Chapter 2

# Integers

## What You'll Learn

- **Lesson 2-1** Compare and order integers, and find the absolute value of an expression.
- **Lessons 2-2 through 2-5** Add, subtract, multiply, and divide integers.
- **Lessons 2-3 and 2-4** Evaluate and simplify algebraic expressions.
- **Lesson 2-5** Find the average of a set of data.
- **Lesson 2-6** Graph points, and show algebraic relationships on a coordinate plane.

## Key Vocabulary

- integer (p. 56)
- inequality (p. 57)
- absolute value (p. 58)
- additive inverse (p. 66)
- quadrants (p. 86)

## Why It's Important

In both mathematics and everyday life, there are many situations where integers are used. Some examples include temperatures, sports such as golf and football, and measuring the elevation of points on Earth or the depth below sea level. *You will represent real-world situations with integers in Lesson 2-1.*

# Getting Started

**Prerequisite Skills** To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 2.

## For Lesson 2-1 — Evaluate Expressions

Evaluate each expression if $a = 4$, $b = 10$, and $c = 8$. *(For review, see Lesson 1-3.)*

**1.** $a + b + c$ **2.** $bc - ab$ **3.** $b + ac$

**4.** $4c + 3b$ **5.** $2b - (a + c)$ **6.** $2c - b + a$

## For Lesson 2-3 — Patterns

Find the next term in each list. *(For review, see Lesson 1-1.)*

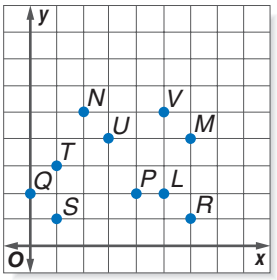
**7.** 34, 28, 22, 16, 10, … **8.** 120, 105, 90, 75, …

## For Lesson 2-6 — Graph Points

Use the grid to name the point for each ordered pair. *(For review, see Lesson 1-6.)*

**9.** (1, 3) **10.** (5, 2) **11.** (5, 5)

**12.** (3, 4) **13.** (0, 2) **14.** (6, 1)

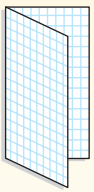



**FOLDABLES™ Study Organizer**

**Operations with Integers** Make this Foldable to help you organize your notes. Begin with a piece of graph paper.

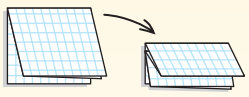
**Step 1 Fold in Half**

Fold the graph paper in half lengthwise.



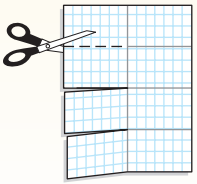
**Step 2 Fold Again in Fourths**

Fold the top to the bottom twice.



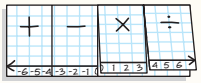
**Step 3 Cut**

Open. Cut along the second fold to make four tabs.



**Step 4 Label**

Fold lengthwise. Draw a number line on the outside. Label each tab as shown.




**Reading and Writing** As you read and study the chapter, write rules and examples for each integer operation under the tabs.

# 2-1 Integers and Absolute Value

## What You'll Learn

- Compare and order integers.
- Find the absolute value of an expression.

## Vocabulary

- negative number
- integers
- coordinate
- inequality
- absolute value

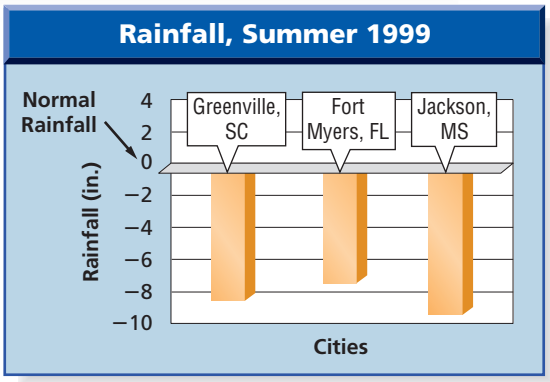
## How are integers used to model real-world situations?

The summer of 1999 was unusually dry in parts of the United States. In the graph, a value of −8 represents 8 inches below the normal rainfall.

**a.** What does a value of −7 represent?

**b.** Which city was farthest from its normal rainfall?

**c.** How could you represent 5 inches above normal rainfall?




## Reading Math

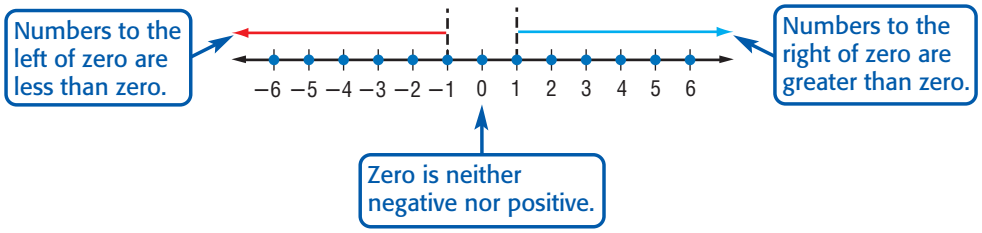
**Integers**

Read −8 as *negative 8*. A positive integer like 6 can be written as +6. It is usually written without the + sign, as 6.

### COMPARE AND ORDER INTEGERS

With normal rainfall as the starting point of 0, you can express 8 inches below normal as 0 − 8, or −8. A **negative number** is a number less than zero.

Negative numbers like −8, positive numbers like +6, and zero are members of the set of **integers**. Integers can be represented as points on a number line.




This set of integers can be written {…, −3, −2, −1, 0, 1, 2, 3, …} where … means continues indefinitely.

### Example 1 Write Integers for Real-World Situations

**Write an integer for each situation.**

**a. 500 feet below sea level** The integer is −500.

**b. a temperature increase of 12°** The integer is +12.

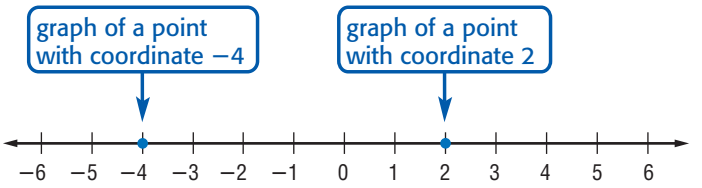
**c. a loss of $240** The integer is −240.

**Concept Check** Which integer is neither positive nor negative?



CONTENTS

To graph integers, locate the points named by the integers on a number line. The number that corresponds to a point is called the **coordinate** of that point.




Notice that the numbers on a number line increase as you move from left to right. This can help you determine which of two numbers is greater.

## Reading Math

**Inequality Symbols**

Read the symbol $<$ as *is less than*. Read the symbol $>$ as *is greater than*.

**Words** $-4$ is less than 2. 2 is greater than $-4$.

OR

**Symbols** $-4 < 2$ $\qquad$ $2 > -4$

The symbol points to the lesser number.

Any mathematical sentence containing $<$ or $>$ is called an inequality. An **inequality** compares numbers or quantities.

### Example 2 Compare Two Integers

**Use the integers graphed on the number line below.**


**a. Write two inequalities involving $-3$ and 4.**

Since $-3$ is to the left of 4, write $-3 < 4$.
Since 4 is to the right of $-3$, write $4 > -3$.

**b. Replace the ● with $<$ or $>$ in $-5$ ● $-1$ to make a true sentence.**

$-1$ is greater since it lies to the right of $-5$. So write $-5 < -1$.

## More About . . .

**Golf**

Annika Sorenstam won the 2003 LPGA Championship at 6 under par. She was the LPGA's leading money winner in 2001 and 2002.

**Source:** www.LPGA.com

Integers are used to compare numbers in many real-world situations.

### Example 3 Order Integers

**GOLF The top ten fourth round scores of the 2003 LPGA Championship tournament were 0, +1, −4, −2, −1, +4, +2, +3, +5, and −3. Order the scores from least to greatest.**

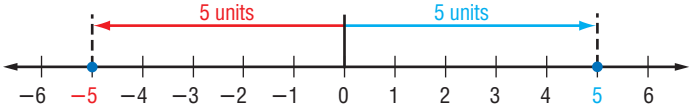
Graph each integer on a number line.


Write the numbers as they appear from left to right.

The scores $-4, -3, -2, -1, 0, +1, +2, +3, +4, +5$ are in order from least to greatest.

**Concept Check** Why is the sentence $5 > 2$ an inequality?

**ABSOLUTE VALUE** On the number line, notice that $-5$ and 5 are on opposite sides of zero, and they are the same distance from zero. In mathematics, we say they have the same **absolute value**, 5.




The symbol for absolute value is two vertical bars on either side of the number.

$|5| = 5$ The absolute value of 5 is 5.
$|-5| = 5$ The absolute value of $-5$ is 5.

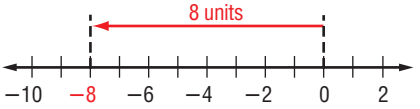
**Key Concept** — **Absolute Value**

- **Words** The absolute value of a number is the distance the number is from zero on the number line. The absolute value of a number is always greater than or equal to zero.
- **Examples** $|5| = 5$ $|-5| = 5$

### Example 4 Expressions with Absolute Value

**Evaluate each expression.**

**a.** $|-8|$




$|-8| = 8$ The graph of $-8$ is 8 units from 0.

**b.** $|9| + |-7|$

$|9| + |-7| = 9 + 7$ The absolute value of 9 is 9. The absolute value of $-7$ is 7.
$= 16$ Simplify.

**c.** $|-4| - |3|$

$|-4| - |3| = 4 - 3$ $|-4| = 4$, $|3| = 3$
$= 1$ Simplify.

**Study Tip**

**Common Misconception**
It is not always true that the absolute value of a number is the opposite of the number. Remember that absolute value is always positive or zero.

**ALGEBRA CONNECTION**

Since variables represent numbers, you can use absolute value notation with algebraic expressions involving variables.

### Example 5 Algebraic Expressions with Absolute Value

**ALGEBRA** **Evaluate $|x| - 3$ if $x = -5$.**

$|x| - 3 = |-5| - 3$ Replace $x$ with $-5$.
$= 5 - 3$ The absolute value of $-5$ is 5.
$= 2$ Simplify.



CONTENTS

## Check for Understanding

**Concept Check**

1. **Explain** how you would graph $-4$ on a number line.
2. **OPEN ENDED** Write two inequalities using integers.
3. **Define** *absolute value.*

**Guided Practice**

**Write an integer for each situation. Then graph on a number line.**

4. 8° below zero
5. a 15-yard gain
6. Graph the set of integers $\{0, -3, 6\}$ on a number line.

**Write two inequalities using the numbers in each sentence. Use the symbols < or >.**

7. $-4°$ is colder than 2°.
8. $-6$ is greater than $-10$.

**Replace each ● with <, >, or = to make a true sentence.**

9. $-18$ ● $-8$
10. $0$ ● $-3$
11. $9$ ● $-9$
12. Order the integers $\{28, -6, 0, -2, 5, -52, 115\}$ from least to greatest.

**Evaluate each expression.**

13. $|-10|$
14. $|10| - |-4|$
15. $|16| + |-5|$

**ALGEBRA** **Evaluate each expression if $a = -8$ and $b = 5$.**

16. $9 + |a|$
17. $|a| - b$
18. $2|a|$

**Application**

19. **WEATHER** The table shows the record low temperatures in °F for selected states. Order the temperatures from least to greatest.

| State | AL | CA | FL | IN | KY | NY | NC | OK | OR |
|---|---|---|---|---|---|---|---|---|---|
| Temperature | −27 | −45 | −2 | −36 | −37 | −52 | −34 | −27 | −54 |

## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 20–25, 66 | 1 |
| 26–43 | 2 |
| 44–47, 67–70 | 3 |
| 48–59 | 4 |
| 60–65 | 5 |

**Extra Practice**
**See page 726.**

**Write an integer for each situation. Then graph on a number line.**

20. a bank withdrawal of $100
21. a loss of 6 pounds
22. a salary increase of $250
23. a gain of 9 yards
24. 12° above zero
25. 5 seconds before liftoff

**Graph each set of integers on a number line.**

26. $\{0, -2, 4\}$
27. $\{-3, 1, 2, 5\}$
28. $\{-2, -4, -5, -8\}$
29. $\{-4, 0, 6, -7, -1\}$

**Write two inequalities using the numbers in each sentence. Use the symbols < or >.**

30. 3 meters is taller than 2 meters.
31. A temperature of $-5°$F is warmer than a temperature of $-10°$F.
32. 55 miles per hour is slower than 65 miles per hour.

