 10^{-1}) + (6 \times 10^{-2}) + (8 \times 10^{-3})$

Express each number in expanded form.

57. 0.9 **58.** 0.24 **59.** 0.173 **60.** 0.5875

Maintain Your Skills

Mixed Review **Find each product or quotient. Express your answer using exponents.** *(Lesson 4-6)*

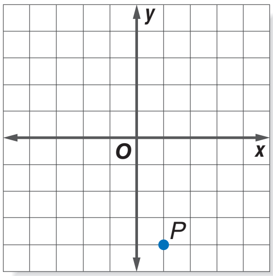
61. $3^6 \cdot 3$ **62.** $x^2 \cdot x^4$ **63.** $\frac{5^5}{5^2}$

64. **ALGEBRA** Write $\frac{16n^3}{8n}$ in simplest form. *(Lesson 4-5)*

ALGEBRA **Use the Distributive Property to rewrite each expression.** *(Lesson 3-1)*

65. $8(y + 6)$ **66.** $(9 + k)(-2)$ **67.** $(n - 3)5$

68. Write the ordered pair that names point P. *(Lesson 2-6)*



Getting Ready for the Next Lesson **PREREQUISITE SKILL** **Find each product.**
*(To review **multiplying decimals**, see page 715.)*

69. 7.2 × 100 **70.** 1.6 × 1000 **71.** 4.05 × 10

72. 3.8 × 0.01 **73.** 5.0 × 0.0001 **74.** 9.24 × 0.1

Practice Quiz 2

Lessons 4-4 through 4-7

Find the GCF of each set of numbers or monomials. *(Lesson 4-4)*

1. 15, 20 **2.** 24, 30 **3.** $2ab, 6a^2$

Write each fraction in simplest form. *(Lesson 4-5)*

4. **SCHOOL** What fraction of days were you absent from school this nine-week period if you were absent twice out of 44 days?

5. **COMMUNICATION** What fraction of E-mail messages did you respond to, if you responded to 6 out of a total of 15 messages?

ALGEBRA **Find each product or quotient. Express using exponents.** *(Lesson 4-6)*

6. $4^2 \cdot 4^4$ **7.** $(n^4)(-2n^3)$ **8.** $\frac{q^9}{q^4}$

9. **ALGEBRA** Write b^{-6} as an expression using a positive exponent. *(Lesson 4-7)*

10. **ALGEBRA** Evaluate x^{-5} if $x = -2$. *(Lesson 4-7)*

4-8 Scientific Notation

What You'll Learn

- Express numbers in standard form and in scientific notation.
- Compare and order numbers written in scientific notation.

Vocabulary

- scientific notation

Why is scientific notation an important tool in comparing real-world data?

Track Length	Track Width
5000 meters	0.5 micron

A compact disc or CD has a single spiral track that stores data. It circles from the inside of the disc to the outside. If the track were stretched out in a straight line, it would be 0.5 micron wide and over 5000 meters long.

a. Write the track length in millimeters.

b. Write the track width in millimeters. (1 micron = 0.001 millimeter.)

SCIENTIFIC NOTATION When you deal with very large numbers like 5,000,000 or very small numbers like 0.0005, it is difficult to keep track of the place value. Numbers such as these can be written in **scientific notation**.

Key Concept — *Scientific Notation*

- **Words** A number is expressed in scientific notation when it is written as the product of a factor and a power of 10. The factor must be greater than or equal to 1 and less than 10.
- **Symbols** $a \times 10^n$, where $1 \le a < 10$ and n is an integer
- **Examples** $5{,}000{,}000 = 5.0 \times 10^6$ $\quad 0.0005 = 5.0 \times 10^{-4}$

Study Tip

Powers of Ten

To multiply by a power of 10,

- move the decimal point to the right if the exponent is positive, and
- move the decimal point to the left if the exponent is negative.

In each case, the exponent tells you how many places to move the decimal point.

Concept Check Is 13.0×10^2 written in scientific notation? Why or why not?

You can express numbers that are in scientific notation in standard form.

Example 1 *Express Numbers in Standard Form*

Express each number in standard form.

a. $\mathbf{3.78 \times 10^6}$

$3.78 \times 10^6 = 3.78 \times 1{,}000{,}000$ $\quad 10^6 = 1{,}000{,}000$

$= 3{,}\underline{780{,}000}$ $\quad$ Move the decimal point 6 places to the right.

b. $\mathbf{5.1 \times 10^{-5}}$

$5.1 \times 10^{-5} = 5.1 \times 0.00001$ $\quad 10^{-5} = 0.00001$

$= 0.\underline{00005}1$ $\quad$ Move the decimal point 5 places to the left.

