Chapter 3

Equations

What You'll Learn

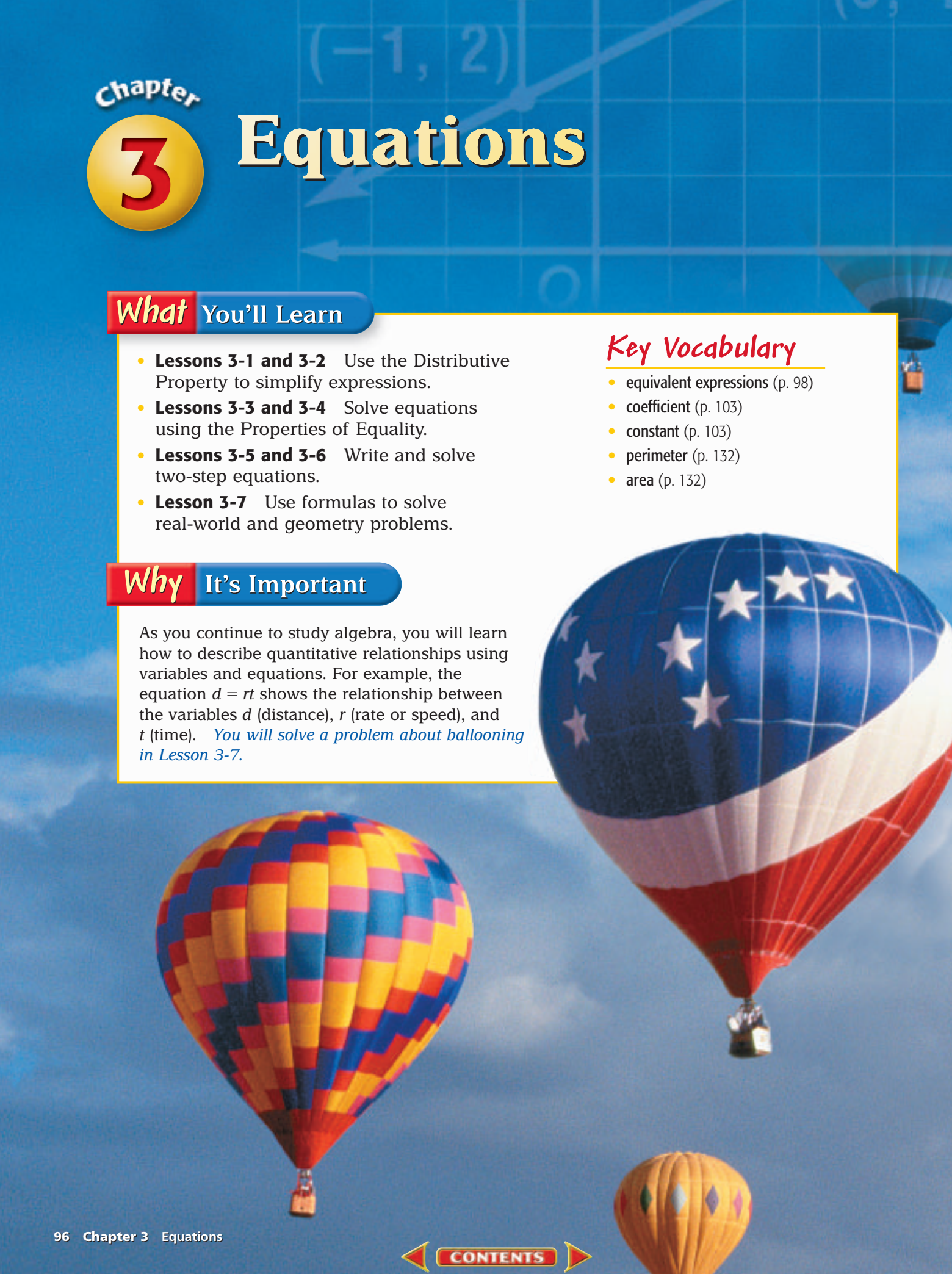
- **Lessons 3-1 and 3-2** Use the Distributive Property to simplify expressions.
- **Lessons 3-3 and 3-4** Solve equations using the Properties of Equality.
- **Lessons 3-5 and 3-6** Write and solve two-step equations.
- **Lesson 3-7** Use formulas to solve real-world and geometry problems.

Key Vocabulary

- equivalent expressions (p. 98)
- coefficient (p. 103)
- constant (p. 103)
- perimeter (p. 132)
- area (p. 132)

Why It's Important

As you continue to study algebra, you will learn how to describe quantitative relationships using variables and equations. For example, the equation $d = rt$ shows the relationship between the variables d (distance), r (rate or speed), and t (time). *You will solve a problem about ballooning in Lesson 3-7.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 3.

For Lesson 3-1 — Multiply Integers

Find each product. *(For review, see Lesson 2-4.)*

1. $2(-3)$ **2.** $-4(3)$ **3.** $-5(-2)$ **4.** $-4(6)$

For Lesson 3-2 — Write Addition Expressions

Write each subtraction expression as an addition expression. *(For review, see Lesson 2-3.)*

5. $5 - 7$ **6.** $6 - 10$ **7.** $-5 - 9$ **8.** $11 - 10$

For Lessons 3-3 and 3-5 — Add Integers

Find each sum. *(For review, see Lesson 2-2.)*

9. $6 + (-9)$ **10.** $-8 + 4$ **11.** $4 + (-4)$ **12.** $7 + (-10)$

For Lesson 3-6 — Write Algebraic Expressions

Write an algebraic expression for each verbal expression. *(For review, see Lesson 1-3.)*

13. five more than twice a number

14. the difference of a number and 15

15. three less than a number

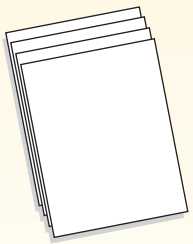
16. the quotient of a number and 10

FOLDABLES™ Study Organizer

Writing and Solving Equations Make this Foldable to help you organize your notes. Begin with four sheets of $8\frac{1}{2}$" × 11" paper.

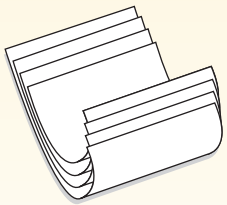
Step 1 **Stack Pages**

Place 4 sheets of paper $\frac{3}{4}$ inch apart.



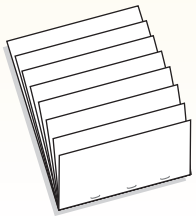
Step 2 **Roll Up Bottom Edges**

All tabs should be the same size.



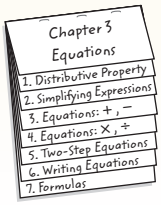
Step 3 **Crease and Staple**

Staple along the fold.



Step 4 **Label**

Label the tabs with topics from the chapter.



Reading and Writing As you read and study the chapter, record examples under each tab.

3-1 The Distributive Property

What You'll Learn

- Use the Distributive Property to write equivalent numerical expressions.
- Use the Distributive Property to write equivalent algebraic expressions.

Vocabulary

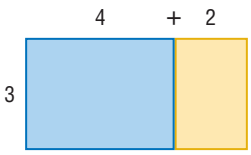
- equivalent expressions

How are rectangles related to the Distributive Property?

To find the area of a rectangle, multiply the length and width. You can find the total area of the blue and yellow rectangles in two ways.

Method 1

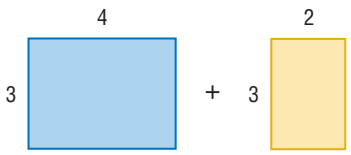
Put them together. Add the lengths, then multiply.



$3(4 + 2) = 3 \cdot 6$ Add.
$= 18$ Multiply.

Method 2

Separate them. Multiply to find each area, then add.



$3 \cdot 4 + 3 \cdot 2 = 12 + 6$ Multiply.
$= 18$ Add.

a. Draw a 2-by-5 and a 2-by-4 rectangle. Find the total area in two ways.

b. Draw a 4-by-4 and a 4-by-1 rectangle. Find the total area in two ways.

c. Draw any two rectangles that have the same width. Find the total area in two ways.

d. What did you notice about the total area in each case?

DISTRIBUTIVE PROPERTY The expressions $3(4 + 2)$ and $3 \cdot 4 + 3 \cdot 2$ are **equivalent expressions** because they have the same value, 18. This example shows how the **Distributive Property** combines addition and multiplication.

Reading Math

Distributive

Root Word: Distribute

To *distribute* means to deliver to each member of a group.

Key Concept — Distributive Property

- **Words** To multiply a number by a sum, multiply each number inside the parentheses by the number outside the parentheses.
- **Symbols** $a(b + c) = ab + ac$ $(b + c)a = ba + ca$
- **Examples** $3(4 + 2) = 3 \cdot 4 + 3 \cdot 2$ $(5 + 3)2 = 5 \cdot 2 + 3 \cdot 2$

Concept Check Name two operations that are combined by the Distributive Property.

Example 1 Use the Distributive Property

Use the Distributive Property to write each expression as an equivalent expression. Then evaluate the expression.

a. 2(6 + 4)

$$\begin{aligned} 2(6 + 4) &= 2 \cdot 6 + 2 \cdot 4 \\ &= 12 + 8 && \text{Multiply.} \\ &= 20 && \text{Add.} \end{aligned}$$

b. (8 + 3)5

$$\begin{aligned} (8 + 3)5 &= 8 \cdot 5 + 3 \cdot 5 \\ &= 40 + 15 && \text{Multiply.} \\ &= 55 && \text{Add.} \end{aligned}$$

Example 2 Use the Distributive Property to Solve a Problem

AMUSEMENT PARKS **A one-day pass to an amusement park costs $40. A round-trip bus ticket to the park costs $5.**

a. Write two equivalent expressions to find the total cost of a one-day pass and a bus ticket for 15 students.

Method 1 Find the cost for 1 person, then multiply by 15.

$$15(\underbrace{\$40 + \$5}_{\text{cost for 1 person}})$$

Method 2 Find the cost of 15 passes and 15 tickets. Then add.

$$\underbrace{15(\$40)}_{\text{cost of 15 passes}} + \underbrace{15(\$5)}_{\text{cost of 15 tickets}}$$

b. Find the total cost.

Evaluate either expression to find the total cost.

$$\begin{aligned} 15(\$40 + 5) &= 15(\$40) + 15(\$5) && \text{Distributive Property} \\ &= \$600 + \$75 && \text{Multiply.} \\ &= \$675 && \text{Add.} \end{aligned}$$

The total cost is $675.

CHECK You can check your results by evaluating 15($45).

More About . . .

Amusement Parks

Attendance at U.S. amusement parks increased 22% in the 1990s. In 1999, over 300 million people attended these parks.

Source: *International Association of Amusement Parks and Attractions*

ALGEBRA CONNECTION

ALGEBRAIC EXPRESSIONS You can also model the Distributive Property by using algebra tiles.

The model shows $2(x + 3)$. There are 2 groups of $(x + 3)$.

Separate the tiles into 2 groups of x and 2 groups of 3.

$$\begin{aligned} 2(x + 3) &= 2x + 2 \cdot 3 \\ &= 2x + 6 \end{aligned}$$

The expressions $2(x + 3)$ and $2x + 6$ are equivalent expressions because no matter what x is, these expressions have the same value.

Concept Check Are $3(x + 1)$ and $3x + 1$ equivalent expressions? Explain.

Example 3 Simplify Algebraic Expressions

Use the Distributive Property to write each expression as an equivalent algebraic expression.

a. $3(x + 1)$

$3(x + 1) = 3x + 3 \cdot 1$

$= 3x + 3$ Simplify.

b. $(y + 4)5$

$(y + 4)5 = y \cdot 5 + 4 \cdot 5$

$= 5y + 20$ Simplify.

Example 4 Simplify Expressions with Subtraction

Use the Distributive Property to write each expression as an equivalent algebraic expression.

a. $2(x - 1)$

$2(x - 1) = 2[x + (-1)]$ Rewrite $x - 1$ as $x + (-1)$.

$= 2x + 2(-1)$ Distributive Property

$= 2x + (-2)$ Simplify.

$= 2x - 2$ Definition of subtraction

b. $-3(n - 5)$

$-3(n - 5) = -3[n + (-5)]$ Rewrite $n - 5$ as $n + (-5)$.

$= -3n + (-3)(-5)$ Distributive Property

$= -3n + 15$ Simplify.

Study Tip

Look Back

To review **subtraction expressions**, see Lesson 2-3.

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an equation using three integers that is an example of the Distributive Property.

2. **FIND THE ERROR** Julia and Catelyn are using the Distributive Property to simplify $3(x + 2)$. Who is correct? Explain your reasoning.

Julia	Catelyn
$3(x + 2) = 3x + 2$	$3(x + 2) = 3x + 6$

Guided Practice

Use the Distributive Property to write each expression as an equivalent expression. Then evaluate it.

3. $5(7 + 8)$ **4.** $2(9 + 1)$ **5.** $(2 + 4)6$

ALGEBRA **Use the Distributive Property to write each expression as an equivalent algebraic expression.**

6. $4(x + 3)$ **7.** $(n + 2)3$ **8.** $8(y - 2)$ **9.** $-6(x - 5)$

Application

MONEY **For Exercises 10 and 11, use the following information.**
Suppose you work in a grocery store 4 hours on Friday and 5 hours on Saturday. You earn $6.25 an hour.

10. Write two different expressions to find your wages.

11. Find the total wages for that weekend.

Practice and Apply

Homework Help

For Exercises	See Examples
12–23	1
24–25	2
26–33	3
34–47	4

Extra Practice
See page 728.

Use the Distributive Property to write each expression as an equivalent expression. Then evaluate it.

12. $2(6 + 1)$ **13.** $5(7 + 3)$ **14.** $(4 + 6)9$
15. $(4 + 3)3$ **16.** $(9 + 2)4$ **17.** $(8 + 8)2$
18. $7(3 - 2)$ **19.** $6(8 - 5)$ **20.** $-5(8 - 4)$
21. $-3(9 - 2)$ **22.** $(8 - 4)(-2)$ **23.** $(10 - 3)(-5)$

24. MOVIES One movie ticket costs $7, and one small bag of popcorn costs $3. Write two equivalent expressions for the total cost of four movie tickets and four bags of popcorn. Then find the cost.

25. SPORTS A volleyball uniform costs $15 for the shirt, $10 for the pants, and $8 for the socks. Write two equivalent expressions for the total cost of 12 uniforms. Then find the cost.