**Write two inequalities using the numbers in each sentence. Use the symbols < or >.**

**33.** Yesterday's pollen count was 248. Today's count is 425.

**34.** Yesterday's low temperature was −2°F. The high temperature was 23°F.

**35.** Water boils at 212°F, and it freezes at 32°F.

**Replace each ● with <, >, or = to make a true sentence.**

**36.** $-6 \bullet -2$ **37.** $-10 \bullet -13$ **38.** $0 \bullet -9$ **39.** $14 \bullet 0$

**40.** $-18 \bullet 8$ **41.** $5 \bullet -23$ **42.** $|9| \bullet |-9|$ **43.** $|-20| \bullet |-4|$

**Order the integers in each set from least to greatest.**

**44.** {5, 0, −8} **45.** {−15, −1, −2, −4}

**46.** {24, 5, −46, 9, 0, −3} **47.** {98, −57, −60, 38, 188}

**Evaluate each expression.**

**48.** $|-15|$ **49.** $|46|$ **50.** $-|20|$ **51.** $-|5|$

**52.** $|0|$ **53.** $|7|$ **54.** $|-5| + |4|$ **55.** $|0| + |-2|$

**56.** $|15| - |-1|$ **57.** $|0 + 9|$ **58.** $-|-24|$ **59.** $-||-6|+|14||$

**ALGEBRA** **Evaluate each expression if $a = 0$, $b = 3$, and $c = -4$.**

**60.** $14 + |b|$ **61.** $|c| - a$ **62.** $a + b + |c|$

**63.** $ab + |-40|$ **64.** $|c| - b$ **65.** $|ab| + b$

**66. GEOGRAPHY** The Caribbean Sea has an average depth of 8685 feet below sea level. Use an integer to express this depth.

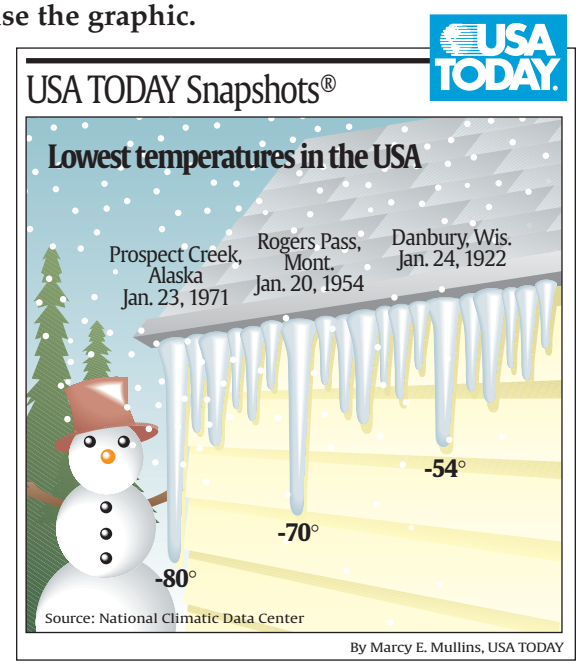
**WEATHER** **For Exercises 67–70, use the graphic.**

**67.** Graph the temperatures on a number line.

**68.** Compare the lowest temperature in the United States and the lowest temperature east of the Mississippi using the < symbol.

**69.** Compare the lowest temperatures of the contiguous 48 states and east of the Mississippi using the > symbol.

**70.** Write the temperatures in order from greatest to least.




**71.** How many units apart are −4 and 3 on a number line?

**72. CRITICAL THINKING** Consider any two points on the number line where $X > Y$. Is it *always*, *sometimes*, or *never* true that $|X| > |Y|$? Explain.

**Study Tip**

**Contiguous States**

Contiguous states are those states that touch each other. Alaska and Hawaii are *not* contiguous states.

**73. CRITICAL THINKING** Consider two numbers $A$ and $B$ on a number line. Is it *always*, *sometimes*, or *never* true that the distance between $A$ and $B$ equals the distance between $|A|$ and $|B|$? Explain.

**74. WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How are integers used to model real-world situations?**

Include the following in your answer:

- an explanation of how integers are used to describe rainfall, and
- some situations in the real world where negative numbers are used.

**Standardized Test Practice**

**75.** Which of the following describes the absolute value of $-2°$?

(A) It is the distance from $-2$ to 2 on a thermometer.

(B) It is the distance from $-2$ to 0 on a thermometer.

(C) It is the actual temperature outside when a thermometer reads $-2°$.

(D) None of these describes the absolute value of $-2°$.

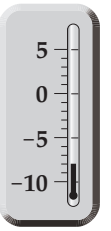
**76.** What is the temperature shown on the thermometer at the right?

(A) 8

(B) 7

(C) $-7$

(D) $-8$




## Maintain Your Skills

**Mixed Review**

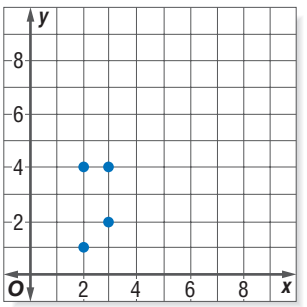
**Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship. Explain your answer.** *(Lesson 1-7)*

**77.** height and arm length

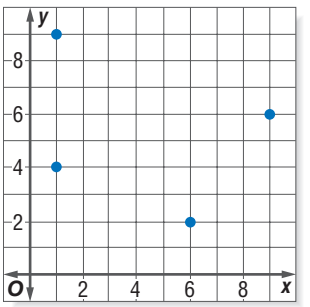
**78.** birth month and weight

**Express each relation as a table and as a list of ordered pairs.** *(Lesson 1-6)*

**79.**




**80.**




**Name the property shown by each statement.** *(Lesson 1-4)*

**81.** $20 \cdot 18 = 18 \cdot 20$

**82.** $9 \cdot 8 \cdot 0 = 0$

**83.** $3ab = 3ba$

**Getting Ready for the Next Lesson**

**BASIC SKILL** **Find each sum or difference.**

**84.** $18 + 29 + 46$

**85.** $232 + 156$

**86.** $451 + 629 + 1027$

**87.** $36 - 19$

**88.** $479 - 281$

**89.** $2011 - 962$

# Algebra Activity

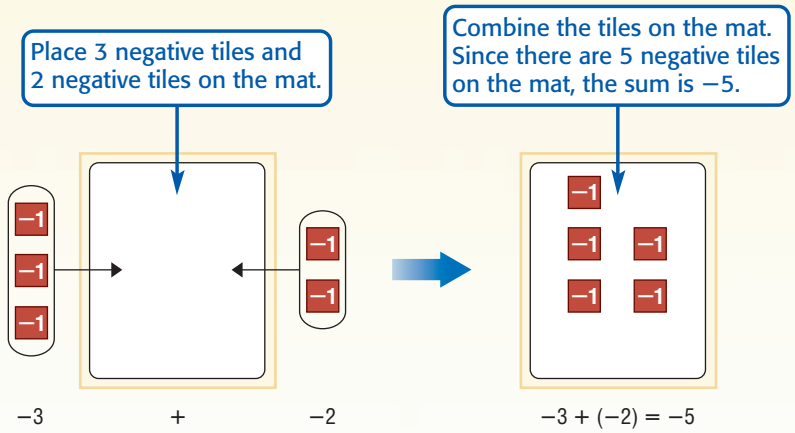
A Preview of Lesson 2-2

# Adding Integers

In a set of algebra tiles, 1 represents the integer 1, and −1 represents the integer −1. You can use algebra tiles and an integer mat to model operations with integers.

## Activity 1

The following example shows how to find the sum $-3 + (-2)$ using algebra tiles. Remember that addition means *combining*. $-3 + (-2)$ tells you to combine a set of 3 negative tiles with a set of 2 negative tiles.

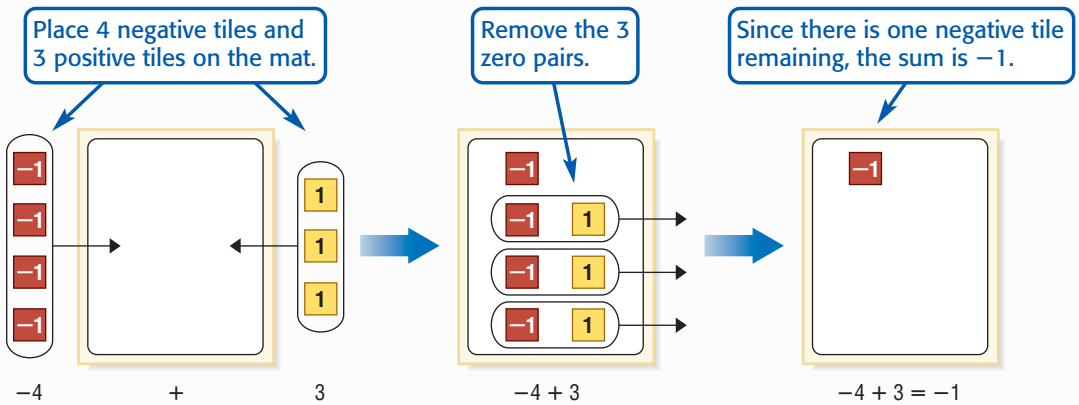



Therefore, $-3 + (-2) = -5$.

There are two important properties to keep in mind when you model operations with integers.

- When one positive tile is paired with one negative tile, the result is called a **zero pair**.
- You can add or remove zero pairs from a mat because removing or adding zero does not change the value of the tiles on the mat.

The following example shows how to find the sum $-4 + 3$.




Therefore, $-4 + 3 = -1$.

## Model

**Use algebra tiles to model and find each sum.**

**1.** $-2 + (-4)$ **2.** $-3 + (-5)$ **3.** $-6 + (-1)$ **4.** $-4 + (-5)$

**5.** $-4 + 2$ **6.** $2 + (-5)$ **7.** $-1 + 6$ **8.** $4 + (-4)$

# Activity 2

The Addition Table was completed using algebra tiles. In the highlighted portion of the table, the addends are $-3$ and 1, and the sum is $-2$. So, $-3 + 1 = -2$. You can use the patterns in the Addition Table to learn more about integers.

**Addition Table**

| + | 4 | 3 | 2 | 1 | 0 | −1 | −2 | −3 | −4 |
|---|---|---|---|---|---|---|---|---|---|
| **4** | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **3** | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 | −1 |
| **2** | 6 | 5 | 4 | 3 | 2 | 1 | 0 | −1 | −2 |
| **1** | 5 | 4 | 3 | 2 | 1 | 0 | −1 | −2 | −3 |
| **0** | 4 | 3 | 2 | 1 | 0 | −1 | −2 | −3 | −4 |
| **−1** | 3 | 2 | 1 | 0 | −1 | −2 | −3 | −4 | −5 |
| **−2** | 2 | 1 | 0 | −1 | −2 | −3 | −4 | −5 | −6 |
| **−3** | 1 | 0 | −1 | −2 | −3 | −4 | −5 | −6 | −7 |
| **−4** | 0 | −1 | −2 | −3 | −4 | −5 | −6 | −7 | −8 |

addends (top) — sums — addends (bottom)

## Make a Conjecture

**9.** Locate all of the positive sums in the table. Describe the addends that result in a positive sum.

**10.** Locate all of the negative sums in the table. Describe the addends that result in a negative sum.

**11.** Locate all of the sums that are zero. Describe the addends that result in a sum of zero.

**12.** The Identity Property says that when zero is added to any number, the sum is the number. Does it appear that this property is true for addition of integers? If so, write two examples that illustrate the property. If not, give a counterexample.

**13.** The Commutative Property says that the order in which numbers are added does not change the sum. Does it appear that this property is true for addition of integers? If so, write two examples that illustrate the property. If not, give a counterexample.

**14.** The Associative Property says that the way numbers are grouped when added does not change the sum. Is this property true for addition of integers? If so, write two examples that illustrate the property. If not, give a counterexample.

# 2-2 Adding Integers

## What You'll Learn

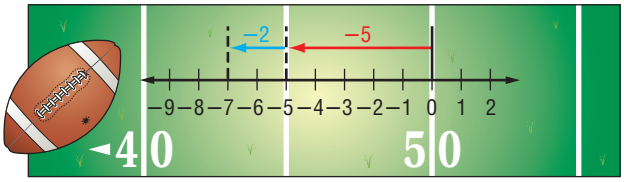
- Add two integers.
- Add more than two integers.

## Vocabulary

- opposites
- additive inverse

## How can a number line help you add integers?

In football, forward progress is represented by a positive integer. Being pushed back is represented by a negative integer. Suppose on the first play a team loses 5 yards and on the second play they lose 2 yards.




**a.** What integer represents the total yardage on the two plays?

**b.** Write an addition sentence that describes this situation.

## Reading Math

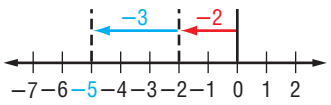
**Addends and Sums**

Recall that the numbers you add are called *addends*. The result is called the *sum*.

**ADD INTEGERS** The equation $-5 + (-2) = -7$ is an example of adding two integers with the same sign. Notice that the sign of the sum is the same as the sign of the addends.

### Example 1 Add Integers on a Number Line

**Find $-2 + (-3)$.**




Start at zero.
Move 2 units to the left.
From there, move 3 more units to the left.

