

UNIT

1

Algebra and Integers

The word *algebra* comes from the Arabic word *al-jabr*, which was part of the title of a book about equations and how to solve them. In this unit, you will lay the foundation for your study of algebra by learning about the language of algebra, its properties, and methods of solving equations.

Chapter 1
The Tools of Algebra

Chapter 2
Integers

Chapter 3
Equations





WebQuest Internet Project

Vacation Travelers Include More Families

“Taking the kids with you is increasingly popular among Americans, according to a travel report that predicts an expanding era of kid-friendly attractions and services.” **Source:** USA TODAY, November 17, 1999

In this project, you will be exploring how graphs and formulas can help you plan a family vacation.



Log on to www.pre-alg.com/webquest.
Begin your WebQuest by reading the Task.

Then continue working on your WebQuest as you study Unit 1.

Lesson	1-7	2-4	3-7
Page	43	79	135

USA TODAY Snapshots®

Spouses are top travel partners

Spouses	58%
Children/grandchildren	34%
Friends	18%
Other family members	14%
Solo	13%
Group tour	8%

Source: Travel Industry Association of America

By Cindy Hall and Sam Ward, USA TODAY



The Tools of Algebra

What You'll Learn

- **Lesson 1-1** Use a four-step plan to solve problems and choose the appropriate method of computation.
- **Lessons 1-2 and 1-3** Translate verbal phrases into numerical expressions and evaluate expressions.
- **Lesson 1-4** Identify and use properties of addition and multiplication.
- **Lesson 1-5** Write and solve simple equations.
- **Lesson 1-6** Locate points and represent relations.
- **Lesson 1-7** Construct and interpret scatter plots.

Key Vocabulary

- order of operations (p. 12)
- variable (p. 17)
- algebraic expression (p. 17)
- ordered pair (p. 33)
- relation (p. 35)

Why It's Important

Algebra is important because it can be used to show relationships among variables and numbers. You can use algebra to describe how fast something grows. For example, the growth rate of bamboo can be described using variables. *You will find the growth rate of bamboo in Lesson 1-6.*



Getting Started

Prerequisite Skills To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 1.

For Lesson 1-1

Add and Subtract Decimals

Find each sum or difference. (For review, see page 713.)

1. $6.6 + 8.2$

2. $4.7 + 8.5$

3. $5.4 - 2.3$

4. $8.6 - 4.9$

5. $2.65 + 0.3$

6. $1.08 + 1.2$

7. $4.25 - 0.7$

8. $4.3 - 2.89$

9. $9.06 - 1.18$

For Lessons 1-1 through 1-5

Estimate with Whole Numbers

Estimate each sum, difference, product, or quotient.

10. $1800 + 285$

11. $328 + 879$

12. $22,431 - 13,183$

13. $659 - 536$

14. 68×12

15. 189×89

16. $3845 \div 82$

17. $21,789 \div 97$

18. $\$1951 \div 49$

For Lessons 1-1 through 1-5

Estimate with Decimals

Estimate each sum, difference, product, or quotient. (For review, see pages 712 and 714.)

19. $8.8 + 5.3$

20. $47.2 + 9.75$

21. $\$7.34 - \2.16

22. $83.6 - 75.32$

23. 4.2×29.3

24. $18.8(5.3)$

25. $7.8 \div 2.3$

26. $54 \div 9.1$

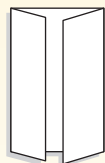
27. $21.3 \div 1.7$

FOLDABLES™ Study Organizer

Problem Solving Make this Foldable to help you organize your notes. Begin with a sheet of unlined paper.

Step 1 Fold

Fold the short sides so they meet in the middle.



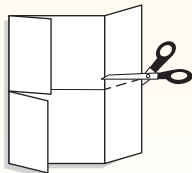
Step 2 Fold Again

Fold the top to the bottom.



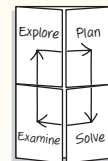
Step 3 Cut

Unfold. Cut along the second fold to make four tabs.



Step 4 Label

Label each of the tabs as shown.



Reading and Writing As you read and study the chapter, you can write examples of each problem-solving step under the tabs.