To write a number in scientific notation, place the decimal point after the first nonzero digit. Then find the power of 10.

Example 2 Express Numbers in Scientific Notation

Express each number in scientific notation.

a. 60,000,000

$60{,}000{,}000 = 6.0 \times 10{,}000{,}000$ The decimal point moves 7 places.

$= 6.0 \times 10^7$ The exponent is positive.

b. 32,800

$32{,}800 = 3.28 \times 10{,}000$ The decimal point moves 4 places.

$= 3.28 \times 10^4$ The exponent is positive.

c. 0.0049

$0.0049 = 4.9 \times 0.001$ The decimal point moves 3 places.

$= 4.9 \times 10^{-3}$ The exponent is negative.

Study Tip

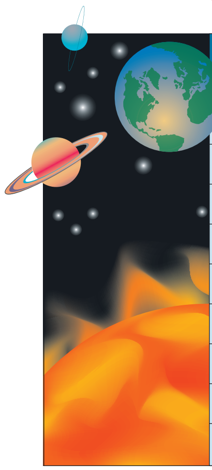
Positive and Negative Exponents

When the number is 1 or greater, the exponent is *positive*. When the number is between 0 and 1, the exponent is *negative*.

People who make comparisons or compute with extremely large or extremely small numbers use scientific notation.

Example 3 Use Scientific Notation to Solve a Problem

SPACE **The table shows the planets and their distances from the Sun. Light travels 300,000 kilometers per second. Estimate how long it takes light to travel from the Sun to Pluto. (*Hint:* Recall that distance = rate × time.)**



Planet	Distance from the Sun (km)
Mercury	5.80×10^7
Venus	1.03×10^8
Earth	1.55×10^8
Mars	2.28×10^8
Jupiter	7.78×10^8
Saturn	1.43×10^9
Uranus	2.87×10^9
Neptune	4.50×10^9
Pluto	5.90×10^9

Explore You know that the distance from the Sun to Pluto is 5.90×10^9 kilometers and that the speed of light is 300,000 kilometers per second.

Plan To find the time, solve the equation $d = rt$. Since you are estimating, round the distance 5.90×10^9 to 6.0×10^9. Write the rate 300,000 as 3.0×10^5.

Solve

$d = rt$ Write the formula.

$6.0 \times 10^9 = (3.0 \times 10^5)t$ Replace d with 6.0×10^9 and r with 3.0×10^5.

$\frac{6.0 \times 10^9}{3.0 \times 10^5} \approx \frac{(3.0 \times 10^5)t}{3.0 \times 10^5}$ Divide each side by 3.0×10^5.

$2.0 \times 10^4 \approx t$ Divide 6.0 by 3.0 and 10^9 by 10^5.

So, it would take about 2.0×10^4 seconds or about 6 hours for light to travel from the Sun to Pluto.

Examine Use estimation to check the reasonableness of these results.

Study Tip

Calculator

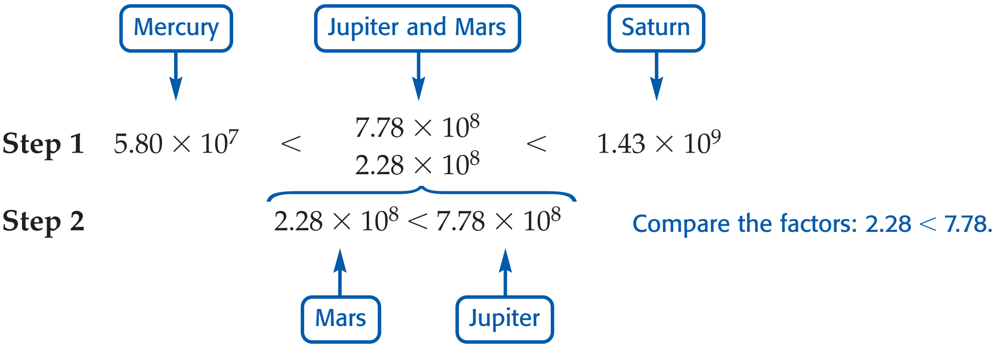
To enter a number in scientific notation on a calculator, enter the decimal portion, press 2nd [EE], then enter the exponent. A calculator in Sci mode will display answers in scientific notation.