ALGEBRA Use the Distributive Property to write each expression as an equivalent algebraic expression.

26. $2(x + 3)$ **27.** $5(y + 6)$ **28.** $3(n + 1)$
29. $7(y + 8)$ **30.** $(x + 3)4$ **31.** $(y + 2)10$
32. $(3 + y)6$ **33.** $(2 + x)5$ **34.** $3(x - 2)$
35. $9(m - 2)$ **36.** $8(z - 3)$ **37.** $15(s - 3)$
38. $(r - 5)6$ **39.** $(x - 3)12$ **40.** $(t - 4)5$
41. $(w - 10)2$ **42.** $-2(z + 4)$ **43.** $-5(a + 10)$
44. $-2(x - 7)$ **45.** $-5(w - 8)$ **46.** $(y - 4)(-2)$
47. $(a - 6)(-5)$ **48.** $2(x + y)$ **49.** $3(a + b)$

SHOPPING For Exercises 50 and 51, use the graphic.

50. Find the total amount spent by two teens and two adults during one average shopping trip.

51. Find the total amount spent by one teen and one adult during five average shopping trips.

52. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are rectangles related to the Distributive Property?

Include the following in your answer:

- a drawing of two rectangles with the same width, and
- two different methods for finding the total area of the rectangles.

53. CRITICAL THINKING Is $3 + (x \cdot y) = (3 + x) \cdot (3 + y)$ a true statement? If so, explain your reasoning. If not, give a counterexample.

Standardized Test Practice

54. One ticket to a baseball game costs t dollars. A soft drink costs s dollars. Which expression represents the total cost of a ticket and soft drink for p people?

Ⓐ pst Ⓑ $p + (ts)$ Ⓒ $t(p + s)$ Ⓓ $p(t + s)$

55. Which equation is always true?

Ⓐ $5(a + b) = 5a + b$ Ⓑ $5(ab) = (5a)(5b)$

Ⓒ $5(a + b) = 5(b + a)$ Ⓓ $5(a + 0) = 5a + 5$

Extending the Lesson

MENTAL MATH The Distributive Property allows you to find certain products mentally. Replace one factor with the sum of a number and a multiple of ten. Then apply the Distributive Property.

Example Find $15 \cdot 12$ mentally.

$15 \cdot 12 = 15(10 + 2)$ Think: 12 is 10 + 2.

$= 15 \cdot 10 + 15 \cdot 2$ Distributive Property

$= 150 + 30$ Multiply mentally.

$= 180$ Add mentally.

Rewrite each product so it is easy to compute mentally. Then find the product.

56. $7 \cdot 14$ **57.** $8 \cdot 23$ **58.** $9 \cdot 32$ **59.** $16 \cdot 11$

60. $14 \cdot 12$ **61.** $9 \cdot 103$ **62.** $11 \cdot 102$ **63.** $12 \cdot 1004$

Maintain Your Skills

Mixed Review

64. The table shows several solutions of the equation $x + y = 4$. *(Lesson 2-6)*

a. Graph the ordered pairs on a coordinate plane.

b. Describe the graph.

$x + y = 4$		
x	y	(x, y)
-1	5	$(-1, 5)$
1	3	$(1, 3)$
2	2	$(2, 2)$

65. ALGEBRA Evaluate $\frac{-4y}{x}$ if $x = 2$ and $y = -3$. *(Lesson 2-5)*

ALGEBRA Find the solution of each equation if the replacement set is {1, 2, 3, 4, 5}. *(Lesson 1-5)*

66. $2n + 3 = 9$ **67.** $3n - 4 = 8$ **68.** $4x - 9 = -5$

Find the next three terms in each pattern. *(Lesson 1-1)*

69. 5, 9, 13, 17, . . . **70.** 20, 22, 26, 32, . . . **71.** 5, 10, 20, 40, . . .

Getting Ready for the Next Lesson

PREREQUISITE SKILL Write each subtraction expression as an addition expression. *(To review **subtraction expressions**, see Lesson 2-3.)*

72. $5 - 3$ **73.** $-8 - 4$ **74.** $10 - 14$

75. $3 - 9$ **76.** $-2 - (-5)$ **77.** $-7 - 10$

3-2 Simplifying Algebraic Expressions

What You'll Learn

- Use the Distributive Property to simplify algebraic expressions.

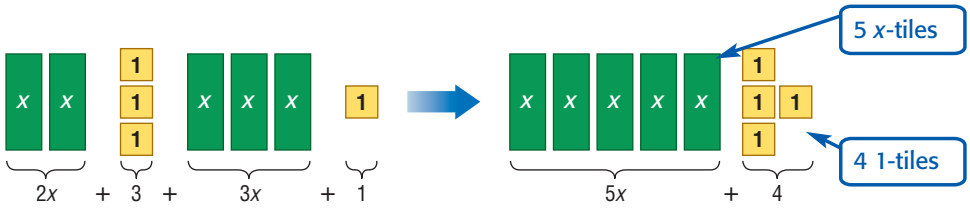
Vocabulary

- term
- coefficient
- like terms
- constant
- simplest form
- simplifying an expression

How can you use algebra tiles to simplify an algebraic expression?

You can use algebra tiles to represent expressions. You can also sort algebra tiles by their shapes and group them.

The drawing on the left represents the expression $2x + 3 + 3x + 1$. On the right, the algebra tiles have been sorted and combined.



Therefore, $2x + 3 + 3x + 1 = 5x + 4$.

Model each expression with algebra tiles or a drawing. Then sort them by shape and write an expression represented by the tiles.

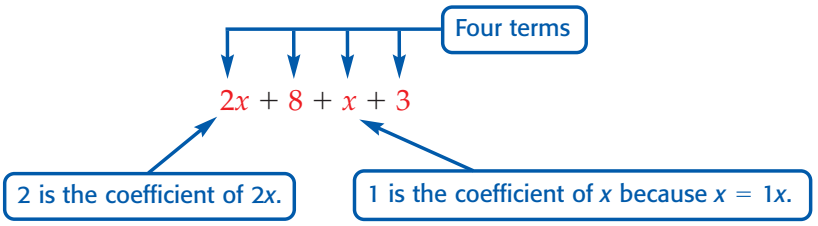
a. $3x + 2 + 4x + 3$

b. $2x + 5 + x$

c. $4x + 5 + 3$

d. $x + 2x + 4x$

SIMPLIFY EXPRESSIONS When plus or minus signs separate an algebraic expression into parts, each part is a **term**. The numerical part of a term that contains a variable is called the **coefficient** of the variable.



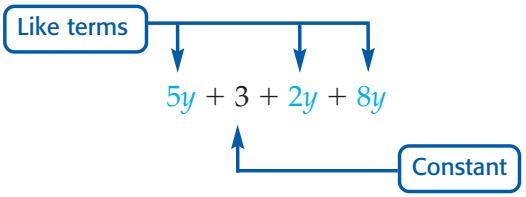
Reading Math

Constant

Everyday Meaning: unchanging

Math Meaning: fixed value in an expression

Like terms are terms that contain the same variables, such as $2n$ and $5n$ or $6xy$ and $4xy$. A term without a variable is called a **constant**. Constant terms are also like terms.



Concept Check Are $5x$ and $5y$ like terms? Explain.

Rewriting a subtraction expression using addition will help you identify the terms of an expression.

Example 1 Identify Parts of Expressions

Identify the terms, like terms, coefficients, and constants in the expression $3x - 4x + y - 2$.

$3x - 4x + y - 2 = 3x + (-4x) + y + (-2)$ Definition of subtraction

$= 3x + (-4x) + 1y + (-2)$ Identity Property

The terms are $3x$, $-4x$, y, and -2. The like terms are $3x$ and $-4x$.

The coefficients are 3, -4, and 1. The constant is -2.

An algebraic expression is in **simplest form** if it has no like terms and no parentheses. When you use the Distributive Property to combine like terms, you are **simplifying the expression**.

Example 2 Simplify Algebraic Expressions

Simplify each expression.

a. $2x + 8x$

$2x$ and $8x$ are like terms.

$2x + 8x = (2 + 8)x$ Distributive Property

$= 10x$ Simplify.

b. $6n + 3 + 2n$

$6n$ and $2n$ are like terms.

$6n + 3 + 2n = 6n + 2n + 3$ Commutative Property

$= (6 + 2)n + 3$ Distributive Property

$= 8n + 3$ Simplify.

c. $3x - 5 - 8x + 6$

$3x$ and $-8x$ are like terms. -5 and 6 are also like terms.

$3x - 5 - 8x + 6 = 3x + (-5) + (-8x) + 6$ Definition of subtraction

$= 3x + (-8x) + (-5) + 6$ Commutative Property

$= [3 + (-8)]x + (-5) + 6$ Distributive Property

$= -5x + 1$ Simplify.

d. $m + 3(n + 4m)$

$m + 3(n + 4m) = m + 3n + 3(4m)$ Distributive Property

$= m + 3n + 12m$ Associative Property

$= 1m + 3n + 12m$ Identity Property

$= 1m + 12m + 3n$ Commutative Property

$= (1 + 12)m + 3n$ Distributive Property

$= 13m + 3n$ Simplify.

Study Tip

Equivalent Expressions

To check whether $2x + 8x$ and $10x$ are equivalent expressions, substitute any value for x and see whether the expressions have the same value.

Concept Check Expressions like $4(x - 3)$, $4x - 12$, and $x + 3x - 12$ are equivalent expressions. Which is in simplest form?

Example 3 Translate Verbal Phrases into Expressions

BASEBALL CARDS **Suppose you and your brother collect baseball cards. He has 15 more cards in his collection than you have. Write an expression in simplest form that represents the total number of cards in both collections.**

Words You have some cards. Your brother has 15 more.

Variables Let x = number of cards you have.

Let $x + 15$ = number of cards your brother has.

Expression To find the total, add the expressions.

$$\begin{aligned} x + (x + 15) &= (x + x) + 15 && \text{Associative Property} \\ &= (1x + 1x) + 15 && \text{Identity Property} \\ &= (1 + 1)x + 15 && \text{Distributive Property} \\ &= 2x + 15 && \text{Simplify.} \end{aligned}$$

The expression $2x + 15$ represents the total number of cards, where x is the number of cards you have.

More About . . .

Baseball Cards

Honus Wagner is considered by many to be baseball's greatest all-around player. In July, 2000, one of his baseball cards sold for $1.1 million.

Source: CMG Worldwide

Check for Understanding

Concept Check

1. **Define** *like terms.*
2. **OPEN ENDED** Write an expression containing three terms that is in simplest form. One of the terms should be a constant.
3. **FIND THE ERROR** Koko and John are simplifying the expression $5x - 4 + x + 2$.

Koko	John
5x - 4 + x + 2 = 6x - 2	5x - 4 + x + 2 = 5x - 2

Who is correct? Explain your reasoning.

Guided Practice

Identify the terms, like terms, coefficients, and constants in each expression.

4. $4x + 3 + 5x + y$
5. $2m - n + 6m$
6. $4y - 2x - 7$

Simplify each expression.

7. $6a + 2a$
8. $x + 9x + 3$
9. $6c + 4 + c + 8$
10. $7m - 2m$
11. $9y + 8 - 8$
12. $2x - 5 - 4x + 8$
13. $5 - 3(y + 7)$
14. $3x + 2y + 4y$
15. $x + 3(x + 4y)$

Application

16. **MONEY** You have saved some money. Your friend has saved $20 more than you. Write an expression in simplest form that represents the total amount of money you and your friend have saved.

Practice and Apply

Homework Help

For Exercises	See Examples
17–22	1
23–49	2
50–53	3

Extra Practice
See page 728.

Identify the terms, like terms, coefficients, and constants in each expression.

17. $3 + 7x + 3x + x$
18. $y + 3y + 8y + 2$
19. $2a + 5c - a + 6a$
20. $5c - 2d + 3d - d$
21. $6m - 2n + 7$
22. $7x - 3y + 3z - 2$

Simplify each expression.

23. $2x + 5x$
24. $7b + 2b$
25. $y + 10y$
26. $5y + y$
27. $2a + 3 + 5a$
28. $4 + 2m + m$
29. $2y + 8 + 5y + 1$
30. $8x + 5 + 7 + 2x$
31. $5x - 3x$
32. $10b - 2b$
33. $4y - 5y$
34. $r - 3r$
35. $8 + x - 5x$
36. $6x + 4 - 7x$
37. $8y - 7 + 7$
38. $9x + 2 - 2$
39. $2x + 3 - 3x + 9$
40. $5t - 3 - t + 2$
41. $3(b + 2) + 2b$
42. $5(x + 3) + 8x$
43. $-3(a + 2) - a$
44. $-2(x + 3) + 2x$
45. $4x - 4(2 + x)$
46. $8a - 2(a - 7)$
47. $6m + 2n + 10m$
48. $-2y + x + 3y$
49. $c + 2(d - 5c)$

More About . . .

Baby-sitting

In a recent survey, 10% of students in grades 6–12 reported that most of their spending money came from baby-sitting.

Source: *USA WEEKEND*

For Exercises 50–53, write an expression in simplest form that represents the total amount in each situation.

50. SCHOOL SUPPLIES You bought 5 folders that each cost x dollars, a calculator for $45, and a set of pens for $3.

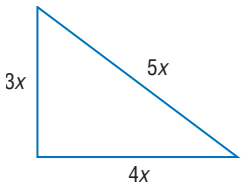
51. SHOPPING Suppose you buy 3 shirts that each cost s dollars, a pair of shoes for $50, and jeans for $30.

52. BIRTHDAYS Today is your friend's birthday. She is y years old. Her sister is 5 years younger.

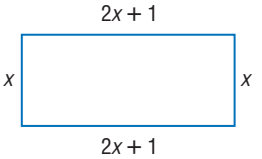
53. BABY-SITTING Alicia earned d dollars baby-sitting. Her friend earned twice as much. You earned $2 less than Alicia's friend earned.

GEOMETRY You can find the perimeter of a geometric figure by adding the measures of its sides. Write an expression in simplest form for the perimeter of each figure.

54.



55.



56. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can you use algebra tiles to simplify an algebraic expression?

Include the following in your answer:

- a drawing that shows how to simplify the expression $4x + 2 + 3x + 1$ using algebra tiles,
- a definition of *like terms*, and
- an explanation of how you use the Commutative and Distributive Properties to simplify $4x + 2 + 3x + 1$.