What You'll Learn

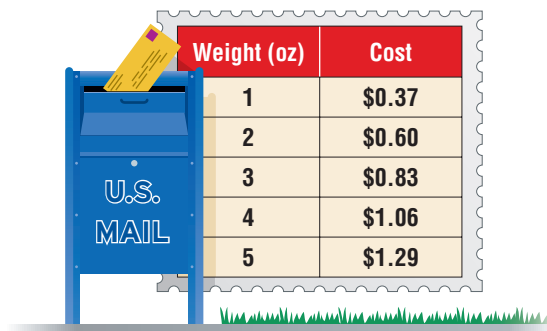
- Use a four-step plan to solve problems.
- Choose an appropriate method of computation.

Vocabulary

- conjecture
- inductive reasoning

Why is it helpful to use a problem-solving plan to solve problems?

The table shows the first-class mail rates in 2004.



Weight (oz)	Cost
1	\$0.37
2	\$0.60
3	\$0.83
4	\$1.06
5	\$1.29

Source: www.ups.com

- Find a pattern in the costs.
- How can you determine the cost to mail a 6-ounce letter?
- Suppose you were asked to find the cost of mailing a letter that weighs 8 ounces. What steps would you take to solve the problem?

FOUR-STEP PROBLEM-SOLVING PLAN It is often helpful to have an organized plan to solve math problems. The following four steps can be used to solve any math problem.

- Explore**
 - Read the problem quickly to gain a general understanding of it.
 - Ask, "What facts do I know?" "What do I need to find out?"
 - Ask, "Is there enough information to solve the problem? Is there extra information?"
- Plan**
 - Reread the problem to identify relevant facts.
 - Determine how the facts relate to each other.
 - Make a plan to solve the problem.
 - Estimate the answer.
- Solve**
 - Use your plan to solve the problem.
 - If your plan does not work, revise it or make a new plan.
- Examine**
 - Reread the problem.
 - Ask, "Is my answer reasonable and close to my estimate?"
 - Ask, "Does my answer make sense?"
 - If not, solve the problem another way.

Study Tip**Problem-Solving Strategies**

Here are a few strategies you will use to solve problems in this book.

- Look for a pattern.
- Solve a simpler problem.
- Guess and check.
- Draw a diagram.
- Make a table or chart.
- Work backward.
- Make a list.

 **Concept Check** Which step involves estimating the answer?

Example 1 Use the Four-Step Problem-Solving Plan

POSTAL SERVICE Refer to page 6. How much would it cost to mail a 9-ounce letter first class?

Explore The table shows the weight of a letter and the respective cost to mail it first class. We need to find how much it will cost to mail a 9-ounce letter.

Plan Use the information in the table to solve the problem. Look for a pattern in the costs. Extend the pattern to find the cost for a 9-ounce letter.

Solve First, find the pattern.

Weight (oz)	1	2	3	4	5
Cost	\$0.37	\$0.60	\$0.83	\$1.06	\$1.29

$+ 0.23$ $+ 0.23$ $+ 0.23$ $+ 0.23$

Each consecutive cost increases by \$0.23. Next, extend the pattern.

Weight (oz)	5	6	7	8	9
Cost	\$1.29	\$1.52	\$1.75	\$1.98	\$2.21

$+ 0.23$ $+ 0.23$ $+ 0.23$ $+ 0.23$

It would cost \$2.21 to mail a 9-ounce letter.

Examine It costs \$0.37 for the first ounce and \$0.23 for each additional ounce. To mail a 9-ounce letter, it would cost \$0.37 for the first ounce and $8 \times \$0.23$ or \$1.84 for the eight additional ounces. Since $\$0.37 + \$1.84 = \$2.21$, the answer is correct.

Study Tip

Reasonableness

Always check to be sure your answer is reasonable. If the answer seems unreasonable, solve the problem again.

A **conjecture** is an educated guess. When you make a conjecture based on a pattern of examples or past events, you are using **inductive reasoning**. In mathematics, you will use inductive reasoning to solve problems.

Example 2 Use Inductive Reasoning

a. Find the next term in 1, 3, 9, 27, 81,



Assuming the pattern continues, the next term is 81×3 or 243.