COMPARE AND ORDER NUMBERS To compare and order numbers in scientific notation, first compare the exponents. With positive numbers, any number with a greater exponent is greater. If the exponents are the same, compare the factors.

Example 4 Compare Numbers in Scientific Notation

SPACE **Refer to the table in Example 3. Order Mars, Jupiter, Mercury, and Saturn from least to greatest distance from the Sun.**

First, order the numbers according to their exponents. Then, order the numbers with the same exponent by comparing the factors.



Step 1 $5.80 \times 10^7 < \begin{matrix} 7.78 \times 10^8 \\ 2.28 \times 10^8 \end{matrix} < 1.43 \times 10^9$

Step 2 $2.28 \times 10^8 < 7.78 \times 10^8$ Compare the factors: $2.28 < 7.78$.

So, the order is Mercury, Mars, Jupiter, and Saturn.

Check for Understanding

Concept Check

1. **Explain** the relationship between a number in standard form and the sign of the exponent when the number is written in scientific notation.

2. **OPEN ENDED** Write a number in standard form and then write the number in scientific notation, explaining each step that you used.

Guided Practice

Express each number in standard form.

3. 3.08×10^{-4}
4. 1.4×10^2
5. 8.495×10^5

Express each number in scientific notation.

6. 80,000,000
7. 697,000
8. 0.059
9. the diameter of a spider's thread, 0.001 inch

Applications

10. **SPACE** Refer to the table in Example 3 on page 187. To the nearest second, how long does it take light to travel from the Sun to Earth?

11. **SPACE** Rank the planets in the table at the right by diameter, from least to greatest.

Planet	Diameter (km)
Earth	1.276×10^4
Mars	6.790×10^3
Venus	1.208×10^4

Practice and Apply

Homework Help

For Exercises	See Examples
12–20	1
21–33	2
34	3
39–41	4

Extra Practice
See page 733.

Express each number in standard form.

12. 4.24×10^2 **13.** 5.72×10^4 **14.** 3.347×10^{-1}

15. 5.689×10^{-3} **16.** 6.1×10^4 **17.** 9.01×10^{-2}

18. 1.399×10^5 **19.** 2.505×10^3 **20.** 1.5×10^{-4}

Express each number in scientific notation.

21. 2,000,000 **22.** 499,000 **23.** 0.006

24. 0.0125 **25.** 50,000,000 **26.** 39,560

27. 5,894,000 **28.** 0.000078 **29.** 0.000425

30. The flow rate of some Antarctic glaciers is 0.00031 mile per hour.

31. Humans blink about 6.25 million times a year.

32. The number of possible ways that a player can play the first four moves in a chess game is 3 billion.

33. A particle of dust floating in the air weighs 0.000000753 gram.

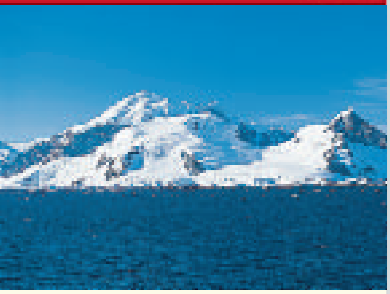
34. **SPACE** Refer to the table in Example 3 on page 187. To the nearest second, how long does it take light to travel from the Sun to Venus?

Choose the greater number in each pair.

35. $2.3 \times 10^5, 1.7 \times 10^5$ **36.** $1.8 \times 10^3, 1.9 \times 10^{-1}$

37. 5.2×10^2, 5000 **38.** $0.012, 1.6 \times 10^{-1}$

More About. . .



Oceans

In 2000, the International Hydrographic Organization named a fifth world ocean near Antarctica, called the *Southern Ocean*. It is larger than the Arctic Ocean and smaller than the Indian Ocean.

Source: www.geography.about.com

39. **OCEANS** Rank the oceans in the table at the right by area from least to greatest.

Ocean	Area (sq mi)
Arctic	5.44×10^6
Atlantic	3.18×10^7
Indian	2.89×10^7
Pacific	6.40×10^7

40. **MEASUREMENT** The table at the right shows the values of different prefixes that are used in the metric system. Write the units attometer, gigameter, kilometer, nanometer, petameter, and picometer in order from greatest to least measure.