57. CRITICAL THINKING You use *deductive reasoning* when you base a conclusion on mathematical rules or properties. Indicate the property that justifies each step that was used to simplify $3(x + 4) + 5(x + 1)$.

a. $3(x + 4) + 5(x + 1) = 3x + 12 + 5x + 5$

b. $= 3x + 5x + 12 + 5$

c. $= 3x + 5x + 17$

d. $= 8x + 17$

Standardized Test Practice

58. Which expression is *not* equivalent to the other three?

Ⓐ $-6(x - 2)$ Ⓑ $x + 12 - 7x$

Ⓒ $-6x - 12$ Ⓓ $-x - 5x + 12$

59. Katie practiced the clarinet for m minutes. Her sister practiced 10 minutes less. Which expression represents the total time they spent practicing?

Ⓐ $m - 10$ Ⓑ $m + 10$ Ⓒ $2m - 10$ Ⓓ $2m + 10$

Maintain Your Skills

Mixed Review

ALGEBRA **Use the Distributive Property to write each expression as an equivalent expression.** *(Lesson 3-1)*

60. $3(a + 5)$ **61.** $-2(y + 8)$ **62.** $-3(x - 1)$

63. Name the quadrant in which $P(-5, -6)$ is located. *(Lesson 2-6)*

64. CRUISES The table shows the number of people who took a cruise in various years. Make a scatter plot of the data. *(Lesson 1-7)*

People Taking Cruises				
Year	1970	1980	1990	2000
Number (millions)	0.5	1.4	3.6	6.5

Source: Cruise Lines International Association

Evaluate each expression. *(Lesson 1-2)*

65. $2 + 3 \cdot 5$ **66.** $8 \div 2 \cdot 4$ **67.** $10 - 2 \cdot 4$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each sum.** *(To review **adding integers**, see Lesson 2-2.)*

68. $-5 + 4$ **69.** $-8 + (-3)$ **70.** $10 + (-1)$

71. $4 + (-9)$ **72.** $11 + (-7)$ **73.** $-4 + (-9)$

Practice Quiz 1

Lessons 3-1 and 3-2

Simplify each expression. *(Lessons 3-1 and 3-2)*

1. $6(x + 2)$ **2.** $5(x - 7)$ **3.** $6y - 4 + y$ **4.** $2a + 4(a - 9)$

5. SCHOOL You spent m minutes studying on Monday. On Tuesday, you studied 15 more minutes than you did on Monday. Write an expression in simplest form that represents the total amount of time spent studying on Monday and Tuesday. *(Lesson 3-2)*

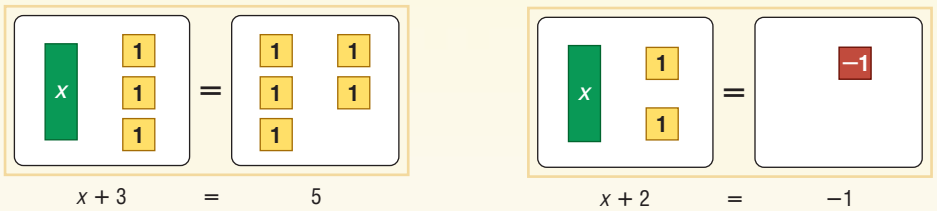
Algebra Activity

A Preview of Lessons 3-3 and 3-4

Solving Equations Using Algebra Tiles

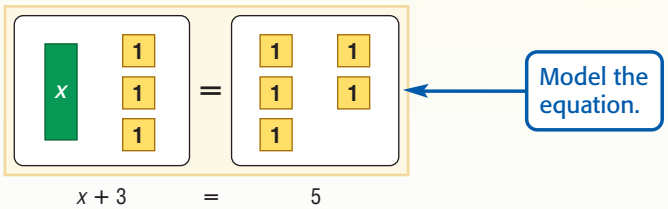
Activity 1

In a set of algebra tiles, x represents the variable x, 1 represents the integer 1, and −1 represents the integer −1. You can use algebra tiles and an equation mat to model equations.



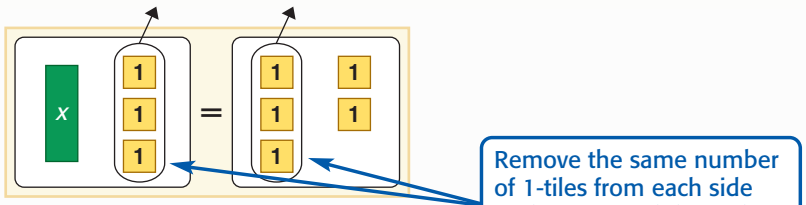
$x + 3 = 5$ $\qquad$ $x + 2 = -1$

When you solve an equation, you are trying to find the value of x that makes the equation true. The following example shows how to solve $x + 3 = 5$ using algebra tiles.



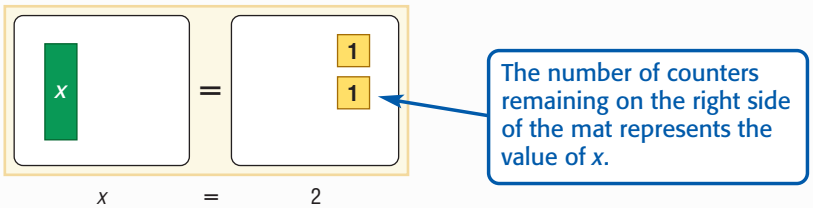
$x + 3 = 5$

Model the equation.



$x + 3 - 3 = 5 - 3$

Remove the same number of 1-tiles from each side of the mat until the x-tile is by itself on one side.



$x = 2$

The number of counters remaining on the right side of the mat represents the value of x.

Therefore, $x = 2$. Since $2 + 3 = 5$, the solution is correct.

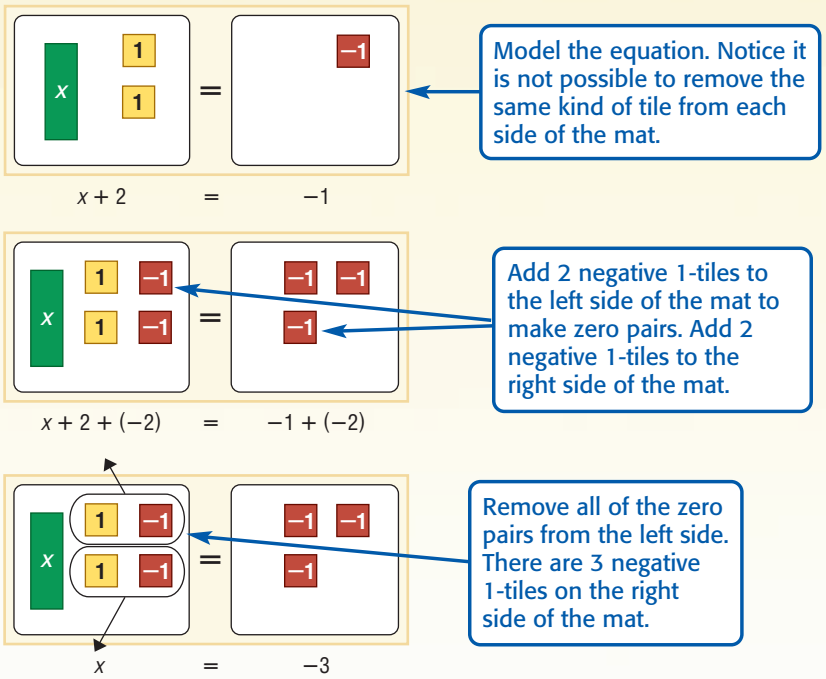
Model

Use algebra tiles to model and solve each equation.

1. $3 + x = 7$ $\qquad$ **2.** $x + 4 = 5$ $\qquad$ **3.** $6 = x + 4$ $\qquad$ **4.** $5 = 1 + x$

Activity 2

Some equations are solved by using zero pairs. You may add or subtract a zero pair from either side of an equation mat without changing its value. The following example shows how to solve $x + 2 = -1$ by using zero pairs.



Therefore, $x = -3$. Since $-3 + 2 = -1$, the solution is correct.

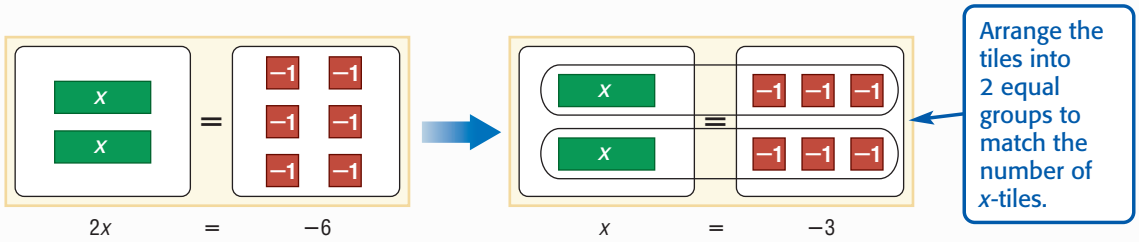
Model

Use algebra tiles to model and solve each equation.

5. $x + 2 = -2$ **6.** $x - 3 = 2$ **7.** $0 = x + 3$ **8.** $-2 = x + 1$

Activity 3

Some equations are modeled using more than one x-tile. The following example shows how to solve $2x = -6$ using algebra tiles.



Therefore, $x = -3$. Since $2(-3) = -6$, the solution is correct.

Model

Use algebra tiles to model and solve each equation.

9. $3x = 3$ **10.** $2x = -8$ **11.** $6 = 3x$ **12.** $-4 = 2x$

3-3 Solving Equations by Adding or Subtracting

What You'll Learn

- Solve equations by using the Subtraction Property of Equality.
- Solve equations by using the Addition Property of Equality.

Vocabulary

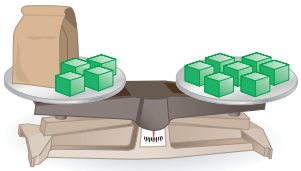
- inverse operation
- equivalent equations

How is solving an equation similar to keeping a scale in balance?

On the balance below, the paper bag contains a certain number of blocks. (Assume that the paper bag weighs nothing.)

a. Without looking in the bag, how can you determine the number of blocks in the bag?

b. Explain why your method works.



SOLVE EQUATIONS BY SUBTRACTING

The equation $x + 4 = 7$ is a model of the situation shown above. You can use inverse operations to solve the equation. **Inverse operations** "undo" each other. For example, to undo the addition of 4 in the expression $x + 4$, you would subtract 4.

To solve the equation $x + 4 = 7$, subtract 4 from each side.

$$x + 4 = 7$$
$$x + 4 - 4 = 7 - 4$$
$$x + 0 = 3$$
$$x = 3$$

Subtract 4 from the left side of the equation to isolate the variable.

Subtract 4 from the right side of the equation to keep it balanced.

The solution is 3.

You can use the **Subtraction Property of Equality** to solve any equation like $x + 4 = 7$.

Key Concept — Subtraction Property of Equality

- **Words** If you subtract the same number from each side of an equation, the two sides remain equal.
- **Symbols** For any numbers a, b, and c, if $a = b$, then $a - c = b - c$.
- **Examples**

$$5 = 5 \qquad\qquad x + 2 = 3$$
$$5 - 3 = 5 - 3 \qquad\qquad x + 2 - 2 = 3 - 2$$
$$2 = 2 \qquad\qquad x = 1$$

Concept Check Which integer would you subtract from each side of $x + 7 = 20$ to solve the equation?

The equations $x + 4 = 7$ and $x = 3$ are **equivalent equations** because they have the same solution, 3. When you solve an equation, you should always check to be sure that the first and last equations are equivalent.

Concept Check Are $x + 4 = 15$ and $x = 4$ equivalent equations? Explain.

Example 1 Solve Equations by Subtracting

Solve $x + 8 = -5$. Check your solution.

$x + 8 = -5$	Write the equation.
$x + 8 - 8 = -5 - 8$	Subtract 8 from each side.
$x + 0 = -13$	$8 - 8 = 0$, $-5 - 8 = -13$
$x = -13$	Identity Property; $x + 0 = x$

To check your solution, replace x with -13 in the original equation.

CHECK	$x + 8 = -5$	Write the equation.
	$-13 + 8 \stackrel{?}{=} -5$	Check to see whether this sentence is true.
	$-5 = -5$ ✓	The sentence is true.

The solution is –13.

Study Tip

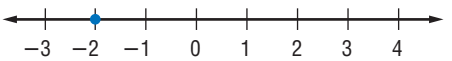
Checking Equations It is always wise to check your solution. You can often use arithmetic facts to check the solutions of simple equations.

Example 2 Graph the Solutions of an Equation

Graph the solution of $16 + x = 14$ on a number line.

$16 + x = 14$	Write the equation.
$x + 16 = 14$	Commutative Property; $16 + x = x + 16$
$x + 16 - 16 = 14 - 16$	Subtract 16 from each side.
$x = -2$	Simplify.

The solution is -2. To graph the solution, draw a dot at -2 on a number line.



Study Tip

Look Back To review **graphing on a number line**, see Lesson 2-1.

SOLVE EQUATIONS BY ADDING Some equations can be solved by adding the same number to each side. This property is called the **Addition Property of Equality**.

Key Concept — **Addition Property of Equality**

- **Words** If you add the same number to each side of an equation, the two sides remain equal.
- **Symbols** For any numbers a, b, and c, if $a = b$, then $a + c = b + c$.
- **Examples**

$6 = 6$	$x - 2 = 5$
$6 + 3 = 6 + 3$	$x - 2 + 2 = 5 + 2$
$9 = 9$	$x = 7$

If an equation has a subtraction expression, first rewrite the expression as an addition expression. Then add the additive inverse to each side.

Example 3 Solve Equations by Adding

Solve $y - 7 = -25$.

$y - 7 = -25$ Write the equation.

$y + (-7) = -25$ Rewrite $y - 7$ as $y + (-7)$.

$y + (-7) + 7 = -25 + 7$ Add 7 to each side.

$y + 0 = -25 + 7$ Additive Inverse Property; $(-7) + 7 = 0$.

$y = -18$ Identity Property; $y + 0 = y$

The solution is -18. Check your solution.

More About . . .

Aviation

On December 17, 1903, the Wright brothers made the first flights in a power-driven airplane. Orville's flight covered 120 feet, which was 732 feet shorter than Wilbur's.