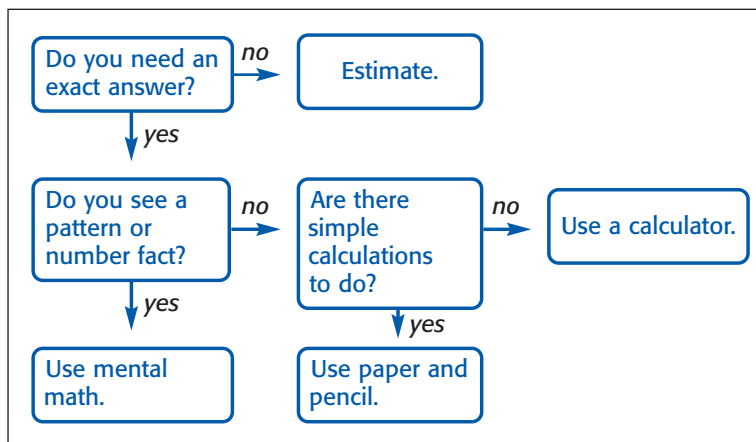
b. Draw the next figure in the pattern.



In the pattern, the shaded square moves counterclockwise. Assuming the pattern continues, the shaded square will be positioned at the bottom left of the figure.

Concept Check What type of reasoning is used when you make a conclusion based on a pattern?

CHOOSE THE METHOD OF COMPUTATION Choosing the method of computation is also an important step in solving problems. Use the diagram below to help you decide which method is most appropriate.



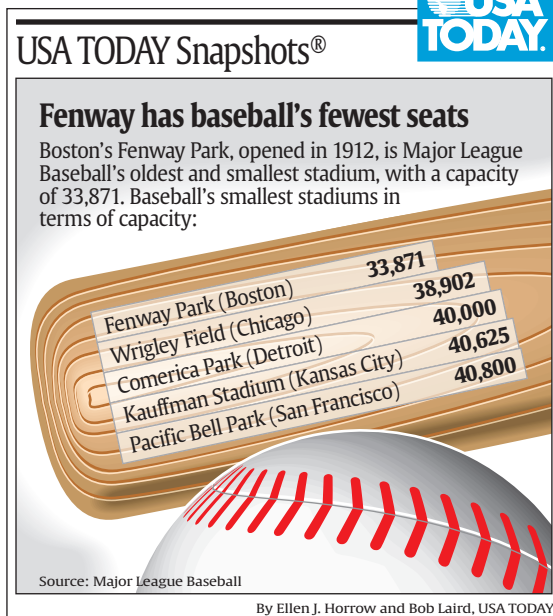
Example 3 Choose the Method of Computation



Log on for:

- Updated data
 - More activities on Using a Problem-Solving Plan
- www.pre-alg.com/usa_today

TRAVEL The graph shows the seating capacity of certain baseball stadiums in the United States. About how many more seats does Comerica Park have than Fenway Park?



Explore You know the seating capacities of Comerica Park and Fenway Park. You need to find how many more seats Comerica Park has than Fenway Park.

Plan The question uses the word *about*, so an exact answer is not needed. We can solve the problem using estimation. Estimate the amount of seats for each park. Then subtract.

Solve

Comerica Park: 40,000 → 40,000
 Fenway Park: 33,871 → 34,000 *Round to the nearest thousand.*
 $40,000 - 34,000 = 6,000$ *Subtract 34,000 from 40,000.*

So, Comerica Park has about 6000 more seats than Fenway Park.

Examine Since $34,000 + 6,000 = 40,000$, the answer makes sense.



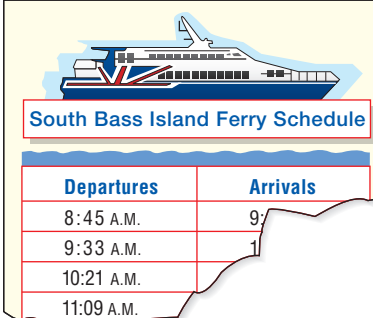
Check for Understanding

Concept Check

- Tell when it is appropriate to solve a problem using estimation.
- OPEN ENDED** Write a list of numbers in which four is added to get each succeeding term.