Metric Measures	
Prefix	**Meaning**
atto	10^{-18}
giga	10^9
kilo	10^3
nano	10^{-9}
peta	10^{15}
pico	10^{-12}

41. Order 6.1×10^4, 6100, 6.1×10^{-5}, 0.0061, and 6.1×10^{-2} from least to greatest.

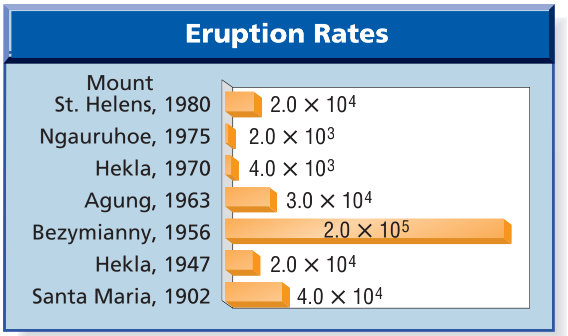
42. Write $(6 \times 10^0) + (4 \times 10^{-3}) + (3 \times 10^{-5})$ in standard form.

43. Write $(4 \times 10^4) + (8 \times 10^3) + (3 \times 10^2) + (9 \times 10^1) + (6 \times 10^0)$ in standard form.

Convert the numbers in each expression to scientific notation. Then evaluate the expression. Express in scientific notation and in decimal notation.

44. $\frac{20{,}000}{0.01}$ **45.** $\frac{(420{,}000)(0.015)}{0.025}$ **46.** $\frac{(0.078)(8.5)}{0.16(250{,}000)}$

PHYSICAL SCIENCE **For Exercises 47 and 48, use the graph.** The graph shows the maximum amounts of lava in cubic meters per second that erupted from seven volcanoes in the last century.



Source: University of Alaska

47. Rank the volcanoes in order from greatest to least eruption rate.

48. How many times larger was the Santa Maria eruption than the Mount St. Helens eruptions?

Online Research **Data Update** How do the eruption rates of other volcanoes compare with those in the graph? Visit www.pre-alg.com/data_update to learn more.

49. **CRITICAL THINKING** In standard form, $3.14 \times 10^{-4} = 0.000314$, and $3.14 \times 10^{4} = 31{,}400$. What is 3.14×10^{0} in standard form?

50. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is scientific notation an important tool in comparing real-world data?

Include the following in your answer:

- some real-world data that is written in scientific notation, and
- the advantages of using scientific notation to compare data.

Standardized Test Practice

51. If the bodies of water in the table are ordered from least to greatest area, which would be third in the list?

Ⓐ Lake Huron　Ⓑ Lake Victoria
Ⓒ Red Sea　Ⓓ Great Salt Lake

Body of Water	Area (km^2)
Lake Huron	5.7×10^4
Lake Victoria	6.9×10^4
Red Sea	4.4×10^5
Great Salt Lake	4.7×10^3

52. Which is 5.80×10^{-4} written in standard form?

Ⓐ 58,000　Ⓑ 5800　Ⓒ 0.58　Ⓓ 0.00058

Maintain Your Skills

Mixed Review **ALGEBRA** **Evaluate each expression if $s = -2$ and $t = 3$.** *(Lesson 4-7)*

53. t^{-4}　**54.** s^{-5}　**55.** 7^{s}

ALGEBRA **Find each product or quotient. Express using exponents.** *(Lesson 4-6)*

56. $4^4 \cdot 4^7$　**57.** $3a^2 \cdot 5a^2$　**58.** $c^5 \div c^2$

59. **BUSINESS** Online Book Distributors add a $2.50 shipping and handling charge to the total price of every order. If the cost of books in an order is c, write an expression for the total cost. *(Lesson 1-3)*

CONTENTS

Chapter 4

Study Guide and Review

Vocabulary and Concept Check

algebraic fraction (p. 170)
base (p. 153)
base two (p. 158)
binary (p. 158)
composite number (p. 159)
divisible (p. 148)
expanded form (p. 154)
exponent (p. 153)
factor (p. 161)
factors (p. 148)
factor tree (p. 160)
greatest common factor (GCF) (p. 164)
monomial (p. 150)
power (p. 153)
prime factorization (p. 160)
prime number (p. 159)
scientific notation (p. 186)
simplest form (p. 169)
standard form (p. 154)
Venn diagram (p. 164)

Determine whether each statement is *true* or *false*. If false, replace the underlined word or number to make a true statement.