Source: www.infoplease.com

Example 4 Use an Equation to Solve a Problem

AVIATION **Use the information at the left. Write and solve an equation to find the length of Wilbur Wright's flight.**

Words Orville's flight was 732 feet shorter than Wilbur's.

Variables Let x = the length of Wilbur's flight.

Equation Orville's flight: 120 | was: $=$ | 732 feet shorter than Wilbur's flight.: $x - 732$

Solve the equation.

$120 = x - 732$ Think of $x - 732$ as $x + (-732)$.

$120 + 732 = x - 732 + 732$ Add 732 to each side.

$852 = x$ Simplify.

Wilbur's flight was 852 feet.

Almost all standardized tests have items involving equations.

Standardized Test Practice A B C D

Example 5 Solve Equations

Multiple-Choice Test Item

What value of x makes $x - 4 = -2$ a true statement?

(A) 6 (B) 2 (C) -2 (D) -6

Read the Test Item To find the value of x, solve the equation.

Solve the Test Item

$x - 4 = -2$ Write the equation.

$x - 4 + 4 = -2 + 4$ Add 4 to each side.

$x = 2$ Simplify.

The answer is B.

Check for Understanding

Concept Check

1. **Tell** what property you would use to solve $x - 15 = -3$.
2. **OPEN ENDED** Write two equations that are equivalent. Then write two equations that are *not* equivalent.

Guided Practice

ALGEBRA **Solve each equation. Check your solution.**

3. $x + 14 = 25$
4. $w + 4 = -10$
5. $16 = y + 20$
6. $n - 8 = 5$
7. $k - 25 = 30$
8. $r - 4 = -18$

ALGEBRA **Graph the solution of each equation on a number line.**

9. $-8 = x - 6$
10. $y - 3 = -1$

Standardized Test Practice

11. What value of x makes $x - 10 = -5$ a true statement?

(A) -5 (B) 15 (C) 5 (D) -15

Practice and Apply

Homework Help

For Exercises	See Examples
12–32	1, 3
37–42	2
43–47	4

Extra Practice See page 729.

ALGEBRA **Solve each equation. Check your solution.**

12. $y + 7 = 21$
13. $x + 5 = 18$
14. $m + 10 = -2$
15. $x + 5 = -3$
16. $a + 10 = -4$
17. $t + 6 = -9$
18. $y + 8 = 3$
19. $9 = 10 + b$
20. $k - 6 = 13$
21. $r - 5 = 10$
22. $8 = r - 5$
23. $19 = g - 5$
24. $x - 6 = -2$
25. $y - 49 = -13$
26. $-15 = x - 16$
27. $-8 = t - 4$
28. $23 + y = 14$
29. $59 = s + 90$
30. $x - 27 = -63$
31. $84 = r - 34$
32. $y - 95 = -18$

ALGEBRA **Write and solve an equation to find each number.**

33. The sum of a number and 9 is –2.
34. The sum of –5 and a number is –15.
35. The difference of a number and 3 is –6.
36. When 5 is subtracted from a number, the result is 16.

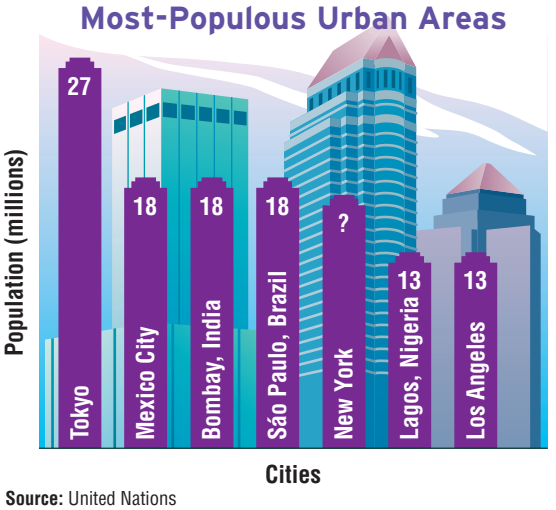
ALGEBRA **Graph the solution of each equation on a number line.**

37. $8 + w = 3$
38. $z - 5 = -8$
39. $8 = x - 2$
40. $9 + x = 12$
41. $-11 = y - 7$
42. $x - (-1) = 0$

43. **ELECTIONS** In the 2000 presidential election, Indiana had 12 electoral votes. That was 20 votes fewer than the number of electoral votes in Texas. Write and solve an equation to find the number of electoral votes in Texas.

44. **WEATHER** The difference between the record high and low temperatures in Charlotte, North Carolina, is 109°F. The record low temperature was −5°F. Write and solve an equation to find the record high temperature.

45. **RESEARCH** Use the Internet or another source to find record temperatures in your state. Use the data to write a problem.



Source: United Nations

POPULATION **For Exercises 46 and 47, use the graph and the following information.**
Tokyo's population is 10 million greater than New York's population. Los Angeles' population is 4 million less than New York's population.

46. Write two different equations to find New York's population.

47. Solve the equations.

Online Research **Data Update** How are the populations of these cities related today? Visit www.pre-alg.com/data_update to learn more.

48. **CRITICAL THINKING** Write two equations in which the solution is -5.

49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is solving an equation similar to keeping a scale in balance?

Include the following in your answer:

- a comparison of an equation and a balanced scale, and
- an explanation of the Addition and Subtraction Properties of Equality.

Standardized Test Practice

50. If $x + 4 = -2$, the numerical value of $-3x - 2$ is

(A) -20. (B) 16. (C) 4. (D) -8.

51. When 7 is subtracted from a number 5 times, the result is 3. What is the number?

(A) 10 (B) -2 (C) 38 (D) 35

Maintain Your Skills

Mixed Review

ALGEBRA **Simplify each expression.** *(Lessons 3-1 and 3-2)*

52. $-2(x + 5)$ **53.** $(t + 4)3$ **54.** $-4(x - 2)$

55. $6z - 3 - 10z + 7$ **56.** $2(x + 6) + 4x$ **57.** $3 - 4(m + 1)$

ALGEBRA **What property is shown by each statement?** *(Lessons 2-2 and 1-4)*

58. $9a + b = b + 9a$ **59.** $x[y + (-y)] = x(0)$ **60.** $6(3x) = (6 \cdot 3)x$

61. **ALGEBRA** Evaluate $9a + 4b$ if $a = 8$ and $b = 3$. *(Lesson 1-3)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Divide.** *(To review **dividing integers**, see Lesson 2-5.)*

62. $-100 \div 10$ **63.** $50 \div (-2)$ **64.** $-49 \div (-7)$

65. $\frac{72}{-8}$ **66.** $\frac{-18}{-6}$ **67.** $\frac{-36}{9}$

3-4 Solving Equations by Multiplying or Dividing

What You'll Learn

- Solve equations by using the Division Property of Equality.
- Solve equations by using the Multiplication Property of Equality.

How are equations used to find the U.S. value of foreign currency?

In Mexico, about 9 *pesos* can be exchanged for $1 of U.S. currency, as shown in the table.

U.S. Value ($)	Number of Pesos
1	9(1) = 9
2	9(2) = 18
3	9(3) = 27
4	9(4) = 36

In general, if we let d represent the number of U.S. dollars and p represent the number of pesos, then $9d = p$.

a. Suppose lunch in Mexico costs 72 pesos. Write an equation to find the cost in U.S. dollars.

b. How can you find the cost in U.S. dollars?

SOLVE EQUATIONS BY DIVIDING The equation $9x = 72$ is a model of the relationship described above. To undo the multiplication operation in $9x$, you would divide by 9.

To solve the equation $9x = 72$, divide each side by 9.

$$9x = 72$$
$$\frac{9x}{9} = \frac{72}{9}$$
$$1x = 8$$
$$x = 8$$

Divide the left side of the equation by 9 to undo the multiplication $9 \cdot x$.

Divide the right side of the equation by 9 to keep it balanced.

The solution is 8.

You can use the **Division Property of Equality** to solve any equation like $9x = 72$.

Key Concept — *Division Property of Equality*

- **Words** When you divide each side of an equation by the same nonzero number, the two sides remain equal.
- **Symbols** For any numbers a, b, and c, where $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
- **Examples**

$14 = 14$	$3x = -12$
$\frac{14}{7} = \frac{14}{7}$	$\frac{3x}{3} = \frac{-12}{3}$
$2 = 2$	$x = -4$

Example 1 Solve Equations by Dividing

Solve $5x = -30$. Check your solution and graph it on a number line.

$5x = -30$ Write the equation.

$\frac{5x}{5} = \frac{-30}{5}$ Divide each side by 5 to undo the multiplication in $5 \cdot x$.

$1x = -6$ $5 \div 5 = 1$, $-30 \div 5 = -6$

$x = -6$ Identity Property; $1x = x$

To check your solution, replace x with -6 in the original equation.

CHECK

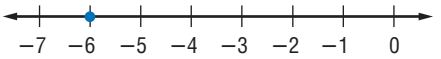
$5x = -30$ Write the equation.

$5(-6) \stackrel{?}{=} -30$ Check to see whether this statement is true.

$-30 = -30$ ✓ The statement is true.

The solution is –6.

To graph the solution, draw a dot at –6 on a number line.



Concept Check Explain how you could find the value of x in $3x = 18$.

Example 2 Use an Equation to Solve a Problem

PARKS It costs $3 per car to use the hiking trails along the Columbia River Highway. If income from the hiking trails totaled $1275 in one day, how many cars entered the park?

Words $3 times the number of cars equals the total.

Variables Let x represent the number of cars.

Equation

The cost per car	times	the number of cars	equals	the total.
$3	·	x	=	$1275

Solve the equation.

$3x = 1275$ Write the equation.

$\frac{3x}{3} = \frac{1275}{3}$ Divide each side by 3.

$x = 425$ Simplify.

CHECK

$3x = 1275$ Write the equation.

$3(425) \stackrel{?}{=} 1275$ Check to see whether this statement is true.

$1275 = 1275$ ✓ The statement is true.

Therefore, 425 cars entered the park.

Concept Check Suppose it cost $5 per car to use the hiking trails and the total income was $1275. What equation would you solve?

More About . . .

Parks

The Columbia River Highway, built in 1913, is a historic route in Oregon that curves around twenty waterfalls through the Cascade Mountains.

Source: *USA TODAY*

SOLVE EQUATIONS BY MULTIPLYING Some equations can be solved by multiplying each side by the same number. This property is called the **Multiplication Property of Equality**.

Key Concept — Multiplication Property of Equality

- **Words** When you multiply each side of an equation by the same number, the two sides remain equal.
- **Symbols** For any numbers a, b, and c, if $a = b$, then $ac = bc$.
- **Examples**

$8 = 8$ | $\frac{x}{6} = 7$

$8(-2) = 8(-2)$ | $\left(\frac{x}{6}\right)6 = (7)6$

$-16 = -16$ | $x = 42$

Example 3 Solve Equations by Multiplying

Study Tip

Division Expressions
Remember, $\frac{y}{-4}$ means *y divided by −4*.

Solve $\frac{y}{-4} = -9$. Check your solution.

$\frac{y}{-4} = -9$ Write the equation.

$\frac{y}{-4}(-4) = -9(-4)$ Multiply each side by -4 to undo the division in $\frac{y}{-4}$.

$y = 36$ Simplify.

CHECK $\frac{y}{-4} = -9$ Write the equation.

$\frac{36}{-4} \stackrel{?}{=} -9$ Check to see whether this statement is true.

$-9 = -9$ ✓ The statement is true.

The solution is 36.

Check for Understanding

Concept Check

1. **State** what property you would use to solve $\frac{x}{-9} = -36$.
2. **Explain** how to find the value of y in $-5y = -45$.
3. **OPEN ENDED** Write an equation of the form $ax = c$ where a and c are integers and the solution is 4.

Guided Practice

ALGEBRA **Solve each equation. Check your solution.**

4. $4x = 24$
5. $-2a = 10$
6. $-42 = -7t$
7. $\frac{k}{3} = 9$
8. $\frac{y}{5} = -8$
9. $-11 = \frac{n}{-6}$

Application

10. **TOYS** A spiral toy that can bounce down a flight of stairs is made from 80 feet of wire. Write and solve an equation to find how many of these toys can be made from a spool of wire that contains 4000 feet.

Practice and Apply

Homework Help

For Exercises	See Examples
11–34, 39–44	1, 3
45–48	2

Extra Practice
See page 729.

ALGEBRA **Solve each equation. Check your solution.**

11. $3t = 21$ **12.** $8x = 72$ **13.** $-32 = 4y$

14. $5n = -95$ **15.** $-56 = -7p$ **16.** $-8j = -64$

17. $\frac{h}{4} = 6$ **18.** $\frac{c}{9} = 4$ **19.** $\frac{g}{-2} = -7$

20. $-42 = \frac{x}{-2}$ **21.** $11 = \frac{b}{-3}$ **22.** $\frac{h}{-7} = 20$

23. $45 = 5x$ **24.** $3u = 51$ **25.** $86 = -2v$

26. $-8a = 144$ **27.** $\frac{m}{45} = -3$ **28.** $\frac{d}{3} = -3$

29. $\frac{f}{-13} = -10$ **30.** $\frac{v}{-11} = -132$ **31.** $-116 = -4w$

32. $-68 = -4m$ **33.** $-21 = \frac{k}{8}$ **34.** $-56 = \frac{t}{9}$

ALGEBRA **Write and solve an equation for each sentence.**

35. The product of a number and 6 is -42.

36. The product of -7 and a number is -35.

37. The quotient of a number and -4 is 8.

38. When you divide a number by -5, the result is -2.

ALGEBRA **Graph the solution of each equation on a number line.**

39. $48 = -6x$ **40.** $-32t = 64$ **41.** $-6r = -18$

42. $-42 = -7x$ **43.** $\frac{n}{12} = 3$ **44.** $\frac{y}{-4} = -1$

More About . . .

Ranching

Some students living in the Outback are so far from schools that they get their education by special radio programming. They mail in their homework and sometimes talk to teachers by two-way radio.

Source: *Kids Discover Australia*

45. RANCHING The largest ranch in the world is in the Australian Outback. It is about 12,000 square miles, which is five times the size of the largest United States ranch. Write and solve an equation to find the size of the largest United States ranch.

46. RANCHING In the driest part of an Outback ranch, each cow needs about 40 acres for grazing. Write and solve an equation to find how many cows can graze on 720 acres of land.