$-2 + (-3) = -5$

This example suggests a rule for adding integers with the same sign.

### Key Concept Adding Integers with the Same Sign

- **Words** To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.
- **Examples** $-5 + (-2) = -7$ $6 + 3 = 9$

### Example 2 Add Integers with the Same Sign

**Find $-4 + (-5)$.**

$-4 + (-5) = -9$ Add $|-4|$ and $|-5|$. Both numbers are negative, so the sum is negative.



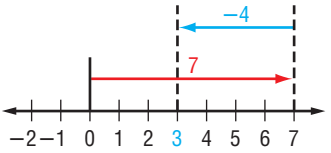
CONTENTS

A number line can also help you understand how to add integers with different signs.

## Example 3 Add Integers on a Number Line

**Find each sum.**

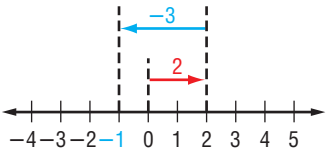
**a. $7 + (-4)$**




Start at zero.
Move 7 units to the right.
From there, move 4 units to the left.

$7 + (-4) = 3$

**b. $2 + (-3)$**




Start at zero.
Move 2 units to the right.
From there, move 3 units to the left.

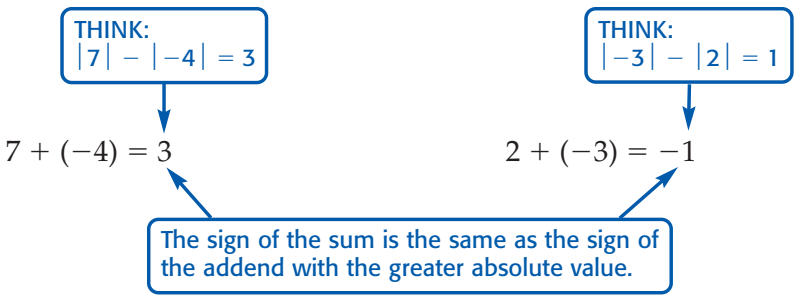
$2 + (-3) = -1$

**Study Tip**

**Adding Integers on a Number Line**

Always start at zero. Move right to model a positive integer. Move left to model a negative integer.

Notice how the sums in Example 3 relate to the addends.




## Key Concept Adding Integers with Different Signs

- **Words** To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.
- **Examples** $7 + (-2) = 5$ $-7 + 2 = -5$

## Example 4 Add Integers with Different Signs

**Find each sum.**

**a. $-8 + 3$**

$-8 + 3 = -5$ To find $-8 + 3$, subtract $|3|$ from $|-8|$.
The sum is negative because $|-8| > |3|$.

**b. $10 + (-4)$**

$10 + (-4) = 6$ To find $10 + |-4|$, subtract $|-4|$ from $|10|$.
The sum is positive because $|10| > |-4|$.

## Example 5 Use Integers to Solve a Problem

**ASTRONOMY** **During the night, the average temperature on the moon is −140°C. By noon, the average temperature has risen 252°C. What is the average temperature on the moon at noon?**

**Words** The temperature at night is −140°C. It increases 252°C by noon. What is the temperature at noon?

**Variables** Let $x$ = the temperature at noon.

**Equation**

| Temperature at night | plus | increase by noon | equals | temperature at noon. |
|---|---|---|---|---|
| $-140$ | $+$ | $252$ | $=$ | $x$ |

Solve the equation.

$-140 + 252 = x$ To find the sum, subtract $|-140|$ from 252.

$112 = x$ The sum is positive because $|252| > |-140|$.

The average temperature at noon is 112°C.

**More About . . .**

**Astronomy**

The temperatures on the moon are so extreme because the moon does not have any atmosphere to trap heat.

## ADD MORE THAN TWO INTEGERS

Two numbers with the same absolute value but different signs are called **opposites**. For example, −4 and 4 are opposites. An integer and its opposite are also called **additive inverses**.

**Key Concept** — **Additive Inverse Property**

- **Words** The sum of any number and its additive inverse is zero.
- **Symbols** $x + (-x) = 0$
- **Example** $6 + (-6) = 0$

**Concept Check** What is the additive inverse of 2?
What is the additive inverse of −6?

The commutative, associative, and identity properties also apply to integers. These properties can help you add more than two integers.

## Example 6 Add Three or More Integers

**Find each sum.**

**a. 9 + (− 3) + (−9)**

$9 + (-3) + (-9) = 9 + (-9) + (-3)$ Commutative Property

$= 0 + (-3)$ Additive Inverse Property

$= -3$ Identity Property of Addition

**b. −4 + 6 + (−3) + 9**

$-4 + 6 + (-3) + 9 = -4 + (-3) + 6 + 9$ Commutative Property

$= [-4 + (-3)] + (6 + 9)$ Associative Property

$= -7 + 15$ or 8 Simplify.

**Study Tip**

**Adding Mentally**

One way to add mentally is to group the positive addends together and the negative addends together. Then add to find the sum. You should also look for addends that are opposites. You can always add in order from left to right.

## Check for Understanding

**Concept Check**

1. **State** whether each sum is positive or negative. Explain your reasoning.
   **a.** $-4 + (-5)$ **b.** $12 + (-2)$
   **c.** $-11 + 9$ **d.** $15 + 10$

2. **OPEN ENDED** Give an example of two integers that are additive inverses.

**Guided Practice**

**Find each sum.**

3. $-2 + (-4)$ 4. $-10 + (-5)$ 5. $7 + (-2)$
6. $11 + (-3)$ 7. $8 + (-5)$ 8. $9 + (-12)$
9. $8 + (-6) + 2$ 10. $-6 + 5 + (-10)$

**Application**

11. **FOOTBALL** A team gained 4 yards on one play. On the next play, they lost 5 yards. Write an addition sentence to find the change in yardage.

## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 12–21 | 1, 2 |
| 22–29 | 3, 4 |
| 32–39 | 6 |
| 40, 41 | 5 |

**Extra Practice**
**See page 726.**

**Find each sum.**

12. $-4 + (-1)$ 13. $-5 + (-2)$ 14. $-4 + (-6)$
15. $-3 + (-8)$ 16. $-7 + (-8)$ 17. $-12 + (-4)$
18. $-9 + (-14)$ 19. $-15 + (-6)$ 20. $-11 + (-15)$
21. $-23 + (-43)$ 22. $8 + (-5)$ 23. $6 + (-4)$
24. $3 + (-7)$ 25. $4 + (-6)$ 26. $-15 + 6$
27. $-5 + 11$ 28. $18 + (-32)$ 29. $-45 + 19$

30. What is the additive inverse of 14?
31. What is the additive inverse of $-21$?

**Find each sum.**

32. $6 + (-9) + 9$ 33. $7 + (-13) + 4$
34. $-9 + 16 + (-10)$ 35. $-12 + 18 + (-12)$
36. $14 + (-9) + 6$ 37. $28 + (-35) + 4$
38. $-41 + 25 + (-10)$ 39. $-18 + 35 + (-17)$

40. **ACCOUNTING** The starting balance in a checking account was \$50. What was the balance after checks were written for \$25 and for \$32?

41. **GOLF** A score of 0 is called *even par*. Two under par is written as $-2$. Two over par is written as $+2$. Suppose a player shot 4 under par, 2 over par, even par, and 3 under par in four rounds of a tournament. What was the player's final score?

**Find each sum.**

42. $|18 + (-13)|$ 43. $|-27 + 19|$
44. $|-25 + (-12)|$ 45. $|-28 + (-12)|$

**POPULATION** For Exercises 46 and 47, use the table below that shows the change in population of several cities from 1990 to 2000.

| City | 1990 Population | Change as of 2000 |
|---|---|---|
| Dallas, TX | 1,006,877 | +181,703 |
| Honolulu, HI | 365,272 | +6385 |
| Jackson, MS | 196,637 | −12,381 |
| Philadelphia, PA | 1,585,577 | −68,027 |

**46.** What was the population in each city in 2000?

**47.** What was the total change in population of these cities?

*Online Research* **Data Update** How have the populations of other cities changed since 2000? Visit www.pre-alg.com/data_update to learn more.

**48. CRITICAL THINKING** *True* or *false*: $-n$ always names a negative number. If false, give a counterexample.

**49. WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How can a number line help you add integers?**

Include the following in your answer:

- an example showing the sum of a positive and a negative integer, and
- an example showing the sum of two negative integers.

*Standardized Test Practice*

**50.** What is the sum of $-32 + 20$?

(A) $-52$ (B) $-18$ (C) $-12$ (D) $12$

**51.** What is the value of $-|-2 + 8|$?

(A) $-10$ (B) $10$ (C) $6$ (D) $-6$

## Maintain Your Skills

*Mixed Review*

**52. CHEMISTRY** The freezing point of oxygen is 219 degrees below zero on the Celsius scale. Use an integer to express this temperature. *(Lesson 2-1)*

**Order the integers in each set from least to greatest.** *(Lesson 2-1)*

**53.** {14, −12, −8, 3, −9, 0}

**54.** {−242, 35, −158, 99, −24}

**Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship.** *(Lesson 1-7)*

**55.** age and family size

**56.** temperature and sales of mittens

**Identify the solution of each equation from the list given.** *(Lesson 1-5)*

**57.** $18 - n = 12$; 6, 16, 30

**58.** $25 = 16 + x$; 9, 11, 41

**59.** $\frac{x}{2} = 10$; 5, 12, 20

**60.** $7a = 49$; 7, 42, 343

*Getting Ready for the Next Lesson*

**PREREQUISITE SKILL** **Evaluate each expression if $a = 6$, $b = 10$, and $c = 3$.** *(To review **evaluating expressions**, see Lesson 1-3.)*

**61.** $a + 19$

**62.** $2b - 6$

**63.** $ab - ac$

**64.** $3a - (b + c)$

**65.** $5b + 5c$

**66.** $\frac{6b}{c}$

# Reading Mathematics

## Learning Mathematics Vocabulary

Some words used in mathematics are also used in English and have similar meanings. For example, in mathematics *add* means *to combine*. The meaning in English is *to join or unite*.

Some words are used only in mathematics. For example, *addend* means *a number to be added to another*.

Some words have more than one mathematical meaning. For example, an *inverse* operation *undoes the effect of another operation*, and an additive *inverse* is *a number that when added to a given number gives zero*.

The list below shows some of the mathematics vocabulary used in Chapters 1 and 2.

| Vocabulary | Meaning | Examples |
|---|---|---|
| algebraic expression | an expression that contains at least one variable and at least one mathematical operation | $2 + x$, $\frac{4}{c}$, $3b$ |
| evaluate | to find the value of an expression | $2 + 5 = 7$ |
| simplify | to find a simpler form of an expression | $3b + 2b = 5b$ |
| integer | a whole number, its inverse, or zero | $-3$, 0, 2 |
| factor | a number that is multiplied by another number | $3(4) = 12$<br>3 and 4 are factors. |
| product | the result of multiplying | $3(4) = 12$ ← product |
| quotient | the result of dividing two numbers | $\frac{12}{4} = 3$ ← quotient |
| dividend | the number being divided | $\frac{12}{4} = 3$ ← dividend |
| divisor | the number being divided into another number | $\frac{12}{4} = 3$ ← divisor |
| coordinate | a number that locates a point | (5, 2) |

### Reading to Learn

**1.** Name two of the words above that are also used in everyday English. Use the Internet, a dictionary, or another reference to find their everyday definition. How do the everyday definitions relate to the mathematical definitions?

**2.** Name two words above that are used only in mathematics.

**3.** Name two words above that have more than one mathematical meaning. List their meanings.

# 2-3 Subtracting Integers

## What You'll Learn

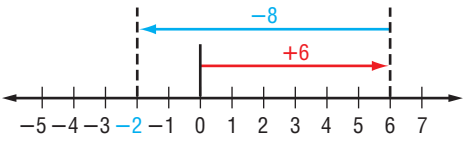
- Subtract integers.
- Evaluate expressions containing variables.

## How are addition and subtraction of integers related?

You can use a number line to subtract integers. The model below shows how to find $6 - 8$.

**Step 1** Start at 0. Move 6 units right to show positive 6.

**Step 2** From there, move 8 units left to subtract positive 8.

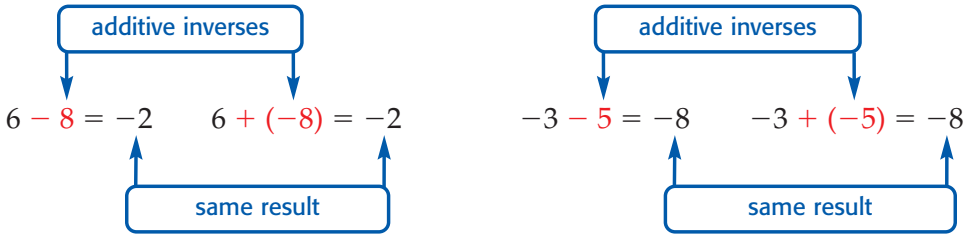



**a.** What is $6 - 8$?

**b.** What direction do you move to indicate subtracting a positive integer?

**c.** What addition sentence is also modeled by the number line above?

**SUBTRACT INTEGERS** When you subtract $6 - 8$, as shown on the number line above, the result is the same as adding $6 + (-8)$. When you subtract $-3 - 5$, the result is the same as adding $-3 + (-5)$.