1. A prime number is a whole number that has exactly two factors, 1 and itself.
2. Numbers expressed using exponents are called powers.
3. The number 7 is a factor of 49 because it divides 49 with a remainder of zero.
4. A monomial is a number, a variable, or a sum of numbers and/or variables.
5. The number 64 is a composite number.
6. The number 9,536 is written in standard form.
7. To write a fraction in simplest form, divide the numerator and the denominator by the GCF.
8. A fraction is in simplest form when the GCF of the numerator and the denominator is 2.

Lesson-by-Lesson Review

4-1 Factors and Monomials

See pages 148–152.

Concept Summary

- Numbers that are multiplied to form a product are called factors.

Example **Determine whether 102 is divisible by 2, 3, 5, 6, or 10.**

2: Yes, the ones digit is divisible by 2.
3: Yes, the sum of the digits is 3, and 3 is divisible by 3.
5: No, the ones digit is not 0 or 5.
6: Yes, the number is divisible by 2 and by 3.
10: No, the ones digit is not 0.

Exercises **Use divisibility rules to determine whether each number is divisible by 2, 3, 5, 6, or 10.** *See Example 1 on page 149.*

9. 111 **10.** 405 **11.** 635 **12.** 863
13. 582 **14.** 2124 **15.** 700 **16.** 4200

www.pre-alg.com/vocabulary_review

4-2 Powers and Exponents

See pages 153–157.

Concept Summary

- An exponent is a shorthand way of writing repeated multiplication.
- A number can be written in expanded form by using exponents.
- Follow the order of operations to evaluate algebraic expressions containing exponents.

Example **Evaluate $4(a + 2)^3$ if $a = -5$.**

$4(a + 2)^3 = 4(-5 + 2)^3$ Replace a with -5.

$= 4(-3)^3$ Simplify the expression inside the parentheses.

$= 4(-27)$ Evaluate $(-3)^3$.

$= -108$ Simplify.

Exercises **Evaluate each expression if $x = -3$, $y = 4$, and $z = -2$.**
See Example 3 on page 155.

17. 3^3 **18.** 10^4 **19.** $(-5)^2$ **20.** y^3

21. $10x^2$ **22.** xy^3 **23.** $7y^0z^4$ **24.** $2(3z + 4)^5$

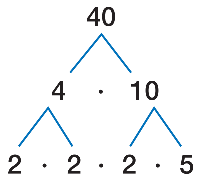
4-3 Prime Factorization

See pages 159–163.

Concept Summary

- A prime number is a whole number that has exactly two factors, 1 and itself.
- A composite number is a whole number that has more than two factors.

Example **Write the prime factorization of 40.**



$40 = 4 \cdot 10$

$4 = 2 \cdot 2$ and $10 = 2 \cdot 5$

The prime factorization of 40 is $2 \cdot 2 \cdot 2 \cdot 5$ or $2^3 \cdot 5$.

Example **Factor $9s^3t^2$.**

$9s^3t^2 = 3 \cdot 3 \cdot s^3 \cdot t^2$ $9 = 3 \cdot 3$

$= 3 \cdot 3 \cdot s \cdot s \cdot s \cdot t \cdot t$ $s^3 \cdot t^2 = s \cdot s \cdot s \cdot t \cdot t$

Exercises **Write the prime factorization of each number. Use exponents for repeated factors.** *See Example 2 on page 160.*

25. 45 **26.** 55 **27.** 68 **28.** 200

Factor each monomial. *See Example 3 on page 161.*

29. $49k$ **30.** $-15n^2$ **31.** $26p^3$ **32.** $10a^2b$

4-4 Greatest Common Factor (GCF)

See pages 164–168.

Concept Summary

- The greatest number or monomial that is a factor of two or more numbers or monomials is the GCF.
- The Distributive Property can be used to factor algebraic expressions.

Examples

1 Find the GCF of $12a^2$ and $15ab$.

$$\begin{aligned} 12a^2 &= 2 \cdot 2 \cdot \boxed{3} \cdot \boxed{a} \cdot a \\ 15ab &= \boxed{3} \cdot 5 \cdot \boxed{a} \cdot b \end{aligned}$$

The GCF of $12a^2$ and $15ab$ is $3 \cdot a$ or $3a$.