47. PAINTING A **person-day** is a unit of measure that represents one person working for one day. A painting contractor estimates that it will take 24 person-days to paint a house. Write and solve an equation to find how many painters the contractor will need to hire to paint the house in 6 days.

48. MEASUREMENT The chart shows several conversions in the customary system. Write and solve an equation to find each quantity.

a. the number of feet in 132 inches

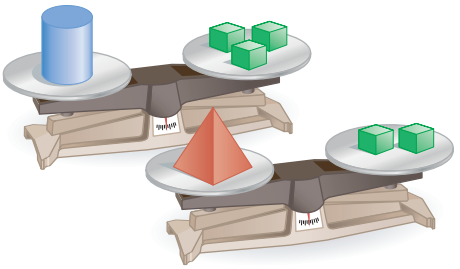
b. the number of yards in 15 feet

c. the number of miles in 10,560 feet

Customary System (length)
1 mile = 5280 feet
1 mile = 1760 yards
1 yard = 3 feet
1 foot = 12 inches
1 yard = 36 inches

CONTENTS

49. CRITICAL THINKING Suppose that one pyramid balances two cubes and one cylinder balances three cubes. Determine whether each statement is *true* or *false*. Justify your answers.



a. One pyramid and one cube balance three cubes.

b. One pyramid and one cube balance one cylinder.

c. One cylinder and one pyramid balance four cubes.

50. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are equations used to find the U.S. value of foreign currency?

Include the following in your answer:

- the cost in U.S. dollars of a 12-*pound* bus trip in Egypt, if 4 *pounds* can be exchanged for one U.S. dollar, and
- the cost in U.S. dollars of a 3040-*schilling* hotel room in Austria, if 16 *schillings* can be exchanged for one U.S. dollar.

Online Research Data Update How many pounds and schillings can be exchanged for a U.S. dollar today? Visit www.pre-alg.com/data_update to learn more.

Standardized Test Practice (A) (B) (C) (D)

51. Solve $\frac{mx}{n} = p$ for x.

(A) $x = \frac{m}{pn}$ (B) $x = \frac{pn}{m}$ (C) $x = \frac{p - n}{m}$ (D) $x = pn - m$

52. A number is divided by -6, and the result is 24. What is the original number?

(A) -4 (B) 4 (C) -144 (D) 144

Maintain Your Skills

Mixed Review

ALGEBRA **Solve each equation.** *(Lesson 3-3)*

53. $3 + y = 16$ **54.** $29 = n + 4$ **55.** $k - 12 = -40$

ALGEBRA **Simplify each expression.** *(Lesson 3-2)*

56. $4x + 7x$ **57.** $2y + 6 + 5y$ **58.** $3 - 2(y + 4)$

59. ALGEBRA Evaluate $3ab$ if $a = -6$ and $b = 2$. *(Lesson 2-4)*

60. Replace ● in -14 ● -4 with $<$, $>$, or $=$ to make a true sentence. *(Lesson 2-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each difference.**

*(To review **subtracting integers**, see Lesson 2-3.)*

61. $8 - (-2)$ **62.** $-5 - 5$ **63.** $-10 - (-8)$

64. $-18 - 4$ **65.** $-3 - (-5)$ **66.** $-45 - (-9)$

67. $-24 - (-5)$ **68.** $-15 - (-15)$ **69.** $-8 - 19$

www.pre-alg.com/self_check_quiz

3-5 Solving Two-Step Equations

What You'll Learn

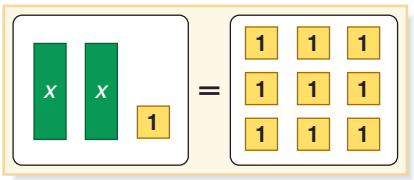
- Solve two-step equations.

How can algebra tiles show the properties of equality?

Vocabulary

- two-step equation

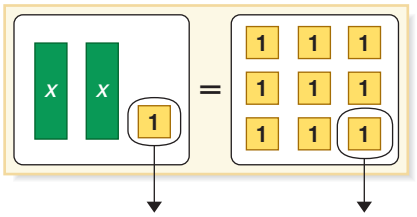
The equation $2x + 1 = 9$, modeled below, can be solved with algebra tiles.



$2x + 1 \quad = \quad 9$

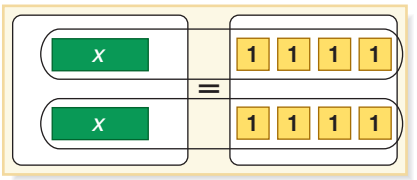
You can use the steps shown at the right to solve the equation.

Step 1 Remove 1 tile from each side of the mat.



$2x + 1 - 1 \quad = \quad 9 - 1$

Step 2 Separate the remaining tiles into two equal groups.



$2x \quad = \quad 8$

a. What property is shown by removing a tile from each side?

b. What property is shown by separating the tiles into two groups?

c. What is the solution of $2x + 1 = 9$?

SOLVE TWO-STEP EQUATIONS A **two-step equation** contains two operations. In the equation $2x + 1 = 9$, x is multiplied by 2 and then 1 is added. To solve two-step equations, use inverse operations to undo each operation in reverse order. You can solve $2x + 1 = 9$ in two steps.

Step 1 First, undo addition.

$$2x + 1 = 9$$
$$2x + 1 - 1 = 9 - 1$$
$$2x = 8$$

Subtract 1 from each side.

Step 2 Then, undo multiplication.

$$2x = 8$$
$$\frac{2x}{2} = \frac{8}{2}$$
$$x = 4$$

Divide each side by 2.

The solution is 4.

CONTENTS

Example 1 Solve Two-Step Equations

a. Solve $5x - 2 = 13$. Check your solution.

$5x - 2 = 13$ Write the equation.

$5x - 2 + 2 = 13 + 2$ Undo subtraction. Add to each side.

$5x = 15$ Simplify.

$\frac{5x}{5} = \frac{15}{5}$ Undo multiplication. Divide each side by 5.

$x = 3$ Simplify.

CHECK $5x - 2 = 13$ Write the equation.

$5(3) - 2 \stackrel{?}{=} 13$ Check to see whether this statement is true.

$13 = 13$ ✓ The statement is true.

The solution is 3.

b. Solve $4 = \frac{n}{6} + 11$.

$4 = \frac{n}{6} + 11$ Write the equation.

$4 - 11 = \frac{n}{6} + 11 - 11$ Undo addition. Subtract 11 from each side.

$-7 = \frac{n}{6}$ Simplify.

$6(-7) = 6\left(\frac{n}{6}\right)$ Undo division. Multiply each side by 6.

$-42 = n$ Simplify.

The solution is -42. Check your solution.

Concept Check Explain how inverse operations can be used to solve a two-step equation.

Many real-world situations can be modeled with two-step equations.

Example 2 Use an Equation to Solve a Problem

SALES Mandy bought a DVD player. The sales clerk says that if she pays \$80 now, her monthly payments will be \$32. The total cost will be \$400. Solve $80 + 32x = 400$ to find how many months she will make payments.

$80 + 32x = 400$ Write the equation.

$80 - 80 + 32x = 400 - 80$ Subtract 80 from each side.

$32x = 320$ Simplify.

$\frac{32x}{32} = \frac{320}{32}$ Divide each side by 32.

$x = 10$ Simplify.

The solution is 10.

Therefore, Mandy will make payments for 10 months.

Study Tip

Checking Your Solution

Use estimation to determine whether your solution is reasonable: $80 + 30(10) = 380$. Since \$380 is close to \$400, the solution is reasonable.

Some two-step equations have terms with negative coefficients.

Example 3 Equations with Negative Coefficients

Solve $4 - x = 10$.

$4 - x = 10$	Write the equation.
$4 - 1x = 10$	Identity Property; $x = 1x$
$4 + (-1x) = 10$	Definition of subtraction
$-4 + 4 + (-1x) = -4 + 10$	Add -4 to each side.
$-1x = 6$	Simplify.
$\frac{-1x}{-1} = \frac{6}{-1}$	Divide each side by -1.
$x = -6$	Simplify.
The solution is -6.	Check your solution.

Sometimes it is necessary to combine like terms before solving.

Example 4 Combine Like Terms Before Solving

Solve $m - 5m + 3 = 47$.

$m - 5m + 3 = 47$	Write the equation.
$1m - 5m + 3 = 47$	Identity Property; $m = 1m$
$-4m + 3 = 47$	Combine like terms, $1m$ and $-5m$.
$-4m + 3 - 3 = 47 - 3$	Subtract 3 from each side.
$-4m = 44$	Simplify.
$\frac{-4m}{-4} = \frac{44}{-4}$	Divide each side by -4.
$m = -11$	Simplify.
The solution is -11.	Check your solution.

Study Tip

Mental Computation

You use the Distributive Property to simplify $1m - 5m$.

$1m - 5m = (1 - 5)m$
$= -4m$

You can also simplify the expression mentally.

Check for Understanding

Concept Check

1. **Explain** how you can work backward to solve a two-step equation.
2. **OPEN ENDED** Write a two-step equation that could be solved by using the Addition and Multiplication Properties of Equality.

Guided Practice

ALGEBRA **Solve each equation. Check your solution.**

3. $2x - 7 = 9$
4. $3t + 5 = 2$
5. $-16 = 6a - 4$
6. $\frac{y}{3} + 2 = 10$
7. $1 + \frac{k}{4} = -9$
8. $8 = \frac{n}{-7} - 5$
9. $3 - c = 7$
10. $2a - 8a = 24$
11. $8y - 9y + 6 = -4$

Application

12. **MEDICINE** For Jillian's cough, her doctor says that she should take eight tablets the first day and then four tablets each day until her prescription runs out. There are 36 tablets. Solve $8 + 4d = 36$ to find the number of days she will take four tablets.

Practice and Apply

Homework Help

For Exercises	See Examples
13–34	1
35–38	3
39–46	4
47–49	2

Extra Practice
See page 729.

ALGEBRA **Solve each equation. Check your solution.**

13. $3x + 1 = 7$ **14.** $5x - 4 = 11$ **15.** $4h + 6 = 22$

16. $8n + 3 = -5$ **17.** $37 = 4d + 5$ **18.** $9 = 15 + 2p$

19. $2n - 5 = 21$ **20.** $3j - 9 = 12$ **21.** $-1 = 2r - 7$

22. $12 = 5k - 8$ **23.** $10 = 6 + \frac{y}{7}$ **24.** $14 = 6 + \frac{n}{5}$

25. $3 + \frac{t}{2} = 35$ **26.** $13 + \frac{p}{3} = -4$ **27.** $\frac{k}{5} - 10 = 3$

28. $\frac{w}{8} - 4 = -7$ **29.** $8 = \frac{c}{-3} + 15$ **30.** $\frac{b}{-4} + 8 = -42$

ALGEBRA **Find each number.**

31. Five more than twice a number is 27. Solve $2n + 5 = 27$.

32. Three less than four times a number is -7. Solve $4n - 3 = -7$.

33. Ten less than the quotient of a number and 2 is 5. Solve $\frac{n}{2} - 10 = 5$.

34. Six more than the quotient of a number and 6 is -3. Solve $\frac{n}{6} + 6 = -3$.

ALGEBRA **Solve each equation. Check your solution.**

35. $8 - t = -25$ **36.** $3 - y = 13$ **37.** $-5 - b = 8$

38. $10 = -9 - x$ **39.** $2w - 4w = -10$ **40.** $3x - 5x = 22$

41. $x + 4x + 6 = 31$ **42.** $5r + 3r - 6 = 10$ **43.** $1 - 3y + y = 5$

44. $16 = w - 2w + 9$ **45.** $23 = 4t - 7 - t$ **46.** $-4 = -a + 8 - 2a$

47. POOLS There were 640 gallons of water in a 1600-gallon pool. Water is being pumped into the pool at a rate of 320 gallons per hour. Solve $1600 = 320t + 640$ to find how many hours it will take to fill the pool.

48. PHONE CALLING CARDS A telephone calling card allows for 25¢ per minute plus a one-time service charge of 75¢. If the total cost of the card is $5, solve $25m + 75 = 500$ to find the number of minutes you can use the card.

49. BUSINESS Twelve-year old Aaron O'Leary of Columbus, Ohio, bought old bikes at an auction for $350. He fixed them and sold them for $50 each. He made a $6200 profit. Solve $6200 = 50b - 350$ to determine how many bikes he sold.

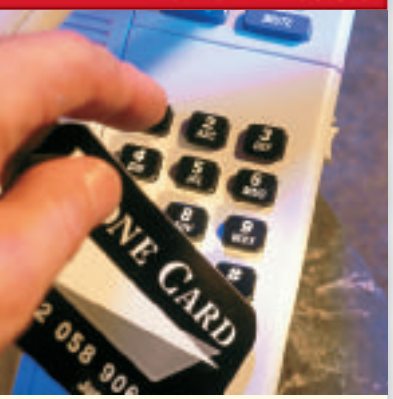
50. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How can algebra tiles show the properties of equality?

Include the following in your answer:

- a drawing that shows how to solve $2x + 3 = 7$ using algebra tiles, and
- a list of the properties of equality that you used to solve $2x + 3 = 7$.

More About . . .



Phone Calling Cards

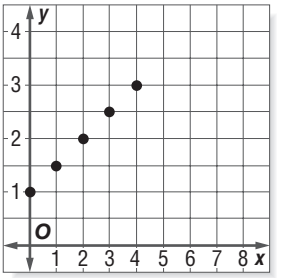
Sales of prepaid phone cards skyrocketed from $12 million in 1992 to $1.9 billion in 1998.

Source: Atlantic ACM

51. CRITICAL THINKING Write a two-step equation with a variable using the numbers 2, 5, and 8, in which the solution is 2.

Standardized Test Practice

52. GRID IN The charge to park at an art fair is a flat rate plus a per hour fee. The graph shows the charge for parking for up to 4 hours. If x represents the number of hours and y represents the total charge, what is the charge for parking for 7 hours?



53. A local health club charges an initial fee of $45 for the first month and then a $32 membership fee each month after the first. The table shows the cost to join the health club for up to 6 months.

Months	1	2	3	4	5	6
Cost (dollars)	45	77	109	141	173	205

What is the cost to join the health club for 10 months?