These examples suggest a method for subtracting integers.

## Key Concept — Subtracting Integers

- **Words** To subtract an integer, add its additive inverse.
- **Symbols** $a - b = a + (-b)$
- **Examples** $5 - 9 = 5 + (-9)$ or $-4$ $\quad -2 - 7 = -2 + (-7)$ or $-9$

**Study Tip**

**Subtracting a Positive Integer**

To subtract a positive integer, think about moving left on a number line from the starting integer. In Example 1a, start at 8, then move left 13. You'll end at $-5$. In Example 1b, start at $-4$, then move left 10. You'll end at $-14$.

## Example 1 Subtract a Positive Integer

**Find each difference.**

**a.** $8 - 13$

$8 - 13 = 8 + (-13)$ To subtract 13, add $-13$.

$= -5$ Simplify.

**b.** $-4 - 10$

$-4 - 10 = -4 + (-10)$ To subtract 10, add $-10$.

$= -14$ Simplify.



CONTENTS

In Example 1, you subtracted a positive integer by adding its additive inverse. Use inductive reasoning to see if the method also applies to subtracting a negative integer.

**Study Tip**

**Look Back**
To review **inductive reasoning**, see Lesson 1-1.

| Subtracting an Integer | ↔ | Adding Its Additive Inverse |
|---|---|---|
| $2 - 2 = 0$ | | $2 + (-2) = 0$ |
| $2 - 1 = 1$ | | $2 + (-1) = 1$ |
| $2 - 0 = 2$ | | $2 + 0 = 2$ |
| $2 - (-1) = ?$ | | $2 + 1 = 3$ |

Continuing the pattern in the first column, $2 - (-1) = 3$. The result is the same as when you add the additive inverse. This suggests that the method also works for subtracting a negative integer.

## Example 2 Subtract a Negative Integer

**Find each difference.**

**a.** $7 - (-3)$

$7 - (-3) = 7 + 3$ To subtract $-3$, add 3.

$= 10$

**b.** $-2 - (-4)$

$-2 - (-4) = -2 + 4$ To subtract $-4$, add 4.

$= 2$

**Concept Check** How do you find the difference $9 - (-16)$?

## Example 3 Subtract Integers to Solve a Problem

**WEATHER** **The table shows the record high and low temperatures recorded in selected states through 1999. What is the range, or difference between the highest and lowest temperatures, for Virginia?**

| State | Lowest Temp. °F | Highest Temp. °F |
|---|---|---|
| Utah | −69 | 117 |
| Vermont | −50 | 105 |
| Virginia | −30 | 110 |
| Washington | −48 | 118 |
| West Virginia | −37 | 112 |
| Wisconsin | −54 | 114 |
| Wyoming | −66 | 114 |

**Source:** *The World Almanac*

**Explore** You know the highest and lowest temperatures. You need to find the range for Virginia's temperatures.

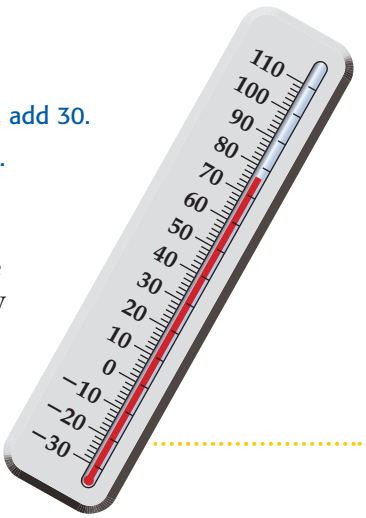
**Plan** To find the range, or difference, subtract the lowest temperature from the highest temperature.

**Solve** $110 - (-30) = 110 + 30$ To subtract $-30$, add 30.

$= 140$ Add 110 and 30.

The range for Virginia is 140°.

**Examine** Think of a thermometer. The difference between 110° above zero and 30° below zero must be 110 + 30 or 140°. The answer appears to be reasonable.



ALGEBRA CONNECTION

**EVALUATE EXPRESSIONS** You can use the rule for subtracting integers to evaluate expressions.

### Example 4 Evaluate Algebraic Expressions

**a. Evaluate $x - (-6)$ if $x = 12$.**

$x - (-6) = 12 - (-6)$ Write the expression. Replace $x$ with 12.

$= 12 + 6$ To subtract $-6$, add its additive inverse, 6.

$= 18$ Add 12 and 6.

**b. Evaluate $s - t$ if $s = -9$ and $t = -3$.**

$s - t = -9 - (-3)$ Replace $s$ with $-9$ and $t$ with $-3$.

$= -9 + 3$ To subtract $-3$, add 3.

$= -6$ Add $-9$ and 3.

**c. Evaluate $a - b + c$ if $a = 15$, $b = 5$, and $c = -8$.**

$a - b + c = 15 - 5 + (-8)$ Replace $a$ with 15, $b$ with 5, and $c$ with $-8$.

$= 10 + (-8)$ Order of operations

$= 2$ Add 10 and $-8$.

**Concept Check** How do you subtract integers using additive inverses?

## Check for Understanding

### Concept Check

1. **OPEN ENDED** Write examples of a positive and a negative integer and their additive inverses.

2. **FIND THE ERROR** José and Reiko are finding $8 - (-2)$.

| José | Reiko |
|---|---|
| $8 - (-2) = 8 + 2$<br>$= 10$ | $8 - (-2) = 8 + (-2)$<br>$= 6$ |

Who is correct? Explain your reasoning.

### Guided Practice

**Find each difference.**

3. $8 - 11$
4. $-9 - 3$
5. $5 - (-4)$
6. $7 - (-10)$
7. $-6 - (-4)$
8. $-2 - (-8)$

**ALGEBRA** **Evaluate each expression if $x = 10$, $y = -4$, and $z = -15$.**

9. $x - (-10)$
10. $y - x$
11. $x + y - z$

### Application

**WEATHER** **For Exercises 12 and 13, use the table in Example 3 on page 71.**

12. Find the range in temperature for Vermont.
13. Name a state that has a greater range than Vermont's.

## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 14–21, 30–33 | 1 |
| 22–29, 34–37 | 2 |
| 38, 39 | 3 |
| 40–51 | 4 |

**Extra Practice**
See page 727.

**Find each difference.**

**14.** $3 - 8$ **15.** $4 - 5$ **16.** $2 - 9$

**17.** $9 - 12$ **18.** $-3 - 1$ **19.** $-5 - 4$

**20.** $-6 - 7$ **21.** $-4 - 8$ **22.** $6 - (-8)$

**23.** $4 - (-6)$ **24.** $7 - (-4)$ **25.** $9 - (-3)$

**26.** $-9 - (-7)$ **27.** $-7 - (-10)$ **28.** $-11 - (-12)$

**29.** $-16 - (-7)$ **30.** $10 - 24$ **31.** $45 - 59$

**32.** $-27 - 14$ **33.** $-16 - 12$ **34.** $48 - (-50)$

**35.** $125 - (-114)$ **36.** $-320 - (-106)$ **37.** $-2200 - (-3500)$

**38. WEATHER** During January, the normal high temperature in Duluth, Minnesota, is 16°F, and the normal low temperature is −2°F. Find the difference between the temperatures.

**39. GEOGRAPHY** The highest point in California is Mount Whitney, with an elevation of 14,494 feet. The lowest point is Death Valley, elevation −282 feet. Find the difference in the elevations.

**ALGEBRA** **Evaluate each expression if $x = -3$, $y = 8$, and $z = -12$.**

**40.** $y - 10$ **41.** $12 - z$ **42.** $3 - x$

**43.** $z - 24$ **44.** $x - y$ **45.** $z - x$

**46.** $y - z$ **47.** $z - y$ **48.** $x + y - z$

**49.** $z - y + x$ **50.** $x - y - z$ **51.** $z - y - x$

**Career Choices**

**Veterinarian**

Veterinarians work with animals to diagnose, treat, and prevent disease, disorders, and injuries.

**Online Research**

For information about a career as a veterinarian, visit: www.pre-alg.com/careers

**PETS** **For Exercises 52 and 53, use the following table.**

**52.** Describe the change in the number of dogs of each breed registered from Year 1 to Year 2.

**53.** What was the total change in the number of dogs of these breeds registered from Year 1 to Year 2?

**Registration in American Kennel Club**

| Breed | Year 1 | Year 2 |
|---|---|---|
| Airedale Terrier | 2891 | 2950 |
| Beagle | 53,322 | 49,080 |
| Chinese Shar-Pei | 8614 | 6845 |
| Chow Chow | 6241 | 4342 |
| Labrador Retriever | 157,936 | 154,897 |
| Pug | 21,487 | 21,555 |

**Source:** www.akc.org

**54. BUSINESS** The formula $P = I - E$ is used to find the profit ($P$) when income ($I$) and expenses ($E$) are known. One month a small business has income of $19,592 and expenses of $20,345.

**a.** What is the profit for the month?

**b.** What does a negative profit mean?

**55. CRITICAL THINKING** Determine whether each statement is *true* or *false*. If false, give a counterexample.

**a.** Subtraction of integers is commutative.

**b.** Subtraction of integers is associative.

**56.** WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

**How are addition and subtraction of integers related?**

Include the following in your answer:

- a model that shows how to find the difference $4 - 10$, and
- the expression $4 - 10$ rewritten as an addition expression.

**Standardized Test Practice**

**57.** The terms in a pattern are given in the table. What is the value of the 5th term?

| Term | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Value | 13 | 8 | 3 | −2 | ? |

(A) $-7$ (B) $-5$

(C) 7 (D) 5

**58.** When 5 is subtracted from a number, the result is $-4$. What is the number?

(A) 9 (B) 1 (C) $-1$ (D) $-9$

## Maintain Your Skills

**Mixed Review**

**59.** OCEANOGRAPHY A submarine at 1300 meters below sea level descends an additional 1150 meters. What integer represents the submarine's position with respect to sea level? *(Lesson 2-2)*

**60.** ALGEBRA Evaluate $|b| - |a|$ if $a = 2$ and $b = -4$. *(Lesson 2-1)*

**ALGEBRA Solve each equation mentally.** *(Lesson 1-5)*

**61.** $x + 9 = 12$ **62.** $18 = w - 2$ **63.** $5a = 35$ **64.** $\frac{64}{b} = 8$

**ALGEBRA Translate each phrase into an algebraic expression.** *(Lesson 1-3)*

**65.** a number divided by 5

**66.** the sum of $t$ and 9

**67.** the quotient of eighty-six and $b$

**68.** $s$ decreased by 8

**Find the value of each expression.** *(Lesson 1-2)*

**69.** $2 \times (5 + 8) - 6$

**70.** $96 \div (6 \times 8) \div 2$

**Getting Ready for the Next Lesson**

**BASIC SKILL Find each product.**

**71.** $5 \cdot 15$ **72.** $8 \cdot 12$ **73.** $3 \cdot 5 \cdot 8$ **74.** $2 \cdot 7 \cdot 5 \cdot 9$

## Practice Quiz 1 — Lessons 2-1 through 2-3

**1.** WEATHER The three states with the lowest recorded temperatures are Alaska at $-80°$F, Utah at $-69°$F, and Montana at $-70°$F. Order the temperatures from least to greatest. *(Lesson 2-1)*

**Find each sum.** *(Lesson 2-2)*

**2.** $-5 + (-15)$ **3.** $-5 + 11$ **4.** $-6 + 9 + (-8)$

**Find each difference.** *(Lesson 2-3)*

**5.** $16 - 23$ **6.** $-15 - 8$ **7.** $25 - (-7)$

**ALGEBRA Evaluate each expression if $x = 5$, $y = -2$, and $z = -3$.** *(Lesson 2-3)*

**8.** $x - y$ **9.** $z - 6$ **10.** $x - y - z$

# 2-4 Multiplying Integers

## What You'll Learn

- Multiply integers.
- Simplify algebraic expressions.