2 Factor $4n + 8$.

Step 1 Find the GCF of $4n$ and 8.

$$\begin{aligned} 4n &= \boxed{2} \cdot \boxed{2} \cdot n \\ 8 &= \boxed{2} \cdot \boxed{2} \cdot 2 \end{aligned}$$

The GCF is $2 \cdot 2$ or 4.

Step 2 Write the product of the GCF and its remaining factors.

$$\begin{aligned} 4n + 8 &= 4(n) + 4(2) && \text{Rewrite each term using the GCF.} \\ &= 4(n + 2) && \text{Distributive Property} \end{aligned}$$

Exercises **Find the GCF of each set of numbers or monomials.**

See Examples 2 and 4 on pages 165 and 166.

33. 6, 48

34. 16, 24

35. $4n$, $5n^2$

36. $20c^3d$, $12cd$

Factor each expression. *See Example 5 on page 166.*

37. $2t + 20$

38. $3x + 24$

39. $30 + 4n$

4-5 Simplifying Algebraic Fractions

See pages 169–173.

Concept Summary

- Algebraic fractions can be written in simplest form by dividing the numerator and the denominator by the GCF.

Example

Simplify $\frac{8np}{18n^2}$.

$$\frac{8np}{18n^2} = \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot 2 \cdot \overset{1}{\cancel{n}} \cdot p}{\underset{1}{\cancel{2}} \cdot 3 \cdot 3 \cdot \underset{1}{\cancel{n}} \cdot n} \qquad \text{Divide the numerator and the denominator by the GCF, } 2 \cdot n.$$

$$= \frac{4p}{9n} \qquad \text{Simplify.}$$

Exercises **Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.** *See Examples 2 and 4 on page 170.*

40. $\frac{6}{21}$

41. $\frac{24}{40}$

42. $\frac{15}{16}$

43. $\frac{30}{51}$

44. $\frac{st}{t^4}$

45. $\frac{23x}{32y}$

46. $\frac{9mn}{18n^2}$

47. $\frac{15ac^2}{24ab}$

Chapter 4 **For More ...**

- Extra Practice, see pages 730–733.
- Mixed Problem Solving, see page 761.

4-6 Multiplying and Dividing Monomials

See pages 175–179.

Concept Summary

- Powers with the same base can be multiplied by adding their exponents.
- Powers with the same base can be divided by subtracting their exponents.

Examples

1 **Find $x^3 \cdot x^2$.**

$x^3 \cdot x^2 = x^{3+2}$ The common base is x.

$= x^5$ Add the exponents.

2 **Find $\frac{4^5}{4^3}$.**

$\frac{4^5}{4^3} = 4^{5-3}$ The common base is 4.

$= 4^2$ Subtract the exponents.

Exercises **Find each product or quotient. Express using exponents.**

See Examples 1–3 on pages 176 and 177.

48. $8^4 \cdot 8^5$ **49.** $c \cdot c^3$ **50.** $\frac{3^7}{3^2}$ **51.** $\frac{r^{11}}{r^9}$ **52.** $7x \cdot 2x^6$

4-7 Negative Exponents

See pages 181–185.

Concept Summary

- For $a \neq 0$ and any integer n, $a^{-n} = \frac{1}{a^n}$.

Example **Write 3^{-4} as an expression using a positive exponent.**

$3^{-4} = \frac{1}{3^4}$ Definition of negative exponent

Exercises **Write each expression using a positive exponent.**

See Example 1 on page 182.

53. 7^{-2} **54.** 10^{-1} **55.** b^{-4} **56.** t^{-8} **57.** $(-4)^{-3}$

4-8 Scientific Notation

See pages 186–190.

Concept Summary

- A number in scientific notation contains a factor and a power of 10.

Examples

1 **Express 3.5×10^{-2} in standard form.**

$3.5 \times 10^{-2} = 3.5 \times 0.01$ $10^{-2} = 0.01$

$= 0.\underbrace{035}$ Move the decimal point 2 places to the left.

2 **Express 269,000 in scientific notation.**

$\underbrace{269{,}000} = 2.69 \times 100{,}000$ The decimal point moves 5 places.

$= 2.69 \times 10^5$ The exponent is positive.