Guided Practice

- TRAVEL** The ferry schedule at the right shows that the ferry departs at regular intervals. Use the four-step plan to find the earliest time a passenger can catch the ferry if he/she cannot leave until 1:30 P.M.



Departures	Arrivals
8:45 A.M.	9:00 A.M.
9:33 A.M.	9:45 A.M.
10:21 A.M.	10:30 A.M.
11:09 A.M.	11:20 A.M.

Find the next term in each list.

- 10, 20, 30, 40, 50, ...
- 37, 33, 29, 25, 21, ...
- 12, 17, 22, 27, 32, ...
- 3, 12, 48, 192, 768, ...

Application

- MONEY** In 1999, the average U.S. household spent \$12,057 on housing, \$1891 on entertainment, \$5031 on food, and \$7011 on transportation. How much was spent on food each month? Round to the nearest cent.

Source: Bureau of Labor Statistics

Practice and Apply

Homework Help

For Exercises	See Examples
9, 10	1
11–20	2
21–26	3

Extra Practice
See page 724.

HEALTH For Exercises 9 and 10, use the table that gives the approximate heart rate a person should maintain while exercising at 85% intensity.

Age	20	25	30	35	40	45
Heart Rate (beats/min)	174	170	166	162	158	154

- Assume the pattern continues. Use the four-step plan to find the heart rate a 15-year-old should maintain while exercising at this intensity.
- What heart rate should a 55-year old maintain while exercising at this intensity?

Find the next term in each list.

- 2, 5, 8, 11, 14, ...
- 4, 8, 12, 16, 20, ...
- 0, 5, 10, 15, 20, ...
- 2, 6, 18, 54, 162, ...
- 54, 50, 46, 42, 38, ...
- 67, 61, 55, 49, 43, ...
- 2, 5, 9, 14, 20, ...
- 3, 5, 9, 15, 23, ...

GEOMETRY Draw the next figure in each pattern.

- 
- 

- MONEY** Ryan needs to save \$125 for a ski trip. He has \$68 in his bank. He receives \$15 for an allowance and earns \$20 delivering newspapers and \$16 shoveling snow. Does he have enough money for the trip? Explain.

22. **MONEY** Using eight coins, how can you make change for 65 cents that will not make change for a quarter?
23. **TRANSPORTATION** A car traveled 280 miles at 55 mph. About how many hours did it take for the car to reach its destination?
24. **CANDY** A gourmet jelly bean company can produce 100,000 pounds of jelly beans a day. One ounce of these jelly beans contains 100 Calories. If there are 800 jelly beans in a pound, how many jelly beans can be produced in a day?



More About . . .

Candy

In 1981, $3\frac{1}{2}$ tons of red, blue, and white jelly beans were sent to the Presidential Inaugural Ceremonies for Ronald Reagan.

Source: www.jellybelly.com

25. **MEDICINE** The number of different types of transplants that were performed in the United States in 1999 are shown in the table. About how many transplants were performed?
26. **COMMUNICATION** A telephone tree is set up so that every person calls three other people. Anita needs to tell her co-workers about a time change for a meeting. Suppose it takes 2 minutes to call 3 people. In 10 minutes, how many people will know about the change of time?

Transplant	Number
heart	2185
liver	4698
kidney	12,483
heart-lung	49
lung	885
pancreas	363
intestine	70
kidney-pancreas	946

Source: *The World Almanac*

27. **CRITICAL THINKING** Think of a 1 to 9 multiplication table.
- Are there more odd or more even products? How can you determine the answer without counting?
 - Is this different from a 1 to 9 addition facts table?
28. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why is it helpful to use a problem-solving plan to solve problems?

Include the following in your answer:

- an explanation of the importance of performing each step of the four-step problem-solving plan, and
- an explanation of why it is beneficial to estimate the answer in the *Plan* step.