## How are the signs of factors and products related?

The temperature drops 7°C for each 1 kilometer increase in altitude. A drop of 7°C is represented by −7. So, the temperature change equals the altitude times −7. The table shows the change in temperature for several altitudes.

| Altitude (km) | Altitude × Rate of Change | Temperature Change (°C) |
|---|---|---|
| 1 | 1(−7) | −7 |
| 2 | 2(−7) | −14 |
| 3 | 3(−7) | −21 |
| ... | ... | ... |
| 11 | 11(−7) | −77 |

**a.** Suppose the altitude is 4 kilometers. Write an expression to find the temperature change.

**b.** Use the pattern in the table to find 4(−7).

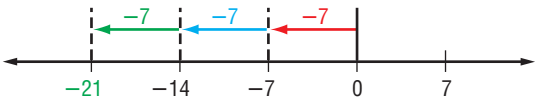
### Reading Math

**Parentheses**

Recall that a product can be written using parentheses. Read 3(−7) as *3 times negative 7.*

**MULTIPLY INTEGERS** Multiplication is repeated addition. So, 3(−7) means that −7 is used as an addend 3 times.

$$3(-7) = (-7) + (-7) + (-7)$$
$$= -21$$




By the Commutative Property of Multiplication, 3(−7) = −7(3).

This example suggests the following rule.

### Key Concept — Multiplying Two Integers with Different Signs

- **Words** The product of two integers with different signs is negative.
- **Examples** 4(−3) = −12 −3(4) = −12

### Example 1 Multiply Integers with Different Signs

**Find each product.**

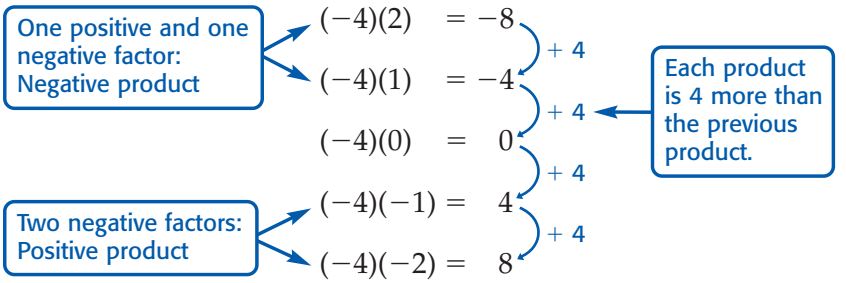
**a. 5(−6)**

5(−6) = −30 The factors have different signs. The product is negative.

**b. −4(16)**

−4(16) = −64 The factors have different signs. The product is negative.

The product of two positive integers is positive. What is the sign of the product of two negative integers? Use a pattern to find $(-4)(-2)$.




This example suggests the following rule.

## Key Concept Multiplying Two Integers with the Same Sign

- **Words** The product of two integers with the same sign is positive.
- **Examples** $4(3) = 12$ $-4(-3) = 12$

## Example 2 Multiply Integers with the Same Sign

**Find $-6(-12)$.**

$-6(-12) = 72$ The two factors have the same sign. The product is positive.

## Example 3 Multiply More Than Two Integers

**Find $-4(-5)(-8)$.**

$-4(-5)(-8) = [(-4)(-5)](-8)$ Associative Property

$= 20(-8)$ $(-4)(-5) = 20$

$= -160$ $20(-8) = -160$

**Study Tip**

**Look Back**

To review the **Associative Property**, see Lesson 1-4.

## Example 4 Use Integers to Solve a Problem

**Standardized Test Practice** (A) (B) (C) (D)

**Multiple-Choice Test Item**

A glacier was receding at a rate of 300 feet per day. What is the glacier's movement in 5 days?

(A) 305 feet (B) −1500 feet (C) −300 feet (D) −60 feet

**Read the Test Item**

The word *receding* means moving backward, so the rate per day is represented by $-300$. Multiply 5 times $-300$ to find the movement in 5 days.

**Solve the Test Item**

$5(-300) = -1500$ The product is negative.

The answer is B.

**Test-Taking Tip**

Read the problem. Try to picture the situation. Look for words that suggest mathematical concepts.

ALGEBRA CONNECTION

**ALGEBRAIC EXPRESSIONS** You can use the rules for multiplying integers to simplify and evaluate algebraic expressions.

## Example 5 Simplify and Evaluate Algebraic Expressions

**a. Simplify $-4(9x)$.**

$-4(9x) = (-4 \cdot 9)x$ Associative Property of Multiplication

$= -36x$ Simplify.

**b. Simplify $-2x(3y)$.**

$-2x(3y) = (-2)(x)(3)(y)$ $-2x = (-2)(x)$, $3y = (3)(y)$

$= (-2 \cdot 3)(x \cdot y)$ Commutative Property of Multiplication

$= -6xy$ $-2 \cdot 3 = -6$, $x \cdot y = xy$

**c. Evaluate $4ab$ if $a = 3$ and $b = -5$.**

$4ab = 4(3)(-5)$ Replace $a$ with 3 and $b$ with $-5$.

$= [4(3)](-5)$ Associative Property of Multiplication

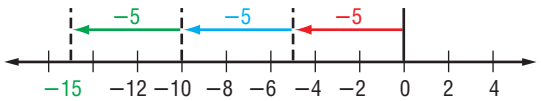
$= 12(-5)$ The product of 4 and 3 is positive.

$= -60$ The product of 12 and $-5$ is negative.

# Check for Understanding

## Concept Check

**1. Write** the product that is modeled on the number line below.




**2. State** whether each product is positive or negative.

**a.** $-5 \cdot 8$ **b.** $6(-4)$ **c.** $8 \cdot 24$

**d.** $-9(-7)$ **e.** $-2(9)(-3)$ **f.** $-7(-5)(-11)$

**3. OPEN ENDED** Give an example of three integers whose product is negative.

## Guided Practice

**Find each product.**

**4.** $-3 \cdot 8$ **5.** $5(-8)$ **6.** $4 \cdot 30$

**7.** $-7(-4)$ **8.** $-4(2)(-6)$ **9.** $-5(-9)(-12)$

**ALGEBRA Simplify each expression.**

**10.** $-4 \cdot 3x$ **11.** $7(-3y)$ **12.** $-8a(-3b)$

**ALGEBRA Evaluate each expression.**

**13.** $-6h$, if $h = -20$ **14.** $-4st$, if $s = -9$ and $t = 3$

## Standardized Test Practice

A B C D

**15.** The research submarine *Alvin*, used to locate the wreck of the *Titanic*, descends at a rate of about 100 feet per minute. Which integer describes the distance *Alvin* travels in 5 minutes?

(A) $-500$ ft (B) $-100$ ft (C) $-20$ ft (D) 100 ft

## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 16–21 | 1 |
| 22–25 | 2 |
| 26–33 | 3 |
| 34, 35, 54, 55 | 4 |
| 36–53 | 5 |

**Extra Practice** See page 727.

**Find each product.**

**16.** $-3 \cdot 4$ **17.** $-7 \cdot 6$ **18.** $4(-8)$

**19.** $9 \cdot (-8)$ **20.** $-12 \cdot 3$ **21.** $14(-5)$

**22.** $6 \cdot 19$ **23.** $4(32)$ **24.** $-8(-11)$

**25.** $-15(-3)$ **26.** $-5(-4)(6)$ **27.** $5(-13)(-2)$

**28.** $-7(-8)(-3)$ **29.** $-11(-4)(-7)$ **30.** $-12(-9)(6)$

**31.** $-6(-8)(11)$ **32.** $2(-8)(-9)(10)$ **33.** $4(-7)(-4)(-12)$

**34. FLOODS** In 1993, the Mississippi River was so high that it caused the Illinois River to flow backward. If the Illinois River flowed at the rate of $-1500$ feet per hour, how far would the water travel in 24 hours?

**35. TEMPERATURE** During a 10-hour period, the temperature in Browning, Montana, changed at a rate of $-10°$F per hour, starting at 44°F. What was the ending temperature?

**ALGEBRA Simplify each expression.**

**36.** $-5 \cdot 7x$ **37.** $-8 \cdot 12y$ **38.** $6(-8a)$

**39.** $5(-11b)$ **40.** $-7s(-8t)$ **41.** $-12m(-9n)$

**42.** $2ab(3)(-7)$ **43.** $3x(5y)(-9)$ **44.** $-4(-p)(-q)$

**45.** $-8(-11b)(-c)$ **46.** $9(-2c)(3d)$ **47.** $-6j(3)(5k)$

**ALGEBRA Evaluate each expression.**

**48.** $-7n$, if $n = -4$ **49.** $9s$, if $s = -11$

**50.** $ab$, if $a = 9$ and $b = 8$ **51.** $-2xy$, if $x = -8$ and $y = 5$

**52.** $-16cd$, if $c = 4$ and $d = -5$ **53.** $18gh$, if $g = -3$ and $h = 4$

**More About . . .**

**Tides**

It takes about 6 hours for the ocean to move from low to high tide. High tide can change the width of the beach at a rate of $-17$ feet an hour.

**TIDES For Exercises 54 and 55, use the information below and at the left.** In Wrightsville, North Carolina, during low tide, the beachfront in some places is about 350 feet from the ocean to the homes. At high tide, the water is much closer to the homes.

**54.** What is the change in the width of the beachfront from low to high tide?

**55.** What is the distance from the ocean to the homes at high tide?

**56. CRITICAL THINKING** Write a rule that will help you determine the sign of the product if you are multiplying two or more integers.

**57. CRITICAL THINKING** Determine whether each statement is *true* or *false*. If false, give a counterexample. If true, give an example.

**a.** Multiplication of integers is commutative.

**b.** Multiplication of integers is associative.

58. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How are the signs of factors and products related?**

Include the following in your answer:

- a model of $2(-4)$,
- an explanation of why the product of a positive and a negative integer must be negative, and
- a pattern that explains why the product $-3(-3)$ is positive.

**Standardized Test Practice** (A) (B) (C) (D)

59. The product of two negative integers is—

(A) always negative. (B) always positive.

(C) sometimes negative. (D) never positive.

60. Which values complete the table at the right for $y = -3x$?

| $x$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|
| $y$ | | | | |

(A) $-6, -3, 0, 3$ (B) $-6, -2, 0, 2$

(C) $6, 2, 0, -2$ (D) $6, 3, 0, -3$

## Maintain Your Skills

**Mixed Review**

**ALGEBRA** **Evaluate each expression if $a = -2$, $b = -6$, and $c = 14$.** *(Lesson 2-3)*

61. $a - c$ 62. $b - a$ 63. $a - b$

64. $a + b + c$ 65. $b - a + c$ 66. $a - b - c$

67. **WEATHER RECORDS** The highest recorded temperature in Columbus, Ohio, is 104°F. The lowest recorded temperature is −22°F. What is the difference between the highest and lowest temperatures? *(Lesson 2-3)*

**Find each sum.** *(Lesson 2-2)*

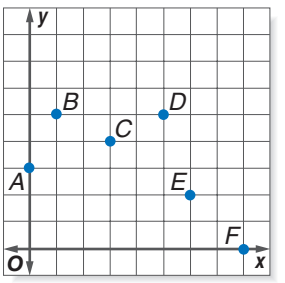
68. $-10 + 8 + 4$ 69. $-4 + (-3) + (-7)$ 70. $9 + (-14) + 2$

**Refer to the coordinate system. Write the ordered pair that names each point.** *(Lesson 1-6)*

71. $E$ 72. $C$

73. $B$ 74. $F$

75. $D$ 76. $A$




**Find each sum or product mentally.** *(Lesson 1-4)*

77. $3 \cdot 8 \cdot 20$ 78. $8 + 98 + 102$ 79. $5 \cdot 11 \cdot 10$

**Getting Ready for the Next Lesson**

**BASIC SKILL** **Find each quotient.**

80. $40 \div 8$ 81. $90 \div 15$ 82. $45 \div 3$

83. $105 \div 7$ 84. $240 \div 6$ 85. $96 \div 24$

**WebQuest**

The cost of a trip to a popular amusement park can be determined with integers. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

www.pre-alg.com/self_check_quiz

# 2-5 Dividing Integers

**What** You'll Learn

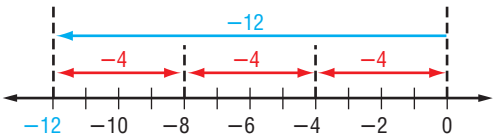
- Divide integers.
- Find the average of a set of data.

## Vocabulary

- average (mean)

**How** is dividing integers related to multiplying integers?

You can find the quotient $-12 \div (-4)$ using a number line. To find how many groups of $-4$ there are in $-12$, show $-12$ on a number line. Then divide it into groups of $-4$.




**a.** How many groups are there?

**b.** What is the quotient of $-12 \div (-4)$?

**c.** What multiplication sentence is also shown on the number line?

**d.** Draw a number line and find the quotient $-10 \div (-2)$.

**DIVIDE INTEGERS** You can find the quotient of two integers by using the related multiplication sentence.

Think of this factor to find this quotient.

$-4 \times 3 = -12 \rightarrow -12 \div (-4) = 3$

$-2 \times 5 = -10 \rightarrow -10 \div (-2) = 5$

## Reading Math

**Parts of a Division Sentence**

In a division sentence, like $15 \div 5 = 3$, the number you are dividing, 15, is called the *dividend*. The number you are dividing by, 5, is called the *divisor*. The result, 3, is called the *quotient*.