Exercises **Express each number in standard form.** *See Example 1 on page 186.*

58. 6.1×10^2 **59.** 2.9×10^{-3} **60.** 1.85×10^{-2} **61.** 7.045×10^4

Express each number in scientific notation. *See Example 2 on page 187.*

62. 1200 **63.** 0.008 **64.** 0.000319 **65.** 45,710,000

Chapter 4

Practice Test

Vocabulary and Concepts

1. **Explain** how to use the divisibility rules to determine whether a number is divisible by 2, 3, 5, 6, or 10.
2. **Explain** the difference between a prime number and a composite number.
3. **OPEN ENDED** Write an algebraic fraction that is in simplest form.

Skills and Applications

Determine whether each expression is a monomial. Explain why or why not.

4. $6xyz$

5. $-2m + 9$

Write each expression using exponents.

6. $3 \cdot 3 \cdot 3 \cdot 3$

7. $-2 \cdot -2 \cdot -2 \cdot a \cdot a \cdot a \cdot a$

Factor each expression.

8. $12r^2$

9. $50xy^2$

10. $7 + 21p$

Find the GCF of each set of numbers or monomials.

11. 70, 28

12. 36, 90, 180

13. $12a^3b$, $40ab^4$

Write each fraction in simplest form. If the fraction is already in simplest form, write *simplified*.

14. $\frac{57}{95}$

15. $\frac{240}{360}$

16. $\frac{56m^3n}{32mn}$

Find each product or quotient. Express using exponents.

17. $5^3 \cdot 5^6$

18. $(4x^7)(-6x^3)$

19. $w^9 \div w^5$

Write each expression using a positive exponent.

20. 4^{-2}

21. t^{-6}

22. $(yz)^{-3}$

Write each number in standard form.

23. 9.0×10^{-2}

24. 5.206×10^{-3}

25. 3.71×10^4

Write each number in scientific notation.

26. 345,000

27. 1,680,000

28. 0.00072

29. **BAKING** A recipe for butter cookies requires 12 tablespoons of sugar for every 16 tablespoons of flour. Write this as a fraction in simplest form.

30. **STANDARDIZED TEST PRACTICE** Earth is approximately 93 million miles away from the Sun. Express this distance in scientific notation.

Ⓐ 9.3×10^7 mi

Ⓑ 9.3×10^6 mi

Ⓒ 93×10^6 mi

Ⓓ 93,000,000 mi

www.pre-alg.com/chapter_test

CONTENTS

Chapter 4 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Andy has 7 fewer computer games than Ling. Carlos has twice as many computer games as Andy. If Ling has x computer games, which of these represents the number of computer games that Carlos has? (Lesson 1-3)

Ⓐ $7 - 2x$ Ⓑ $x - 7$
Ⓒ $2x - 7$ Ⓓ $2(x - 7)$

2. Which of the following statements is *false*, when r, s, and t are different integers? (Lesson 1-4)

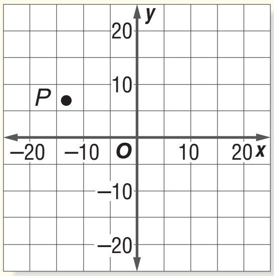
Ⓐ $(rs)t = r(st)$ Ⓑ $r + s = s + r$
Ⓒ $rs = sr$ Ⓓ $r - s = s - r$

3. The low temperatures during the past five days are given in the table. Find the average (mean) of the temperatures. (Lesson 2-5)

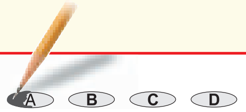
Day	1	2	3	4	5
Temperature (°F)	−2	0	4	5	4

Ⓐ 3°F Ⓑ 2.2°F Ⓒ 13°F Ⓓ 2.75°F

4. Which coordinates are most likely to be the coordinates of point P? (Lesson 2-6)



Ⓐ $(-13, 7)$
Ⓑ $(7, -13)$
Ⓒ $(13, 7)$
Ⓓ $(7, 13)$



Test-Taking Tip

Question 2
When a multiple-choice question asks you to verify statements that include variables, you can substitute numbers into the variables to determine which statements are true and which are false.