Ⓐ $215 Ⓑ $320 Ⓒ $333 Ⓓ $450

Maintain Your Skills

Mixed Review

ALGEBRA **Solve each equation. Check your solution.** *(Lessons 3-3 and 3-4)*

54. $5y = 60$ **55.** $14 = -2n$ **56.** $\frac{x}{3} = -9$

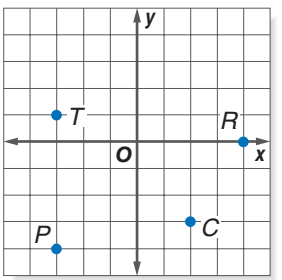
57. $x - 4 = -6$ **58.** $-13 = y + 5$ **59.** $18 = 20 + x$

ALGEBRA **Simplify each expression.** *(Lesson 3-1)*

60. $4(x + 1)$ **61.** $-5(y + 3)$ **62.** $3(k - 10)$

63. $-9(y - 4)$ **64.** $7(a - 2)$ **65.** $-8(r - 5)$

Name the ordered pair for each point graphed on the coordinate plane at the right. *(Lesson 2-6)*



66. T

67. C

68. R

69. P

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Write an algebraic expression for each verbal expression.** *(To review **algebraic expressions**, see Lesson 1-3.)*

70. two times a number less six

71. the quotient of a number and 15

72. the difference between twice a number and 8

73. twice a number increased by 10

74. the sum of $2x$, $7x$, and 4

Translating Verbal Problems into Equations

An important skill in algebra is translating verbal problems into equations. To do this accurately, analyze the statements until you completely understand the relationships among the given information. Look for key words and phrases.

> Jennifer is 6 years older than Akira. The sum of their ages is 20.

You can explore a problem situation by asking and answering questions.

Questions	Answers
a. Who is older?	**a.** Jennifer
b. How many years older?	**b.** 6 years
c. If Akira is x years old, how old is Jennifer?	**c.** $x + 6$
d. What expression represents the phrase *the sum of their ages*?	**d.** $x + (x + 6)$
e. What equation represents the sentence *the sum of their ages is 20*?	**e.** $x + (x + 6) = 20$

Reading to Learn

For each verbal problem, answer the related questions.

1. Lucas is 5 inches taller than Tamika, and the sum of their heights is 137 inches.

a. Who is taller?

b. How many inches taller?

c. If x represents Tamika's height, how tall is Lucas?

d. What expression represents *the sum of their heights*?

e. What equation represents the sentence *the sum of their heights is 137*?

2. There are five times as many students as teachers on the field trip, and the sum of students and teachers is 132.

a. Are there more students or teachers?

b. How many times more?

c. If x represents the number of teachers, how many students are there?

d. What expression represents *the sum of students and teachers*?

e. What equation represents *the sum of students and teachers is 132*?

3-6 Writing Two-Step Equations

What You'll Learn

- Write verbal sentences as two-step equations.
- Solve verbal problems by writing and solving two-step equations.

How are equations used to solve real-world problems?

A phone company advertises that you can call anywhere in the United States for 4¢ per minute plus a monthly fee of 99¢ with their calling card. The table shows how to find your total monthly cost.

Time (minutes)	Monthly Cost (cents)
0	4(0) + 99 = 99
5	4(5) + 99 = 119
10	4(10) + 99 = 139
15	4(15) + 99 = 159
20	4(20) + 99 = 179

a. Let n represent the number of minutes. Write an expression that represents the cost when your call lasts n minutes.

b. Suppose your monthly cost was 299¢. Write and solve an equation to find the number of minutes you used the calling card.

c. Why is your equation considered to be a two-step equation?

WRITE TWO-STEP EQUATIONS

In Chapter 1, you learned how to write verbal phrases as expressions.

Phrase the sum of 4 times some number and 99

Expression $4n + 99$

Study Tip

Expression
An expression is any combination of numbers and operations.

An equation is a statement that two expressions are equal. The expressions are joined with an equals sign. Look for the words *is*, *equals*, or *is equal to* when you translate sentences into equations.

Sentence The sum of 4 times some number and 99 is 299.

Equation $4n + 99 = 299$

Example 1 Translate Sentences into Equations

Translate each sentence into an equation.

	Sentence	Equation
a.	Six more than twice a number is −20.	$2n + 6 = -20$
b.	Eighteen is 6 less than four times a number.	$18 = 4n - 6$
c.	The quotient of a number and 5, increased by 8, is equal to 14.	$\frac{n}{5} + 8 = 14$

Concept Check What is the difference between an expression and an equation?

Example 2 Translate and Solve an Equation

Seven more than three times a number is 31. Find the number.

Words Seven more than three times a number is 31.

Variables Let n = the number.

Equation

$$3n + 7 = 31$$ Write the equation.

$$3n + 7 - 7 = 31 - 7$$ Subtract 7 from each side.

$$3n = 24$$ Simplify.

$$n = 8$$ Mentally divide each side by 3.

Therefore, the number is 8.

Study Tip

Solving Equations Mentally

When solving a simple equation like $3n = 24$, mentally divide each side by 3.

TWO-STEP VERBAL PROBLEMS There are many real-world situations in which you start with a given amount and then increase it at a certain rate. These situations can be represented by two-step equations.

Example 3 Write and Solve a Two-Step Equation

SCOOTERS **Suppose you are saving money to buy a scooter that costs $100. You have already saved $60 and plan to save $5 each week. How many weeks will you need to save?**

Explore You have already saved $60. You plan to save $5 each week until you have $100.

Plan Organize the data for the first few weeks in a table. Notice the pattern.

Week	Amount
0	5(0) + 60 = 60
1	5(1) + 60 = 65
2	5(2) + 60 = 70
3	5(3) + 60 = 75

Write an equation to represent the situation.

Let x = the number of weeks.

$5 each week for x weeks	plus	amount already saved	equals	$100.
$5x$	+	60	=	100

Solve

$$5x + 60 = 100$$ Write the equation.

$$5x + 60 - 60 = 100 - 60$$ Subtract 60 from each side.

$$5x = 40$$ Simplify.

$$x = 8$$ Mentally divide each side by 5.

You need to save $5 each week for 8 weeks.

Examine If you save $5 each week for 8 weeks, you'll have an additional $40. The answer appears to be reasonable.

More About . . .

Scooters

Most scooters are made of aircraft aluminum. They can transport over 200 pounds easily, yet are light enough to carry.

Source: www.emazing.com

Example 4 Write and Solve a Two-Step Equation

OLYMPICS **In the 2000 Summer Olympics, the United States won 9 more medals than Russia. Together they won 185 medals. How many medals did the United States win?**

Words Together they won 185 medals.

Variables Let x = number of medals won by Russia.
Then $x + 9$ = number of medals won by United States.

Equation

$x + (x + 9) = 185$	Write the equation.
$(x + x) + 9 = 185$	Associative Property
$2x + 9 = 185$	Combine like terms.
$2x + 9 - 9 = 185 - 9$	Subtract 9 from each side.
$2x = 176$	Simplify.
$\frac{2x}{2} = \frac{176}{2}$	Divide each side by 2.
$x = 88$	Simplify.

Since x represents the number of medals won by Russia, Russia won 88 medals. The United States won 88 + 9 or 97 medals.

Study Tip

Alternative Method
Let x = number of U.S. medals. Then $x - 9$ = number of Russian medals.
$x + (x - 9) = 185$
$x = 97$
In this case, x is the number of U.S. medals, 97.

Check for Understanding

Concept Check

1. **List** three words or phrases in a verbal problem that can be translated into an equals sign in an equation.

2. **OPEN ENDED** Write a verbal sentence involving an unknown number and two operations.

3. **FIND THE ERROR** Alicia and Ben are translating the following sentence into an equation: *Three less than two times a number is 15.*

Alicia	Ben
$3 - 2x = 15$	$2x - 3 = 15$

Who is correct? Explain your reasoning.

Guided Practice

Translate each sentence into an equation. Then find each number.

4. Three more than four times a number is 23.

5. Four less than twice a number is –2.

Applications

Solve each problem by writing and solving an equation.

6. **METEOROLOGY** Suppose the current temperature is 17°F. It is expected to rise 3°F each hour for the next several hours. In how many hours will the temperature be 32°F?

7. **AGES** Lawana is five years older than her brother Cole. The sum of their ages is 37. How old is Lawana?

Practice and Apply

Homework Help

For Exercises	See Examples
8–19	1, 2
20, 21	3
22–24	4

Extra Practice
See page 730.

Translate each sentence into an equation. Then find each number.

8. Seven more than twice a number is 17.
9. Twenty more than three times a number is -4.
10. Four less than three times a number is 20.
11. Eight less than ten times a number is 82.
12. Ten more than the quotient of a number and -2 is three.
13. The quotient of a number and -4, less 8, is -42.
14. The difference between twice a number and 9 is 17.
15. The difference between three times a number and 8 is -2.
16. If 5 is decreased by 3 times a number, the result is -4.
17. If 17 is decreased by twice a number, the result is 5.
18. Three times a number plus twice the number plus 1 is -4.
19. Four times a number plus five more than three times the number is 47.

Solve each problem by writing and solving an equation.

20. **WILDLIFE** Your friend bought 3 bags of wild bird seed and an $18 bird feeder. Each bag of birdseed costs the same amount. If your friend spent $45, find the cost of one bag of birdseed.

21. **METEOROLOGY** The temperature is 8°F. It is expected to fall 5° each hour for the next several hours. In how many hours will the temperature be -7°F?

22. **FOOD SERVICE** You and your friend spent a total of $15 for lunch. Your friend's lunch cost $3 more than yours did. How much did you spend for lunch?

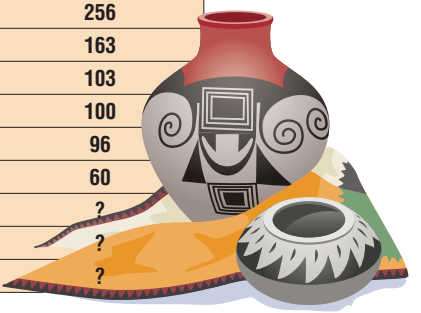
23. **POPULATION** By 2020, California is expected to have 2 million more senior citizens than Florida, and the sum of the number of senior citizens in the two states is expected to be 12 million. Find the expected senior citizen population of Florida in 2020.

24. **NATIVE AMERICANS** North Carolina's Native-American population is 22,000 greater than New York's. New York's Native-American population is 187,000 less than Oklahoma's. If the total population of all three is 437,000, find each state's Native-American population.

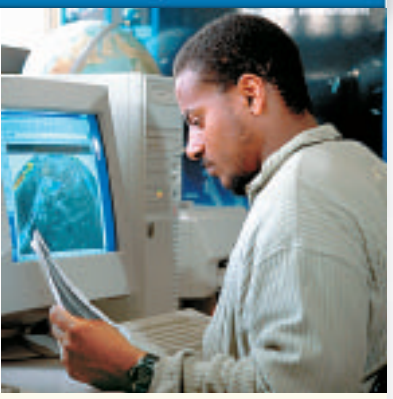
Native-American Populations (thousands)

California	309
Arizona	256
New Mexico	163
Washington	103
Alaska	100
Texas	96
Michigan	60
Oklahoma	?
New York	?
North Carolina	?

Source: U.S. Census Bureau



Career Choices



Meteorologist

Meteorologists are best known for forecasting the weather. However, they also work in the fields of air pollution, agriculture, and defense.

Online Research

For information about a career as a meteorologist, visit: www.pre-alg.com/careers

www.pre-alg.com/self_check_quiz

25. WRITE A PROBLEM The table shows the expected population age 85 or older for certain states in 2020. Use the data to write a problem that can be solved by using a two-step equation.

Population (age 85 or older)	
State	Number (thousands)
CA	809
FL	735
TX	428
NY	418

26. CRITICAL THINKING If you begin with an even integer and count by two, you are counting *consecutive even integers*. Write and solve an equation to find two consecutive even integers whose sum is 50.

27. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are equations used to solve real-world problems?

Include the following in your answer:

- an example that starts with a given amount and increases, and
- an example that involves the sum of two quantities.

Standardized Test Practice

28. Which verbal expression represents the phrase *three less than five times a number*?

(A) $3 - 5n$ (B) $n - 3$ (C) $5n - 3$ (D) $5 + n - 3$

29. The Bank of America building in San Francisco is 74 feet shorter than the Transamerica Pyramid. If their combined height is 1632 feet, how tall is the Transamerica Pyramid?

(A) 41 ft (B) 779 ft (C) 781 ft (D) 853 ft

Maintain Your Skills

Mixed Review

ALGEBRA **Solve each equation.** *(Lessons 3-3, 3-4, and 3-5)*

30. $6 - 2x = 10$ **31.** $-4x = -16$ **32.** $y - 7 = -3$

Evaluate each expression if $x = -12$, $y = 4$, and $z = -1$. *(Lesson 2-1)*

33. $|x| - 7$ **34.** $|x| + |y|$ **35.** $|z| - |x|$

36. Name the property shown by $(2 + 6) + 9 = 2 + (6 + 9)$. *(Lesson 1-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation. Check your solution.**
*(To review **solving equations**, see Lesson 3-4.)*

37. $2x = -8$ **38.** $24 = 6y$ **39.** $5w = -25$

40. $15s = 75$ **41.** $108 = 18x$ **42.** $25z = 175$

Practice Quiz 2

Lessons 3-3 through 3-6

ALGEBRA **Solve each equation.** *(Lessons 3-3, 3-4, and 3-5)*

1. $4h = -52$ **2.** $\frac{x}{-3} = 4$ **3.** $y - 5 = -23$ **4.** $2v - 11 = -5$

5. ALGEBRA Twenty more than three times a number is 32. Write and solve an equation to find the number. *(Lesson 3-6)*

3-7 Using Formulas

What You'll Learn

- Solve problems by using formulas.
- Solve problems involving the perimeters and areas of rectangles.

Vocabulary

- formula
- perimeter
- area

Why are formulas important in math and science?

The top recorded speed of a mallard duck in level flight is 65 miles per hour. You can make a table to record the distances that a mallard could fly at that rate.

a. Write an expression for the distance traveled by a duck in t hours.

b. What disadvantage is there in showing the data in a table?

c. Describe an easier way to summarize the relationship between the speed, time, and distance.

Speed (mph)	Time (hr)	Distance (mi)
65	1	65
65	2	130
65	3	195
65	t	?

FORMULAS A **formula** is an equation that shows a relationship among certain quantities. A formula usually contains two or more variables. One of the most commonly-used formulas shows the relationship between distance, rate (or speed), and time.