In the division sentences $-12 \div (-4) = 3$ and $-10 \div (-2) = 5$, notice that the dividends and divisors are both negative. In both cases, the quotient is positive.

negative dividend and divisor

$-12 \div (-4) = 3$ $\quad -10 \div (-2) = 5$

positive quotient

You already know that the quotient of two positive integers is positive.

$12 \div 4 = 3 \qquad 10 \div 2 = 5$

These and similar examples suggest the following rule for dividing integers with the same sign.

**Key Concept** *Dividing Integers with the Same Sign*

- **Words** The quotient of two integers with the same sign is positive.
- **Examples** $-12 \div (-3) = 4$ $\qquad 12 \div 3 = 4$

## Example 1 Divide Integers with the Same Sign

**Find each quotient.**

**a. $-32 \div (-8)$**

$-32 \div (-8) = 4$ The dividend and the divisor have the same sign. The quotient is positive.

**b. $\frac{75}{5}$**

$\frac{75}{5} = 75 \div 5$ The dividend and divisor have the same sign.

$= 15$ The quotient is positive.

What is the sign of the quotient of a positive and a negative integer? Look for a pattern in the following related sentences.

Think of this factor to find this quotient.

$-4 \times (-6) = 24 \rightarrow 24 \div (-4) = -6$

$2 \times (-9) = -18 \rightarrow -18 \div 2 = -9$

Notice that the signs of the dividend and divisor are different. In both cases, the quotient is negative.

different signs

$24 \div (-4) = -6$

$-18 \div 2 = -9$

negative quotient

different signs

These and other similar examples suggest the following rule.

## Key Concept Dividing Integers with Different Signs

- **Words** The quotient of two integers with different signs is negative.
- **Examples** $-12 \div 4 = -3$ $12 \div (-4) = -3$

**Concept Check** How do you know the sign of the quotient of two integers?

## Example 2 Divide Integers with Different Signs

**Find each quotient.**

**a. $-42 \div 3$**

$-42 \div 3 = -14$ The signs are different. The quotient is negative.

**b. $\frac{48}{-6}$**

$\frac{48}{-6} = 48 \div (-6)$ The signs are different. The quotient is negative.

$= -8$ Simplify.

ALGEBRA CONNECTION

You can use the rules for dividing integers to evaluate algebraic expressions.

### Example 3 Evaluate Algebraic Expressions

**Evaluate $ab \div (-4)$ if $a = -6$ and $b = -8$.**

$ab \div (-4) = -6(-8) \div (-4)$ Replace $a$ with $-6$ and $b$ with $-8$.

$= 48 \div (-4)$ The product of $-6$ and $-8$ is positive.

$= -12$ The quotient of 48 and $-4$ is negative.

**AVERAGE (MEAN)** Division is used in statistics to find the **average**, or **mean**, of a set of data. To find the mean of a set of numbers, find the sum of the numbers and then divide by the number in the set.

### Example 4 Find the Mean

**a. Rachel had test scores of 84, 90, 89, and 93. Find the average (mean) of her test scores.**

$\frac{84 + 90 + 89 + 93}{4} = \frac{356}{4}$ Find the sum of the test scores. Divide by the number of scores.

$= 89$ Simplify.

The average of her test scores is 89.

**b. Find the average (mean) of $-2$, 8, 5, $-9$, $-12$, and $-2$.**

$\frac{-2 + 8 + 5 + (-9) + (-12) + (-2)}{6} = \frac{-12}{6}$ Find the sum of the set of integers. Divide by the number in the set.

$= -2$ Simplify.

The average is $-2$.

**Study Tip**

*Checking Reasonableness*

The average must be between the greatest and least numbers in the set. Are the averages in Examples 4a and 4b reasonable?

You can refer to the following table to review operations with integers.

**Concept Summary** — *Operations with Integers*

| Words | Examples |
|---|---|
| **Adding Integers**<br>To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.<br>To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value. | $-5 + (-4) = -9$ $5 + 4 = 9$<br>$-5 + 4 = -1$ $5 + (-4) = 1$ |
| **Subtracting Integers**<br>To subtract an integer, add its additive inverse. | $5 - 9 = 5 + (-9)$ or $-4$<br>$5 - (-9) = 5 + 9$ or 14 |
| **Multiplying Integers**<br>The product of two integers with the same sign is positive.<br>The product of two integers with different signs is negative. | $5 \cdot 4 = 20$ $-5 \cdot (-4) = 20$<br>$-5 \cdot 4 = -20$ $5 \cdot (-4) = -20$ |
| **Dividing Integers**<br>The quotient of two integers with the same sign is positive.<br>The quotient of two integers with different signs is negative. | $20 \div 5 = 4$ $-20 \div (-5) = 4$<br>$-20 \div 5 = -4$ $20 \div (-5) = -4$ |

## Check for Understanding

**Concept Check**

1. **OPEN ENDED** Write an equation with three integers that illustrates dividing integers with different signs.
2. **Explain** how to find the average of a set of numbers.

**Guided Practice**

**Find each quotient.**

3. $88 \div 8$
4. $-20 \div (-5)$
5. $-18 \div 6$
6. $\frac{-36}{-4}$
7. $\frac{70}{-7}$
8. $\frac{-81}{9}$

**ALGEBRA** **Evaluate each expression.**

9. $x \div 4$, if $x = -52$
10. $\frac{s}{t}$, if $s = -45$ and $t = 5$

**Application**

11. **WEATHER** The low temperatures for 7 days in January were $-2$, 0, 5, $-1$, $-4$, 2, and 0. Find the average for the 7-day period.

## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 12–17, 24, 25 | 1 |
| 18–23 | 2 |
| 26–31 | 3 |
| 32, 33 | 4 |

**Extra Practice** See page 727.

**Find each quotient.**

12. $54 \div 9$
13. $45 \div 5$
14. $-27 \div (-3)$
15. $-64 \div (-8)$
16. $-72 \div (-9)$
17. $-60 \div (-6)$
18. $-77 \div 7$
19. $-300 \div 6$
20. $480 \div (-12)$
21. $\frac{132}{-12}$
22. $\frac{175}{-25}$
23. $\frac{143}{-13}$
24. What is $-91$ divided by $-7$?
25. Divide $-76$ by $-4$.

**ALGEBRA** **Evaluate each expression.**

26. $\frac{x}{-5}$, if $x = 85$
27. $\frac{108}{m}$, if $m = -9$
28. $\frac{c}{d}$, if $c = -63$ and $d = -7$
29. $\frac{s}{t}$, if $s = 52$ and $t = -4$
30. $xy \div (-3)$ if $x = 9$ and $y = -7$
31. $ab \div 6$ if $a = -12$ and $b = -8$
32. **STATISTICS** Find the average (mean) of 4, $-8$, 9, $-3$, $-7$, 10, and 2.
33. **BASKETBALL** In their first five games, the Jefferson Middle School basketball team scored 46, 52, 49, 53, and 45 points. What was their average number of points per game?

**ENERGY** **For Exercises 34–36, use the information below.**

The formula $d = \left|65 - \frac{h + l}{2}\right|$ can be used to find degree days, where $h$ is the high and $l$ is the low temperature.

34. If Baltimore had a high of 81° and a low of 65°, find the degree days.
35. If Milwaukee had a high of 8° and a low of 0°, find the degree days.
36. **RESEARCH** Use the Internet or another resource to find the high and low temperature for your city for a day in January. Find the degree days.

**37. CRITICAL THINKING** Find values for $x$, $y$, and $z$, so that all of the following statements are true.

- $y > x$, $z < y$, and $x < 0$
- $z \div 2$ and $z \div 3$ are integers.
- $x \div z = -z$
- $x \div y = z$

**38. CRITICAL THINKING** Addition and multiplication are said to be closed for whole numbers, but subtraction and division are not. That is, when you add or multiply any two whole numbers, the result is a whole number. Which operations are closed for integers?

**39. WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How is dividing integers related to multiplying integers?**

Include the following in your answer:

- two related multiplication and division sentences, and
- an example of each case (same signs, different signs) of dividing integers.

## Standardized Test Practice

**40.** On Saturday, the temperature fell 10° in 2 hours. Which expresses the temperature change per hour?

(A) 5° (B) −2° (C) −5° (D) −10°

**41.** Mark has quiz scores of 8, 7, 8, and 9. What is the lowest score he can get on the remaining quiz to have a final average (mean) score of at least 8?

(A) 7 (B) 8 (C) 9 (D) 10

# Maintain Your Skills

## Mixed Review

**Find each difference or product.** *(Lessons 2-3 and 2-4)*

**42.** $-8 - (-25)$ **43.** $75 - 114$ **44.** $2ab \cdot (-2)$ **45.** $(-10c)(5d)$

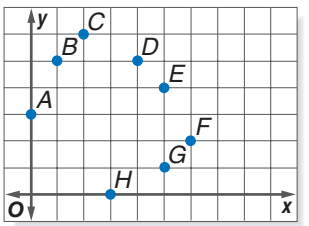
**46. PATTERNS** Find the next two numbers in the pattern 5, 4, 2, −1, … *(Lesson 1-1)*

## Getting Ready for the Next Lesson

**PREREQUISITE SKILL** **Use the grid to name the point for each ordered pair.** *(To review **ordered pairs**, see Lesson 1-6.)*

**47.** (1, 5) **48.** (6, 2)

**49.** (4, 5) **50.** (0, 3)




## Practice Quiz 2

*Lessons 2-4 and 2-5*

**Find each product.** *(Lesson 2-4)*

**1.** $-12 \cdot 7$ **2.** $-6(-15)$ **3.** $-3\,(-7)(-6)$ **4.** $3(-8)(-5)$

**Find each quotient.** *(Lesson 2-5)*

**5.** $-124 \div 4$ **6.** $-90 \div (-6)$ **7.** $125 \div (-5)$ **8.** $-126 \div (-9)$

**9.** Simplify $4x(-5y)$. *(Lesson 2-4)*

**10.** Evaluate $-9a$ if $a = -6$. *(Lesson 2-4)*

# 2-6 The Coordinate System

## What You'll Learn

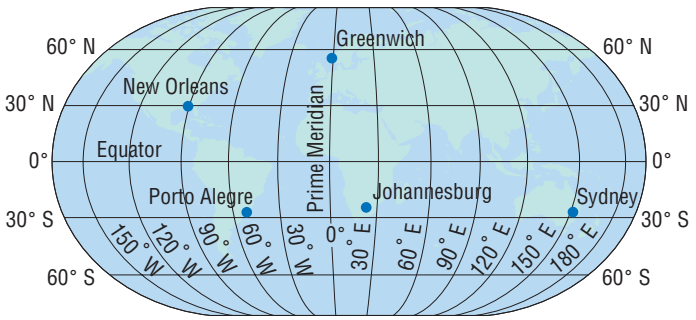
- Graph points on a coordinate plane.
- Graph algebraic relationships.

### Vocabulary

- quadrants

## How is a coordinate system used to locate places on Earth?

A GPS, or Global Positioning System, can be used to find a location anywhere on Earth by identifying its latitude and longitude. Several cities are shown on the map below. For example, Sydney, Australia, is located at approximately 30°S, 150°E.

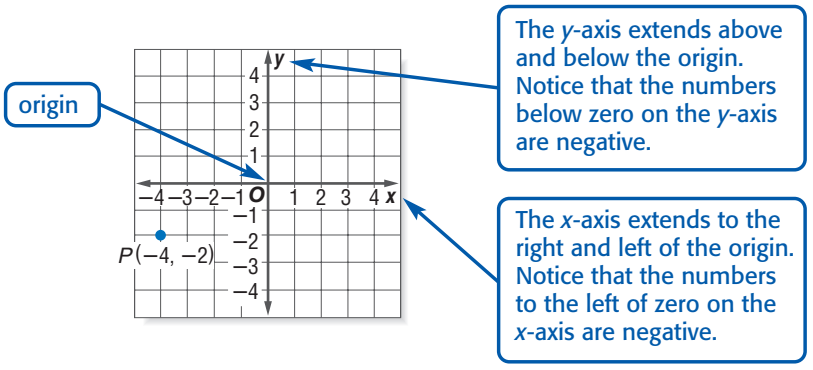



**a.** Latitude is measured north and south of the equator. What is the latitude of New Orleans?

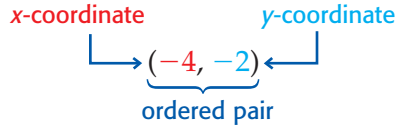
**b.** Longitude is measured east and west of the prime meridian. What is the longitude of New Orleans?

**c.** What does the location 30°N, 90°W mean?

**GRAPH POINTS** Latitude and longitude are a kind of coordinate system. The coordinate system you used in Lesson 1-6 can be extended to include points below and to the left of the origin.