29. Find the next figure in the pattern shown below.



30. A wagon manufacturing plant in Chicago, Illinois, can produce 8000 wagons a day at top production. Which of the following is a reasonable amount of wagons that can be produced in a year?
- (A) 24,000 (B) 240,000 (C) 2,400,000 (D) 240,000,000

Getting Ready for the Next Lesson

BASIC SKILL Round each number to the nearest whole number.

31. 2.8 32. 5.2 33. 35.4
34. 49.6 35. 109.3 36. 999.9



Reading Mathematics

Translating Expressions Into Words

Translating numerical expressions into verbal phrases is an important skill in algebra. Key words and phrases play an essential role in this skill.

The following table lists some words and phrases that suggest addition, subtraction, multiplication, and division.

Addition	Subtraction	Multiplication	Division
plus	minus	times	divided
sum	difference	product	quotient
more than	less than	multiplied	per
increased by	subtract	each	rate
in all	decreased by	of	ratio
	less	factors	separate

A few examples of how to write an expression as a verbal phrase are shown.

Expression	Key Word	Verbal Phrase
5×8	times	5 times 8
$2 + 4$	sum	the sum of 2 and 4
$16 \div 2$	quotient	the quotient of 16 and 2
$8 - 6$	less than	6 less than 8
2×5	product	the product of 2 and 5
$5 - 2$	less	5 less 2

Reading to Learn

1. Refer to the table above. Write a different verbal phrase for each expression.

Choose the letter of the phrase that best matches each expression.

- | | |
|----------------|----------------------------|
| 2. $9 - 3$ | a. the sum of 3 and 9 |
| 3. $3 \div 9$ | b. the quotient of 9 and 3 |
| 4. $9 \cdot 3$ | c. 3 less than 9 |
| 5. $3 + 9$ | d. 9 multiplied by 3 |
| 6. $9 \div 3$ | e. 3 divided by 9 |

Write two verbal phrases for each expression.

- | | |
|-----------------|--------------------|
| 7. $5 + 1$ | 8. $8 + 6$ |
| 9. 9×5 | 10. $2(4)$ |
| 11. $12 \div 3$ | 12. $\frac{20}{4}$ |
| 13. $8 - 7$ | 14. $11 - 5$ |

1-2

Numbers and Expressions

What You'll Learn

- Use the order of operations to evaluate expressions.
- Translate verbal phrases into numerical expressions.

Vocabulary

- numerical expression
- evaluate
- order of operations

Why do we need to agree on an order of operations?

Scientific calculators are programmed to find the value of an expression in a certain order.

Expression	$1 + 2 \times 5$	$8 - 4 \div 2$	$10 \div 5 + 14 \times 2$
Value	11	6	30

- Study the expressions and their respective values. For each expression, tell the order in which the calculator performed the operations.
- For each expression, does the calculator perform the operations in order from left to right?
- Based on your answer to parts **a** and **b**, find the value of each expression below. Check your answer with a scientific calculator.
 $12 - 3 \times 2$ $16 \div 4 - 2$ $18 + 6 - 8 \div 2 \times 3$
- Make a **conjecture** as to the order in which a scientific calculator performs operations.

ORDER OF OPERATIONS Expressions like $1 + 2 \times 5$ and $10 \div 5 + 14 \div 2$ are **numerical expressions**. Numerical expressions contain a combination of numbers and operations such as addition, subtraction, multiplication, and division.

When you **evaluate** an expression, you find its numerical value. To avoid confusion, mathematicians have agreed upon the following **order of operations**.

Study Tip

Grouping Symbols

Grouping symbols include:

- parentheses (),
- brackets [], and
- fraction bars, as in $\frac{6+4}{2}$, which means $(6 + 4) \div 2$.

Concept Summary

Order of Operations

- Step 1** Simplify the expressions inside grouping symbols.
- Step 2** Do all multiplications and/or divisions from left to right.
- Step 3** Do all additions and/or subtractions from left to right.

Numerical expressions have only one value. Consider $6 + 4 \times 3$.

$$6 + 4 \times 3 = 6 + 12 = 18$$

Multiply, then add.

~~$$6 + 4 \times 3 = 10 \times 3 = 30$$~~

Add, then multiply.

Which is the correct value, 18 or 30? Using the order of operations, the correct value of $6 + 4 \times 3$ is 18.