Words Distance equals the rate multiplied by the time.

Variables Let d = distance, r = rate, and t = time.

Equation $d = rt$

Example 1 Use the Distance Formula

SCIENCE **What is the rate in miles per hour of a dolphin that travels 120 miles in 4 hours?**

$d = rt$ Write the formula.

$120 = r \cdot 4$ Replace d with 120 and t with 4.

$\frac{120}{4} = \frac{r \cdot 4}{4}$ Divide each side by 4.

$30 = r$ Simplify.

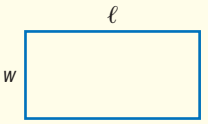
The dolphin travels at a rate of 30 miles per hour.

Concept Check Name an advantage of using a formula to show a relationship among quantities.

PERIMETER AND AREA The distance around a geometric figure is called the **perimeter**. One method of finding the perimeter P of a rectangle is to add the measures of the four sides.

Key Concept — Perimeter of a Rectangle

- **Words** The perimeter of a rectangle is twice the sum of the length and width.
- **Symbols** $P = \ell + \ell + w + w$
  $P = 2\ell + 2w$ or $2(\ell + w)$
- **Model**



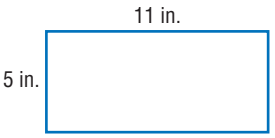
Study Tip

Common Misconception
Although the length of a rectangle is usually greater than the width, it does not matter which side you choose to be the length.

Example 2 Find the Perimeter of a Rectangle

Find the perimeter of the rectangle.

$P = 2(\ell + w)$ Write the formula.
$P = 2(11 + 5)$ Replace ℓ with 11 and w with 5.
$P = 2(16)$ Add 11 and 5.
$P = 32$ Simplify.



The perimeter is 32 inches.

Example 3 Find a Missing Length

The perimeter of a rectangle is 28 meters. Its width is 8 meters. Find the length.

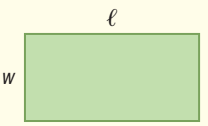
$P = 2(\ell + w)$ Write the formula.
$P = 2\ell + 2w$ Distributive Property
$28 = 2\ell + 2(8)$ Replace P with 28 and w with 8.
$28 = 2\ell + 16$ Simplify.
$28 - 16 = 2\ell + 16 - 16$ Subtract 16 from each side.
$12 = 2\ell$ Simplify.
$6 = \ell$ Mentally divide each side by 2.

The length is 6 meters.

The measure of the surface enclosed by a figure is its **area**.

Key Concept — Area of a Rectangle

- **Words** The area of a rectangle is the product of the length and width.
- **Symbols** $A = \ell w$
- **Model**



CONTENTS

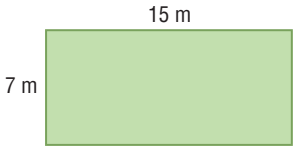
Example 4 Find the Area of a Rectangle

Find the area of a rectangle with length 15 meters and width 7 meters.

$A = \ell w$ Write the formula.

$A = 15 \cdot 7$ Replace ℓ with 15 and w with 7.

$A = 105$ Simplify.



The area is 105 square meters.

Example 5 Find a Missing Width

The area of a rectangle is 45 square feet. Its length is 9 feet. Find its width.

$A = \ell w$ Write the formula.

$45 = 9w$ Replace A with 45 and ℓ with 9.

$5 = w$ Mentally divide each side by 9.

The width is 5 feet.

Concept Check Which is measured in square units, area or perimeter?

Check for Understanding

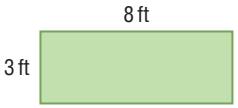
Concept Check

1. **Write** the formula that shows the relationship among distance, rate, and time.
2. **Explain** the difference between the perimeter and area of a rectangle.
3. **OPEN ENDED** Draw and label a rectangle that has a perimeter of 18 inches.

Guided Practice

GEOMETRY **Find the perimeter and area of each rectangle.**

4.



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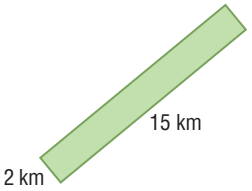
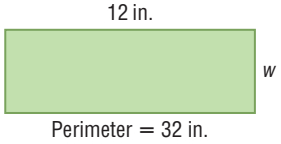


6. a rectangle with length 15 feet and width 6 feet

GEOMETRY **Find the missing dimension in each rectangle.**

7.



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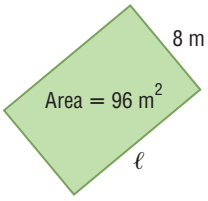


Application

9. **MILITARY** How long will it take an Air Force jet fighter to fly 5200 miles at 650 miles per hour?

Practice and Apply

Homework Help

For Exercises	See Examples
10–11	1
12–19	2, 4
20–27	3, 5

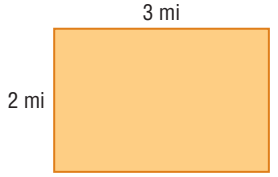
Extra Practice
See page 730.

10. TRAVEL Find the distance traveled by driving at 55 miles per hour for 3 hours.

11. BALLOONING What is the rate, in miles per hour, of a balloon that travels 60 miles in 4 hours?

GEOMETRY Find the perimeter and area of each rectangle.

12.



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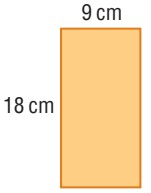

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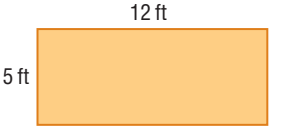

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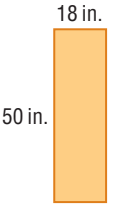

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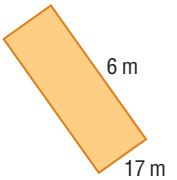

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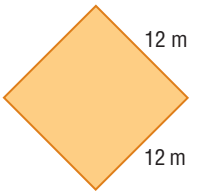
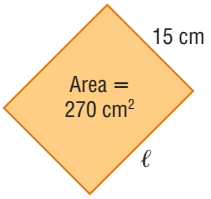


18. a rectangle that is 38 meters long and 10 meters wide

19. a square that is 5 meters on each side

GEOMETRY Find the missing dimension in each rectangle.

20.



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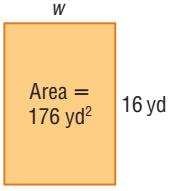

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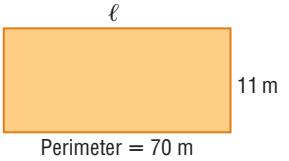

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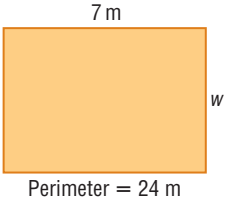

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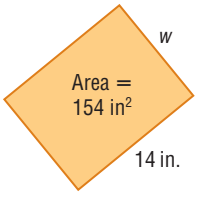

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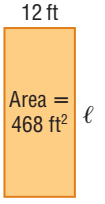


26. GEOMETRY The perimeter of a rectangle is 46 centimeters. Its width is 5 centimeters. Find the length.

27. GEOMETRY The area of a rectangle is 323 square yards. Its length is 17 yards. Find the width.

28. COMMUNITY SERVICE Each participant in a community garden is allotted a rectangular plot that measures 18 feet by 45 feet. How much fencing is needed to enclose each plot?

29. SOCCER Find the perimeter and area of the soccer field described at the left.

More About . . .

Soccer

Mia Hamm led her team in the Women's World Cup games in 2003. Soccer is played on a rectangular field that is usually 120 yards long and 75 yards wide.

Source: www.fifaworldcup.com

CONTENTS

WebQuest

Using a formula can help you find the cost of a vacation. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

For Exercises 30 and 31, translate each sentence into a formula.

30. SALES The sale price of an item s is equal to the list price ℓ minus the discount d.

31. GEOMETRY In a circle, the diameter d is twice the length of the radius r.

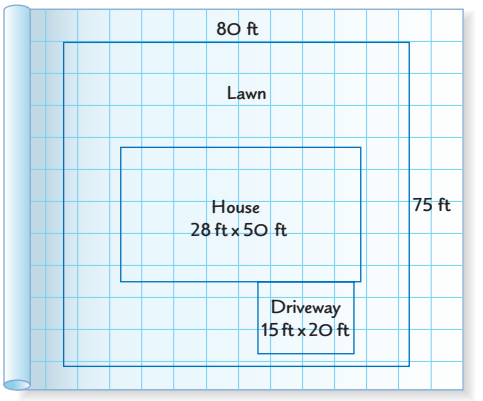
32. RUNNING The *stride rate* r of a runner is the number of strides n that he or she takes divided by the amount of time t, or $r = \frac{n}{t}$. The best runners usually have the greatest stride rate. Use the table to determine which runner has the greater stride rate.

Runner	Number of Strides	Time (s)
A	20	5
B	30	10

LANDSCAPING **For Exercises 33 and 34, use the figure at the right.**

33. What is the area of the lawn?

34. Suppose your family wants to fertilize the lawn that is shown. If one bag of fertilizer covers 2500 square feet, how many bags of fertilizer should you buy?



BICYCLING **For Exercises 35 and 36, use following information.**
American Lance Armstrong won the 2000 Tour De France, completing the 2178-mile race in 92 hours 33 minutes 8 seconds.

35. Find Armstrong's average rate in miles per hour for the race.

36. Armstrong also won the 1999 Tour de France. He completed the 2213-mile race in 91 hours 32 minutes 16 seconds. Without calculating, determine which race was completed with a faster average speed. Explain.

GEOMETRY **Draw and label the dimensions of each rectangle whose perimeter and area are given.**

37. $P = 14$ ft, $A = 12$ ft^2

38. $P = 16$ m, $A = 12$ m^2

39. $P = 16$ cm, $A = 16$ cm^2

40. $P = 12$ in., $A = 8$ in^2

41. CRITICAL THINKING Is it *sometimes*, *always*, or *never* true that the perimeter of a rectangle is numerically greater than its area? Give an example.

42. CRITICAL THINKING An airplane flying at a rate of 500 miles per hour leaves Los Angeles. One-half hour later, a second airplane leaves Los Angeles in the same direction flying at a rate of 600 miles per hour. How long will it take the second airplane to overtake the first?

43. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

Why are formulas important in math and science?

Include the following in your answer:

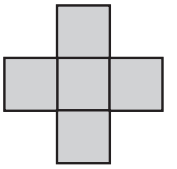
- an example of a formula from math or science that you have used, and
- an explanation of how you used the formula.

Standardized Test Practice

44. The formula $d = rt$ can be rewritten as $\frac{d}{t} = r$. How is the rate affected if the time t increases and the distance d remains the same?

(A) It increases. (B) It decreases.
(C) It remains the same. (D) There is not enough information.

45. The area of each square in the figure is 16 square units. Find the perimeter.



(A) 16 units (B) 32 units
(C) 48 units (D) 64 units

Maintain Your Skills

Mixed Review

46. Eight more than five times a number is 78. Find the number. *(Lesson 3-6)*

ALGEBRA **Solve each equation. Check your solution.** *(Lessons 3-3, 3-4, and 3-5)*

47. $-5x + 8 = 53$ 48. $4y = -24$ 49. $m + 5 = -3$

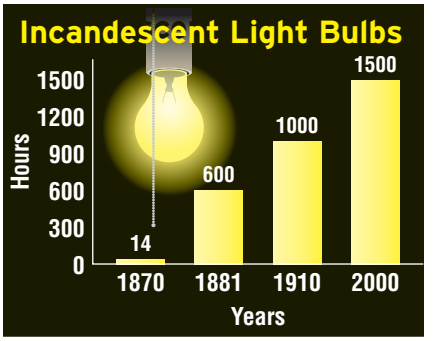
ALGEBRA **Simplify each expression.** *(Lesson 3-2)*

50. $3y + 5 - 2y$ 51. $9 + x - 5x$ 52. $3(r + 2) + 6r$

53. **LIGHT BULBS** The table shows the average life of an incandescent bulb for selected years. *(Lesson 1-6)*

a. Write a set of ordered pairs for the data.

b. State the domain and the range of the relation.



WebQuest Internet Project

Vacation Travelers Include More Families

It's time to complete your Internet project. Use the information and data you have gathered about the costs of lodging, transportation, and entertainment for each of the vacations. Prepare a brochure or Web page to present your project. Be sure to include graphs and/or tables in the presentation.

www.pre-alg.com/webquest

Spreadsheet Investigation

A Follow-Up of Lesson 3-7

Perimeter and Area

A spreadsheet allows you to use formulas to investigate problems. When you change a numerical value in a cell, the spreadsheet recalculates the formula and automatically updates the results.

Example

Suppose a gardener wants to enclose a rectangular garden using part of a wall as one side and 20 feet of fencing for the other three sides. What are the dimensions of the largest garden she can enclose?

If w represents the length of each side attached to the wall, $20 - 2w$ represents the length of the side opposite the wall. These values are listed in column B. The areas are listed in column C.

Garden Dimensions

	A	B	C
1	Length of Fence	20	
2	Length of Side Attached to Wall	Length of Side Opposite Wall	Area
3	1	18	18
4	2	16	32
5	3	14	42
6	4	12	48
7	5	10	50
8	6	8	48
9	7	6	42
10	8	4	32
11	9	2	18
12			

Sheet1 Sheet2

The spreadsheet evaluates the formula B1 − 2 × A3.

The spreadsheet evaluates the formula A9 × B9.

The greatest possible area is 50 square feet. It occurs when the length of each side attached to the wall is 5 feet, and the length of the side opposite the wall is 10 feet.

Exercises

1. What is the area if the length of the side attached to the wall is 10 feet? 11 feet?
2. Are the answers to Exercise 1 reasonable? Explain.
3. Suppose you want to find the greatest area that you can enclose with 30 feet of fencing. Which cell should you modify to solve this problem?
4. Use a spreadsheet to find the dimensions of the greatest area you can enclose with 40 feet, 50 feet, and 60 feet of fencing.
5. **MAKE A CONJECTURE** Use any pattern you may have observed in your answers to Exercise 4 to find the dimensions of the greatest area you can enclose with 100 feet of fencing. Explain.