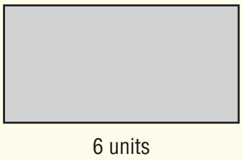
5. Solve $3n - 6 = -39$ for n. (Lesson 3-5)

Ⓐ -15 Ⓑ -11
Ⓒ 11 Ⓓ 15

6. For every order purchased from an Internet bookstore, the shipping and handling charges include a base fee of \$5 plus a fee of \$3 per item purchased in the order. Which equation represents the shipping and handling charge for ordering n items? (Lesson 3-6)

Ⓐ $S = 5(3n)$ Ⓑ $S = 3n - 5$
Ⓒ $S = 5 + \frac{3}{n}$ Ⓓ $S = 5 + 3n$

7. The area of the rectangle below is 18 square units. Use the formula $A = \ell w$ to find its width. (Lesson 3-7)



Ⓐ $\frac{1}{3}$ unit
Ⓑ 2 units
Ⓒ 3 units
Ⓓ 12 units

8. Write the prime factorization of 84. (Lesson 4-3)

Ⓐ $2 \cdot 3 \cdot 7$
Ⓑ $4 \cdot 21$
Ⓒ $3 \cdot 4 \cdot 7$
Ⓓ $2 \cdot 2 \cdot 3 \cdot 7$

9. What is the greatest common factor of 28 and 42? (Lesson 4-4)

Ⓐ 2 Ⓑ 7
Ⓒ 14 Ⓓ 28

10. Write 3^{-3} as a fraction. (Lesson 4-7)

Ⓐ $-\frac{1}{9}$ Ⓑ $-\frac{1}{27}$
Ⓒ $\frac{1}{27}$ Ⓓ $\frac{1}{9}$

11. Asia is the largest continent. It has an area of 17,400,000 square miles. What is 17,400,000 expressed in scientific notation? (Lesson 4-8)

Ⓐ 174×10^5 Ⓑ 1.74×10^7
Ⓒ 174×10^7 Ⓓ 1.74×10^8

CONTENTS

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 771-788.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

12. A health club charges an initial fee of $60 for the first month, and then a $28 membership fee each month after the first month, as shown in the table. What is the total membership cost for 9 months? (Lesson 1-1)

Number of Months	Total Cost ($)
1	60
2	88
3	116
4	144
5	172

13. The temperature in Concord at 5 P.M. was −3 degrees. By midnight, the temperature had dropped 9 degrees. What was the temperature at midnight? (Lesson 2-3)

14. A skating rink charges $2.00 to rent a pair of skates and $1.50 per hour of skating. Jeff wants to spend no more than $8.00 and he needs to rent skates. How many hours can he skate? (Lesson 3-5)

15. At a birthday party, Maka gave 30 gel pens to her friends as prizes. Everyone got at least 1 gel pen. Six friends got just 1 gel pen each, 4 friends got 3 gel pens each for winning games, and the rest of the friends got 2 gel pens each. How many friends got 2 gel pens? (Lesson 3-6)

16. A table 8 feet long and 2 feet wide is to be covered for the school bake sale. If organizers want the covering to hang down 1 foot on each side, what is the area of the covering that they need? (Lesson 3-7)

17. What is the least 3-digit number that is divisible by 3 and 5? (Lesson 4-1)

18. Write $10 \cdot 10 \cdot 10 \cdot 10$ using an exponent. (Lesson 4-2)

19. Find the greatest common factor of 18, 44, and 12. (Lesson 4-4)

20. Write $\frac{1}{5 \cdot 5 \cdot 5}$ using a negative exponent. (Lesson 4-7)

21. The Milky Way galaxy is made up of about 200 billion stars, including the Sun. Write this number in scientific notation. (Lesson 4-8)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

22. Chandra plans to order CDs from an Internet shopping site. She finds that the CD prices are the same at three different sites, but that the shipping costs vary. The shipping costs include a fee per order, plus an additional fee for each item in the order, as shown in the table below. (Lesson 3-6)

Company	Shipping Cost	
	Per Order	Per Item
CDBargains	$4.00	$1.00
WebShopper	$6.00	$3.00
EverythingStore	$2.50	$1.50

a. For each company, write an equation that represents the shipping cost. In each of your three equations, use S to represent the shipping cost and n to represent the number of items purchased.

b. If Chandra orders 2 CDs, which company will charge the least for shipping? Use the equations you wrote and show your work.

c. If Chandra orders 10 CDs, which company will charge the least for shipping? Use the equations you wrote and show your work.

d. For what number of CDs do both CDBargains and EverythingStore charge the same amount for shipping and handling costs?

www.pre-alg.com/standardized_test

CONTENTS