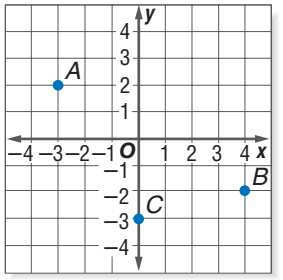



Recall that a point graphed on the coordinate system has an $x$-coordinate and a $y$-coordinate. The dot at the ordered pair $(-4, -2)$ is the graph of point $P$.



## Example 1 Write Ordered Pairs

**Write the ordered pair that names each point.**




**a.** *A*

The *x*-coordinate is −3.
The *y*-coordinate is 2.
The ordered pair is (−3, 2).

**b.** *B*

The *x*-coordinate is 4.
The *y*-coordinate is –2.
The ordered pair is (4, −2).

**c.** *C*

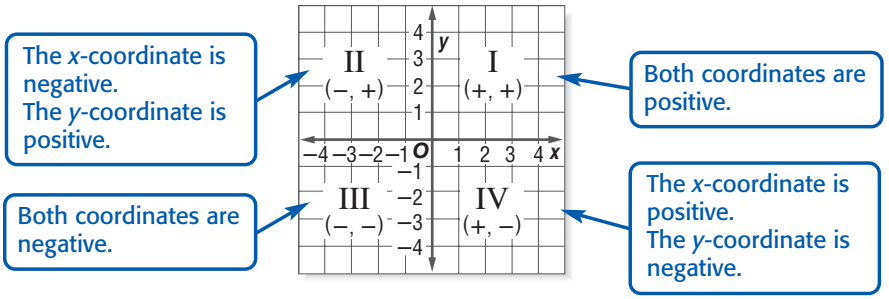
The point lies on the *y*-axis, so its *x*-coordinate is 0.
The *y*-coordinate is −3. The ordered pair is (0, −3).

**Study Tip**

**Ordered Pairs**

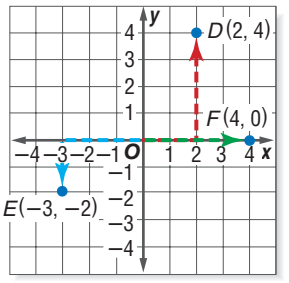
Notice that the axes in an ordered pair (*x*, *y*) are listed in alphabetical order.

The *x*-axis and the *y*-axis separate the coordinate plane into four regions, called **quadrants**. The axes and points on the axes are not located in any of the quadrants.




## Example 2 Graph Points and Name Quadrant

**Graph and label each point on a coordinate plane. Name the quadrant in which each point lies.**




**a.** *D*(2, 4)

Start at the origin. Move 2 units right.
Then move 4 units up and draw a dot.
Point *D*(2, 4) is in Quadrant I.

**b.** *E*(−3, −2)

Start at the origin. Move 3 units left.
Then move 2 units down and draw a dot.
Point *E*(−3, −2) is in Quadrant III.

**c.** *F*(4, 0)

Start at the origin. Move 4 units right. Since the *y*-coordinate is 0, the point lies on the *x*-axis. Point *F*(4, 0) is not in any quadrant.

**Concept Check** What parts of a coordinate graph do not lie in any quadrant?



CONTENTS

**GRAPH ALGEBRAIC RELATIONSHIPS** You can use a coordinate graph to show relationships between two numbers.

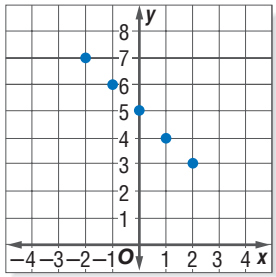
## Example 3 Graph an Algebraic Relationship

**The sum of two numbers is 5. If $x$ represents the first number and $y$ represents the second number, make a table of possible values for $x$ and $y$. Graph the ordered pairs and describe the graph.**

First, make a table. Choose values for $x$ and $y$ that have a sum of 5.

| $x + y = 5$ | | |
|---|---|---|
| $x$ | $y$ | $(x, y)$ |
| 2 | 3 | (2, 3) |
| 1 | 4 | (1, 4) |
| 0 | 5 | (0, 5) |
| −1 | 6 | (−1, 6) |
| −2 | 7 | (−2, 7) |

Then graph the ordered pairs on a coordinate plane.




The points on the graph are in a line that slants downward to the right. The line crosses the $y$-axis at $y = 5$.

# Check for Understanding

### Concept Check

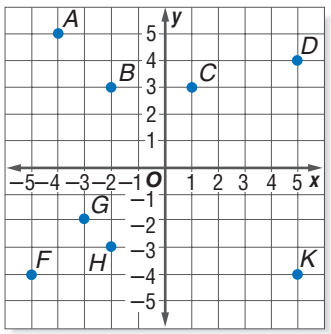
1. **Explain** why the point (3, 6) is different from the point (6, 3).

2. **OPEN ENDED** Name two ordered pairs whose graphs are *not* located in one of the four quadrants.

3. **FIND THE ERROR** Keisha says that if you interchange the coordinates of any point in Quadrant I, the new point would still be in Quadrant 3. Jason says the new point would be in Quadrant 3. Who is correct? Explain your reasoning.

### Guided Practice

**Name the ordered pair for each point graphed at the right.**




4. $A$
5. $C$
6. $G$
7. $K$

**Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.**

8. $J(3, -4)$
9. $K(-2, 2)$
10. $L(0, 4)$
11. $M(-1, -2)$

### Application

12. **ALGEBRA** Make a table of values and graph six ordered integer pairs where $x + y = 3$. Describe the graph.

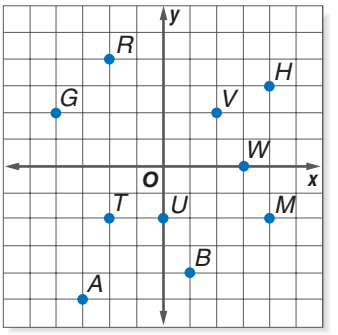
## Practice and Apply

**Homework Help**

| For Exercises | See Examples |
|---|---|
| 13–22 | 1 |
| 23–34, 41, 42 | 2 |
| 35–40, 43, 44 | 3 |

**Extra Practice**
**See page 728.**

**Name the ordered pair for each point graphed at the right.**




**13.** $R$ **14.** $G$

**15.** $M$ **16.** $B$

**17.** $V$ **18.** $H$

**19.** $U$ **20.** $W$

**21.** $A$ **22.** $T$

**Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.**

**23.** $A(4, 5)$ **24.** $K(-5, 1)$ **25.** $M(4, -2)$

**26.** $B(-5, -5)$ **27.** $S(2, -5)$ **28.** $R(-3, 5)$

**29.** $E(0, 3)$ **30.** $H(0, -3)$ **31.** $G(5, 0)$

**32.** $C(6, -1)$ **33.** $D(0, 0)$ **34.** $F(-4, 0)$

**ALGEBRA** **Make a table of values and graph six sets of ordered integer pairs for each equation. Describe the graph.**

**35.** $x + y = 5$ **36.** $x + y = -2$ **37.** $y = 2x$

**38.** $y = -2x$ **39.** $y = x + 2$ **40.** $y = x - 1$

**Graph each point. Then connect the points in alphabetical order and identify the figure.**

**41.** $A(0, 6)$, $B(4, -6)$, $C(-6, 2)$, $D(6, 2)$, $E(-4, -6)$, $F(0, 6)$

**42.** $A(5, 8)$, $B(1, 13)$, $C(5, 18)$, $D(9, 13)$, $E(5, 8)$, $F(5, 6)$, $G(3, 7)$, $H(3, 5)$, $I(7, 7)$, $J(7, 5)$, $K(5, 6)$, $L(5, 3)$, $M(3, 4)$, $N(3, 2)$, $P(7, 4)$, $Q(7, 2)$, $R(5, 3)$, $S(5, 1)$

**43.** Graph eight ordered integer pairs where $|x| > 3$. Describe the graph.

**44.** Graph all ordered integer pairs that satisfy the condition $|x| < 4$ and $|y| < 3$.

### Reading Math

**Vertex, Vertices**

A *vertex* of a triangle is a point where two sides of a triangle meet. *Vertices* is the plural of *vertex*.

**GEOMETRY** **On a coordinate plane, draw a triangle $ABC$ with vertices at $A(3, 1)$, $B(4, 2)$, and $C(2, 4)$. Then graph and describe each new triangle formed in Exercises 45–48.**

**45.** Multiply each coordinate of the vertices in triangle $ABC$ by 2.

**46.** Multiply each coordinate of the vertices in triangle $ABC$ by $-1$.

**47.** Add 2 to each coordinate of the vertices in triangle $ABC$.

**48.** Subtract 4 from each coordinate of the vertices in triangle $ABC$.

**49.** **MAPS** Find a map of your school and draw a coordinate grid on the map with the library as the center. Locate the cafeteria, principal's office, your math classroom, gym, counselor's office, and the main entrance on your grid. Write the coordinates of these places. How can you use these points to help visitors find their way around your school?

**50. CRITICAL THINKING** If the graph of A$(x, y)$ satisfies the given condition, name the quadrant in which point $A$ is located.

**a.** $x > 0, y > 0$ **b.** $x < 0, y < 0$ **c.** $x < 0, y > 0$

**51. CRITICAL THINKING** Graph eight sets of integer coordinates that satisfy $|x| + |y| > 3$. Describe the location of the points.

**52. WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

**How is a coordinate system used to locate places on Earth?**

Include the following in your answer:

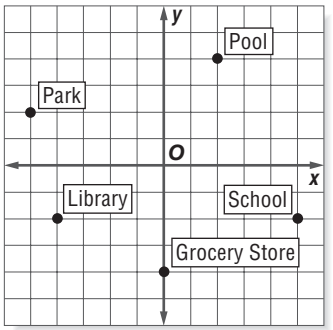
- an explanation of how coordinates can describe a location, and
- a description of how latitude and longitude are related to the $x$- and $y$-axes on a coordinate plane. Include what corresponds to the origin on a coordinate plane.

**Standardized Test Practice** A B C D

**53.** On the coordinate plane at the right, what are the coordinates of the point that shows the location of the library?

(A) $(4, -2)$ (B) $(-2, -4)$

(C) $(4, 2)$ (D) $(-4, -2)$




**54.** On the coordinate plane at the right, what location has coordinates $(5, -2)$?

(A) Park (B) School

(C) Library (D) Grocery Store

## Maintain Your Skills

**Mixed Review** **Find each quotient.** *(Lesson 2-5)*

**55.** $-24 \div 8$ **56.** $105 \div (-5)$ **57.** $-400 \div (-50)$

**ALGEBRA** **Evaluate each expression if $f = -9$, $g = -6$, and $h = 8$.** *(Lesson 2-4)*

**58.** $-5fg$ **59.** $2gh$ **60.** $-10fh$

**61. WEATHER** In the newspaper, Amad read that the low temperature for the day was expected to be $-5°$F and the high temperature was expected to be $8°$F. What was the difference in the expected high and low temperature? *(Lesson 2-3)*

**ALGEBRA** **Simplify each expression.** *(Lesson 1-4)*

**62.** $(a + 8) + 6$ **63.** $4(6h)$ **64.** $(n \cdot 7) \cdot 8$

**65.** $(b \cdot 9) \cdot 5$ **66.** $(16 + 3y) + y$ **67.** $0(4z)$

Chapter 2

# Study Guide and Review

## Vocabulary and Concept Check

absolute value (p. 58)
additive inverse (p. 66)
average (p. 82)
coordinate (p. 57)
inequality (p. 57)
integers (p. 56)
mean (p. 82)
negative number (p. 56)
opposites (p. 66)
quadrants (p. 86)

**Complete each sentence with the correct term. Choose from the list above.**

1. A(n) _____ is a number less than zero.
2. The four regions separated by the axes on a coordinate plane are called _____.
3. The number that corresponds to a point on the number line is called the _____ of that point.
4. An integer and its opposite are also called _____ of each other.
5. The set of _____ includes positive whole numbers, their opposites, and zero.
6. The _____ of a number is the distance the number is from zero on the number line.
7. A(n) _____ is a mathematical sentence containing $<$ or $>$.

## Lesson-by-Lesson Review

### 2-1 Integers and Absolute Value

See pages 56–61.

**Concept Summary**

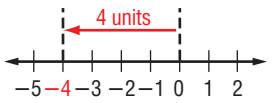
- Numbers on a number line increase as you move from left to right.
- The absolute value of a number is the distance the number is from zero on the number line.

**Examples**

**1** **Replace the ● with $<$, $>$, or $=$ in $-3$ ● 2 to make a true sentence.**


Since $-3$ is to the left of 2, write $-3 < 2$.

**2** **Evaluate $|-4|$.**




The graph of $-4$ is 4 units from 0. So, $|-4| = 4$.