Chapter 3

Study Guide and Review

Vocabulary and Concept Check

area (p. 132)
coefficient (p. 103)
constant (p. 103)
Distributive Property (p. 98)
equivalent equations (p. 111)
equivalent expression (p. 98)
formula (p. 131)
inverse operations (p. 110)
like terms (p. 103)
perimeter (p. 132)
Properties of Equality
  Addition (p. 111)
  Division (p. 115)
  Multiplication (p. 117)
  Subtraction (p. 110)
simplest form (p. 104)
simplifying an expression (p. 104)
term (p. 103)
two-step equation (p. 120)

Complete each sentence with the correct term.

1. Terms that contain the same variables are called _____ .
2. The _____ of a geometric figure is the measure of the distance around it.
3. The _____ states that when you multiply each side of an equation by the same number, the two sides remain equal.
4. The equations $x + 3 = 8$ and $x = 5$ are _____ because they have the same solution.
5. You could use the _____ Property to rewrite $9(t - 2)$ as $9t - 18$.
6. In the term $4b$, 4 is the _____ of the expression.
7. The solution of $2y + 5 = 13$ is a _____ of a point on a number line.
8. The measure of the surface enclosed by a geometric figure is its _____ .
9. In the expression $10x + 6$, the number 6 is the _____ term.
10. Addition and subtraction are _____ because they "undo" each other.

Lesson-by-Lesson Review

3-1 The Distributive Property

See pages 98–102.

Concept Summary

- The Distributive Property combines addition and multiplication.
- For any numbers a, b, and c, $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$.

Example **Use the Distributive Property to rewrite $2(t - 3)$.**

$2(t - 3) = 2[(t + (-3)]$ Rewrite $t - 3$ as $t + (-3)$.
$= 2t + 2(-3)$ Distributive Property
$= 2t + (-6)$ Simplify.
$= 2t - 6$ Definition of subtraction

Exercises **Use the Distributive Property to rewrite each expression.**

See Examples 3 and 4 on page 100.

11. $3(h + 6)$ **12.** $7(x + 2)$ **13.** $-5(k + 1)$ **14.** $-2(a + 8)$
15. $(t - 5)9$ **16.** $(x - 3)7$ **17.** $-2(b - 4)$ **18.** $-6(y - 3)$

www.pre-alg.com/vocabulary_review
CONTENTS

3-2 Simplifying Algebraic Expressions

See pages 103–107.

Concept Summary

- Simplest form means no like terms and no parentheses.

Example

Simplify $9x + 3 - 7x$.

$$\begin{aligned} 9x + 3 - 7x &= 9x + 3 + (-7x) && \text{Definition of subtraction} \\ &= 9x + (-7x) + 3 && \text{Commutative Property} \\ &= [9 + (-7)]x + 3 && \text{Distributive Property} \\ &= 2x + 3 && \text{Simplify.} \end{aligned}$$

Exercises **Simplify each expression.** *See Example 2 on page 104.*

19. $4a + 5a$ **20.** $3x + 7 + x$ **21.** $8(n - 1) - 10n$ **22.** $6w + 2(w + 9)$

3-3 Solving Equations by Adding or Subtracting

See pages 110–114.

Concept Summary

- When you add or subtract the same number from each side of an equation, the two sides remain equal.

Examples

1 **Solve $x + 3 = 7$.**

$$\begin{aligned} x + 3 &= 7 \\ x + 3 - 3 &= 7 - 3 && \text{Subtract 3 from each side.} \\ x &= 4 \end{aligned}$$

2 **Solve $y - 5 = -2$.**

$$\begin{aligned} y - 5 &= -2 \\ y - 5 + 5 &= -2 + 5 && \text{Add 5 to each side.} \\ y &= 3 \end{aligned}$$

Exercises **Solve each equation.** *See Examples 1 and 3 on pages 111 and 112.*

23. $t + 5 = 8$ **24.** $12 = x + 4$ **25.** $k - 1 = 4$ **26.** $-7 = n - 6$

3-4 Solving Equations by Multiplying or Dividing

See pages 115–119.

Concept Summary

- When you multiply or divide each side of an equation by the same nonzero number, the two sides remain equal.

Examples

1 **Solve $-5x = -30$.**

$$\begin{aligned} -5x &= -30 && \text{Write the equation.} \\ \frac{-5x}{-5} &= \frac{-30}{-5} && \text{Divide each side by } -5. \\ x &= 6 && \text{Simplify.} \end{aligned}$$

2 **Solve $\frac{a}{-8} = 3$.**

$$\begin{aligned} \frac{a}{-8} &= 3 && \text{Write the equation.} \\ -8\left(\frac{a}{-8}\right) &= -8(3) && \text{Multiply each side by } -8. \\ a &= -24 && \text{Simplify.} \end{aligned}$$

Exercises **Solve each equation.** *See Examples 1 and 3 on pages 116 and 117.*

27. $6n = 48$ **28.** $-3x = 30$ **29.** $\frac{t}{2} = -9$ **30.** $\frac{r}{-5} = -2$

3-5 Solving Two-Step Equations

See pages 120–124.

Concept Summary

- To solve a two-step equation undo operations in reverse order.

Example

Solve $6k - 4 = 14$.

$6k - 4 = 14$	Write the equation.
$6k - 4 + 4 = 14 + 4$	Undo subtraction. Add 4 to each side.
$6k = 18$	Simplify.
$k = 3$	Mentally divide each side by 6.

Exercises **Solve each equation.** *See Examples 1, 3, and 4 on pages 121 and 122.*

31. $6 + 2y = 8$ **32.** $3n - 5 = -17$ **33.** $\frac{t}{3} + 4 = 2$ **34.** $\frac{c}{9} - 3 = 2$

3-6 Writing Two-Step Equations

See pages 126–130.

Concept Summary

- The words *is*, *equals*, or *is equal to*, can be translated into an equals sign.

Example

Seven less than three times a number is −22. Find the number.

$3n - 7 = -22$	Write the equation.
$3n - 7 + 7 = -22 + 7$	Add 7 to each side.
$3n = -15$	Simplify.
$n = -5$	Mentally divide each side by 3.

Exercises **Translate the sentence into an equation. Then find the number.** *See Examples 1 and 2 on pages 126 and 127.*

35. Three more than twice n is 53. **36.** Four times x minus 16 is 52.

3-7 Using Formulas

See pages 131–136.

Concept Summary

- Perimeter of a rectangle: $P = 2(\ell + w)$
- Area of a rectangle: $A = \ell w$

Example

Find the perimeter and area of a 14-meter by 6-meter rectangle.

$P = 2(\ell + w)$	Formula for perimeter
$P = 2(14 + 6)$	$\ell = 14$ and $w = 6$.
$P = 40$	Simplify.

$A = \ell w$	Formula for area
$A = 14 \cdot 6$	$\ell = 14$ and $w = 6$.
$A = 84$	Simplify.

Exercises **Find the perimeter and area of each rectangle whose dimensions are given.** *See Examples 2 and 4 on pages 132 and 133.*

37. 8 feet by 9 feet **38.** 5 meters by 15 meters

Chapter 3

Practice Test

Vocabulary and Concepts

1. **OPEN ENDED** Give an example of two terms that are like terms and two terms that are *not* like terms.
2. **State** the Distributive Property in your own words.
3. **Explain** the difference between perimeter and area.

Skills and Applications

Simplify each expression.

4. $9x + 5 - x + 3$
5. $-3(a - 8)$
6. $10(y + 3) - 4y$

Solve each equation. Check your solution.

7. $19 = f + 5$
8. $-15 + z = 3$
9. $x - 7 = 16$
10. $g - 9 = -10$
11. $-8y = 72$
12. $\frac{n}{-30} = -6$
13. $25 = 2d - 9$
14. $4w - 18 = -34$
15. $6v + 10 = -62$
16. $-7 = \frac{d}{-5} + 1$
17. $7 - x = 18$
18. $b - 7b + 6 = -30$

Translate each sentence into an equation. Then find each number.

19. The quotient of a number and 8, decreased by 17 is -15.
20. Five less than 3 times a number is 25.

Find the perimeter and area of each rectangle.

21.

22.

23. **ENTERTAINMENT** Suppose you pay \$15 per hour to go horseback riding. You ride 2 hours today and plan to ride 4 more hours this weekend.
    a. Write two different expressions to find the total cost of horseback riding.
    b. Find the total cost.

24. **HEIGHT** Todd is 5 inches taller than his brother. The sum of their heights is 139 inches. Find Todd's height.

25. **STANDARDIZED TEST PRACTICE** A carpet store advertises 16 square yards of carpeting for \$300, which includes the \$60 installation charge. Which equation could be used to determine the cost of one square yard of carpet x?
    (A) $16x = 300$ (B) $x + 60 = 300$
    (C) $60x + 16 = 300$ (D) $16x + 60 = 300$

Chapter 3 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. Ms. Bauer notices that her car's gas tank is nearly empty. Gasoline costs $1.59 a gallon. *About* how many gallons can she buy with a $20 bill? (Prerequisite Skills, p. 714)

Ⓐ 10 Ⓑ 30

Ⓒ 12 Ⓓ 20

2. Which is equivalent to $3 \times 8 - 6 \div 2$? (Lesson 1-2)

Ⓐ 3 Ⓑ 9

Ⓒ 13 Ⓓ 21

3. Which statement illustrates the Commutative Property of Multiplication? (Lesson 1-4)

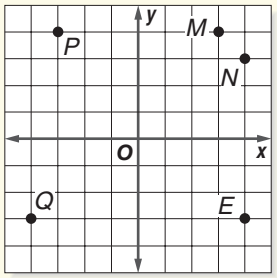
Ⓐ $5 + w + 8 = 5 + 8 + w$

Ⓑ $5 \cdot w \cdot 8 = 5 \cdot 8 \cdot w$

Ⓒ $(5 \cdot w) \cdot 8 = 5 \cdot (w \cdot 8)$

Ⓓ $(5 + w) + 8 = 5 + (w + 8)$

4. Which point on the graph below represents the ordered pair (4, 3)? (Lesson 1-6)



Ⓐ M Ⓑ N Ⓒ P Ⓓ Q

5. The temperature at 6:00 A.M. was −5°F. What was the temperature at 8:00 A.M. if it had risen 7 degrees? (Lesson 2-2)

Ⓐ 2°F Ⓑ −2°F

Ⓒ 12°F Ⓓ −12°F

6. Suppose points at (x, y) are graphed using the values in the table. Which statement is true about the graphs? (Lesson 2-6)

x	y
−1	5
−3	10
−5	6

Ⓐ The graphs are located in Quadrant I.

Ⓑ The graphs are located in Quadrant II.

Ⓒ The graphs are located in Quadrant III.

Ⓓ The graphs are located in Quadrant IV.

7. Which expression is equivalent to $5 \times 3 + 5 \times 12$? (Lesson 3-1)

Ⓐ $5 \times 8 \times 12$ Ⓑ $3 + (5 \times 12)$

Ⓒ $5 \times (3 + 12)$ Ⓓ $5 + (3 \times 12)$

8. Simplify $x - 4(x + 3)$. (Lesson 3-2)

Ⓐ $5x + 12$

Ⓑ $-3x - 12$

Ⓒ $-3x + 12$

Ⓓ $5x - 12$

9. Solve $y - (-4) = 6 - 8$. (Lesson 3-3)

Ⓐ −6 Ⓑ −2

Ⓒ 2 Ⓓ 6

10. Mr. Samuels is a car sales associate. He makes a salary of $400 per week. He also earns a bonus of $100 for each car he sells. Which equation represents the total amount of money Mr. Samuels earns in a week when he sells n cars? (Lesson 3-6)

Ⓐ $T = 400n + 100$

Ⓑ $T = n(100 + 400)$

Ⓒ $T = 100n + 400$

Ⓓ $T = 400 + 100 + n$

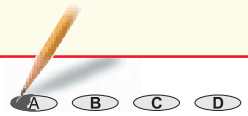
11. Tiffany's Gift Shop has fixed monthly expenses, E, of $1850. If the owner wants to make a profit, P, of $4000 next month, how many dollars in sales, S, does the shop need to earn? Use the formula $P = S - E$. (Lesson 3-7)

Ⓐ 2150 Ⓑ 4000

Ⓒ 5850 Ⓓ 7400

Preparing for Standardized Tests
For test-taking strategies and more practice, see pages 771–788.

Test-Taking Tip



Question 15
This problem does not include a drawing. Make one. Your drawing will help you see how to solve the problem.

Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

12. The charge to enter a nature preserve is a flat amount per vehicle plus a fee for each person in the vehicle. The table shows the charge for vehicles holding up to 4 people.

Number of People	Charge (dollars)
1	1.50
2	2.00
3	2.50
4	3.00

What is the charge, in dollars, for a vehicle holding 8 people? (Lesson 1-1)

13. Evaluate $-2(-8 + 5)$. (Lesson 1-2)

14. Last week, the stock market rose 10 points in two days. What number expresses the average change in the stock market per day? (Lesson 2-1)

15. The ordered pairs $(-7, -2)$, $(-3, 5)$, and $(-3, -2)$ are coordinates of three of the vertices of a rectangle. What is the y-coordinate of the ordered pair that represents the fourth vertex? (Lesson 2-6)

16. What value of x makes $x - 4 = -2$ a true statement? (Lesson 3-3)

17. Cara filled her car's gas tank with 15 gallons of gas. Her car usually gets 24 miles per gallon. How many miles can she drive using 15 gallons of gas? (Lesson 3–7)

18. A mail-order greeting card company charges \$3 for each box of greeting cards plus a handling charge of \$2 per order. How many boxes of cards can you order from this company if you want to spend \$26? (Lesson 3-6)

19. Mr. Ruiz owns a health club and is planning to increase the floor area of the weight room. In the figure below, the rectangle with the solid border represents the floor area of the existing room, and the rectangle with the dashed border represents the floor area to be added. What will be the length, in feet, of the new weight room? (Lesson 3-7)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

20. The overnight low in Fargo, North Dakota, was $-14°$F. The high the next day was $6°$F. (Lesson 3-3)

a. Draw a number line to represent the increase in temperature.

b. How many degrees did the temperature rise from the low to the high?

c. Explain how the concept of absolute value relates to this question.

21. In a school basketball game, each field goal is worth 2 points, and each free throw is worth 1 point. Josh heard the Springdale Stars scored a total of 63 points in their last game. Soledad says that they made a total of 12 free throws in that game. (Lesson 3-6)

a. Write an equation to represent the total points scored p. Use f for the number of free throws and g for the number of field goals.

b. Can both Josh and Soledad be correct? Explain.