**Exercises** **Replace each ● with $<$, $>$, or $=$ to make a true sentence.**

*See Example 2 on page 57.*

**8.** $8 \bullet -8$ **9.** $-3 \bullet -3$ **10.** $-2 \bullet 0$ **11.** $-12 \bullet -21$

**Evaluate each expression.** *See Example 4 on page 58.*

**12.** $|-32|$ **13.** $|25|$ **14.** $-|15|$ **15.** $|-8| + |-14|$



www.pre-alg.com/vocabulary_review

CONTENTS

## 2-2 Adding Integers

See pages 64–68.

### Concept Summary

- To add integers with the same sign, add their absolute values. Give the result the same sign as the integers.
- To add integers with different signs, subtract their absolute values. Give the result the same sign as the integer with the greater absolute value.

**Examples** **Find each sum.**

**1** $-3 + (-4)$

$-3 + (-4) = -7$ The sum is negative.

**2** $5 + (-2)$

$5 + (-2) = 3$ The sum is positive.

**Exercises** **Find each sum.** *See Examples 2, 4, and 6 on pages 64–66.*

**16.** $-6 + (-3)$ **17.** $-4 + (-1)$ **18.** $-2 + 7$

**19.** $4 + (-8)$ **20.** $6 + (-9) + (-8)$ **21.** $4 + (-7) + (-3) + (-4)$

## 2-3 Subtracting Integers

See pages 70–74.

### Concept Summary

- To subtract an integer, add its additive inverse.

**Examples** **Find each difference.**

**1** $-5 - 2$

$-5 - 2 = -5 + (-2)$ To subtract 2, add $-2$.

$= -7$

**2** $8 - (-4)$

$8 - (-4) = 8 + 4$ To subtract $-4$, add 4.

$= 12$

**Exercises** **Find each difference.** *See Examples 1 and 2 on pages 70–71.*

**22.** $4 - 9$ **23.** $-3 - 5$ **24.** $7 - (-2)$ **25.** $-1 - (-6)$

**26.** $-7 - 8$ **27.** $6 - 10$ **28.** $-3 - (-7)$ **29.** $6 - (-3)$

## 2-4 Multiplying Integers

See pages 75–79.

### Concept Summary

- The product of two integers with different signs is negative.
- The product of two integers with the same sign is positive.

**Examples** **Find each product.**

**1** $6(-4)$

$6(-4) = -24$ The factors have different signs, so the product is negative.

**2** $-8(-2)$

$-8(-2) = 16$ The factors have the same sign, so the product is positive.

**Exercises** **Find each product.** *See Examples 1 and 2 on pages 75–76.*

**30.** $-9(5)$ **31.** $11(-6)$ **32.** $-4(-7)$ **33.** $-3(-16)$

**34.** Simplify $-2a(4b)$. *See Example 5 on page 77.*

## 2-5 Dividing Integers

See pages 80–84.

### Concept Summary

- The quotient of two integers with the same sign is positive.
- The quotient of two integers with different signs is negative.

**Examples** **Find each quotient.**

**1** $-30 \div (-5)$.

$-30 \div (-5) = 6$ The signs are the same, so the quotient is positive.

**2** $27 \div (-3)$

$27 \div (-3) = -9$ The signs are different, so the quotient is negative.

**Exercises** **Find each quotient.** *See Examples 1 and 2 on page 81.*

**35.** $-14 \div (-2)$ **36.** $-52 \div (-4)$ **37.** $-36 \div 9$ **38.** $88 \div (-4)$

**39.** Find the average (mean) of $-3, -6, 9, -3$, and 13. *See Example 4 on page 82*

## 2-6 The Coordinate System

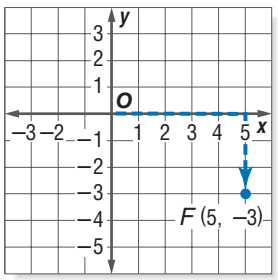
See pages 85–89.

### Concept Summary

- The $x$-axis and the $y$-axis separate the coordinate plane into four quadrants.
- The axes and points on the axes are not located in any of the quadrants.

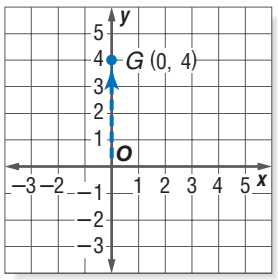
**Examples** **Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.**

**1** $F(5, -3)$




Point $F(5, -3)$ is in quadrant IV.

**2** $G(0, 4)$




Point $G(0, 4)$ is not in any quadrant.

**Exercises** **Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.** *See Example 2 on page 86.*

**40.** $A(4, 3)$ **41.** $J(-2, -4)$ **42** $K(-1, 3)$ **43.** $R(3, 0)$

Chapter 2

# Practice Test

## Vocabulary and Concepts

1. **Explain** how to add two integers with different signs.
2. **State** a rule used for subtracting integers.
3. **Graph** the set of integers $\{-6, 2, -1, 1\}$ on a number line.

## Skills and Applications

**Write two inequalities using the numbers in each sentence. Use the symbols < and >.**

4. $-5$ is less than 2.
5. 12 is greater than $-15$.

**Replace each ● with <, >, or = to make a true sentence.**

6. $-5$ ● $-3$
7. $-5$ ● $-14$
8. $4$ ● $|-7|$

**Find each sum or difference.**

9. $-4 + (-8)$
10. $-9 + 15$
11. $12 + (-15)$
12. $14 + (-7) + -11$
13. $4 - 13$
14. $8 - (-6)$
15. $-6 - (-10)$
16. $-14 - (-7)$

**Find each product or quotient.**

17. $6(-8)$
18. $-9(8)$
19. $-7(-5)$
20. $2(-4)(11)$
21. $54 \div (-9)$
22. $-64 \div (-4)$
23. $-250 \div 25$
24. $-144 \div (-6)$

**ALGEBRA** **Evaluate each expression if $a = -5$, $b = 3$, and $c = -10$.**

25. $ab - c$
26. $c \div a$
27. $4c + |a|$

**Graph and label each point on a coordinate plane. Name the quadrant in which each point is located.**

28. $D(-2, 4)$
29. $E(3, -4)$
30. $F(-1, -3)$

31. **WEATHER** The table shows the low temperatures during one week in Anchorage, Alaska. Find the average low temperature for the week.

| Day | S | M | T | W | T | F | S |
|---|---|---|---|---|---|---|---|
| Temperature (°F) | −12 | 3 | −7 | 0 | −4 | 1 | −2 |

32. **SPORTS** During the first play of the game, the Brownville Tigers football team lost seven yards. On each of the next three plays, an additional four yards were lost. Express the total yards lost at the end of the first four plays as an integer.

33. **STANDARDIZED TEST PRACTICE** Suppose Jason's home represents the origin on a coordinate plane. If Jason leaves his home and walks two miles west and then four miles north, what is the location of his destination as an ordered pair? In which quadrant is his destination?

Ⓐ $(-2, 4)$; II  Ⓑ $(2, 4)$; I  Ⓒ $(-2, -4)$; II  Ⓓ $(4, -2)$; IV

# Chapter 2 Standardized Test Practice

## Part 1 Multiple Choice

**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**

**1.** The table below shows the number of cells present after a certain form of bacteria multiplies for a number of hours. How many cells will be present in five hours? (Lesson 1-1)

| Number of Hours | Number of Cells |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |
| 3 | 27 |

(A) 81 (B) 91
(C) 243 (D) 279

**2.** Suppose your sister has 3 more CDs than you do. Which equation represents the number of CDs that you have? Let $y$ represent your CDs and $s$ represent your sister's CDs. (Lesson 1-5)

(A) $y = s + 3$ (B) $y = s - 3$
(C) $y = 3 - s$ (D) $y = 3s$

**3.** Which expression represents the greatest integer? (Lesson 1-6)

(A) $|4|$ (B) $|-3|$
(C) $|-8|$ (D) $-9$

**4.** The water level of a local lake is normally 0 feet above sea level. In a flood, the water level rose 4 feet above normal. A month later, the water level had gone down 5 feet. Which integer best represents the water level at that time? (Lesson 2-1)

(A) $-3$ (B) $-1$
(C) 4 (D) 9

**5.** What is the sum of $-5$ and 2? (Lesson 2-2)

(A) $-7$ (B) $-3$
(C) 3 (D) 7

**6.** Find the value of $x$ if $x = 7 - (-3)$. (Lesson 2-3)

(A) $-10$ (B) $-4$
(C) 4 (D) 10

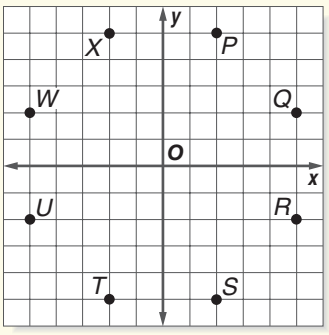
**7.** If $t = -5$, what is the value of the expression $-3t + 7$? (Lesson 2-4)

(A) $-8$ (B) $-6$
(C) 8 (D) 22

**8.** If $a = -2$ and $b = 5$, what is the value of $\frac{b - 13}{a}$? (Lesson 2-5)

(A) $-4$ (B) $-9$
(C) 9 (D) 4

**For Questions 9 and 10, use the following graph.**




**9.** Which letter represents the ordered pair $(-2, 5)$? (Lesson 2-6)

(A) R (B) X
(C) T (D) W

**10.** Which ordered pair represents point $U$? (Lesson 2-6)

(A) $(5, -2)$ (B) $(-2, -5)$
(C) $(-5, -2)$ (D) $(-2, 5)$



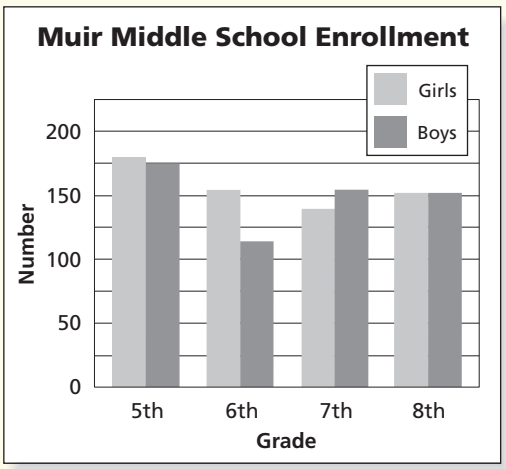
CONTENTS

**Preparing for Standardized Tests**
For test-taking strategies and more practice, see pages 771–788.

## Part 2 Short Response/Grid In

**Record your answers on the answer sheet provided by your teacher or on a sheet of paper.**

**11.** The bar graph shows the numbers of girls and boys in each grade at Muir Middle School. In which grade is the difference between the number of girls and the number of boys the greatest? (Prerequisite Skill, p. 722)




**12.** Nine less than a number is 15. Find the number. (Lesson 1-5)

**13.** The Springfield High School football team gained 7 yards on one play. On the next play, they lost 11 yards. Write an integer that represents the net result of these two plays. (Lesson 2-2)

**14.** The low temperature one winter night in Bismarck, North Dakota, was −15°F. The next day the high temperature was 3°F. How many degrees had the temperature risen? (Lesson 2-3)

**15.** The table below was used to change values of $x$ into values of $y$.

| $x$ | $y = x - 7$ |
|---|---|
| 6 | −1 |
| 7 | 0 |
| 8 | 1 |

What value of $x$ can be used to obtain a $y$-value equal to 5? (Lesson 2-3)

**16.** The low temperatures in Minneapolis during four winter days were +2°F, −7°F, −12°F, and +9°F. What was the average low temperature during these four days? (Lesson 2-5)

## Part 3 Extended Response

**Record your answers on a sheet of paper. Show your work.**

**17.** On graph paper, graph the points $A(4, 2)$, $B(-3, 7)$, and $C(-3, 2)$. Connect the points to form a triangle. (Lesson 2-6)

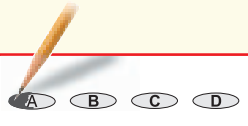
**a.** Add 6 to the $x$-coordinate of each coordinate pair. Graph and connect the new points to form a new figure. Is the new figure the same size and shape as the original triangle? Describe how the size, shape, and position of the new triangle relate to the size, shape, and position of the original triangle.

**b.** If you add −6 to each original $x$-coordinate, and graph and connect the new points to create a new figure, how will the position of the new figure relate to that of the original one?

**c.** Multiply the $y$-coordinate of each original ordered pair by −1. Graph and connect the new points to form a new figure. Describe how the size, shape, and position of the new triangle relate to the size, shape, and position of the original triangle.

**d.** If you multiply each original $x$-coordinate by −1, and graph and connect the new points to create a new figure, how will the position of the new figure relate to that of the original one?

### Test-Taking Tip




**Question 17**
When answering open-ended items on standardized tests, follow these steps:
1. Read the item carefully.
2. Show all of your work. You may receive points for items that are only partially correct.
3. Check your work.