Concept Check

Which operation should you perform first to evaluate $10 - 2 + 3$?

Example 1 Evaluate Expressions

Find the value of each expression.

a. $3 + 4 \times 5$

$$\begin{aligned} 3 + 4 \times 5 &= 3 + 20 && \text{Multiply 4 and 5.} \\ &= 23 && \text{Add 3 and 20.} \end{aligned}$$

b. $18 \div 3 \times 2$

$$\begin{aligned} 18 \div 3 \times 2 &= 6 \times 2 && \text{Divide 18 by 3.} \\ &= 12 && \text{Multiply 6 and 2.} \end{aligned}$$

c. $6(2 + 9) - 3 \cdot 8$

$$\begin{aligned} 6(2 + 9) - 3 \cdot 8 &= 6(11) - 3 \cdot 8 && \text{Evaluate } (2 + 9) \text{ first.} \\ &= 66 - 3 \cdot 8 && 6(11) \text{ means } 6 \times 11. \\ &= 66 - 24 && 3 \cdot 8 \text{ means 3 times 8.} \\ &= 42 && \text{Subtract 24 from 66.} \end{aligned}$$

d. $4[(15 - 9) + 8(2)]$

$$\begin{aligned} 4[(15 - 9) + 8(2)] &= 4[6 + 8(2)] && \text{Evaluate } (15 - 9). \\ &= 4(6 + 16) && \text{Multiply 8 and 2.} \\ &= 4(22) && \text{Add 6 and 16.} \\ &= 88 && \text{Multiply 4 and 22.} \end{aligned}$$

e. $\frac{53 + 15}{17 - 13}$

$$\begin{aligned} \frac{53 + 15}{17 - 13} &= (53 + 15) \div (17 - 13) && \text{Rewrite as a division expression.} \\ &= 68 \div 4 && \text{Evaluate } 53 + 15 \text{ and } 17 - 13. \\ &= 17 && \text{Divide 68 by 4.} \end{aligned}$$

Study Tip

Multiplication and Division Notation

A raised dot or parentheses represents multiplication. A fraction bar represents division.

TRANSLATE VERBAL PHRASES INTO NUMERICAL EXPRESSIONS

You have learned to translate numerical expressions into verbal phrases. It is often necessary to translate verbal phrases into numerical expressions.

Example 2 Translate Phrases into Expressions

Write a numerical expression for each verbal phrase.

a. the product of eight and seven

Phrase the product of eight and seven

Key Word product

Expression 8×7

b. the difference of nine and three

Phrase the difference of nine and three

Key Word difference

Expression $9 - 3$

Study Tip

Differences and Quotients

In this book, the *difference of 9 and 3* means to start with 9 and subtract 3, so the expression is $9 - 3$. Similarly, the *quotient of 9 and 3* means to start with 9 and divide by 3, so the expression is $9 \div 3$.



Example 3 Use an Expression to Solve a Problem

TRANSPORTATION A taxicab company charges a fare of \$4 for the first mile and \$2 for each additional mile. Write and then evaluate an expression to find the fare for a 10-mile trip.

Words $\underbrace{\$4 \text{ for the first mile}}_4$ and $\underbrace{\$2 \text{ for each additional mile}}_{2 \times 9}$
Expression 4 $+$ 2×9

$$\begin{aligned} 4 + 2 \times 9 &= 4 + 18 && \text{Multiply.} \\ &= 22 && \text{Add.} \end{aligned}$$

The fare for a 10-mile trip is \$22.

Check for Understanding

Concept Check

- OPEN ENDED** Give an example of an expression involving multiplication and subtraction, in which you would subtract first.
- Tell whether $2 \times 4 + 3$ and $2 \times (4 + 3)$ have the same value. Explain.
- FIND THE ERROR** Emily and Marcus are evaluating $24 \div 2 \times 3$.

Emily

$$\begin{aligned} 24 \div 2 \times 3 &= 12 \times 3 \\ &= 36 \end{aligned}$$

Marcus

$$\begin{aligned} 24 \div 2 \times 3 &= 24 \div 6 \\ &= 4 \end{aligned}$$

Who is correct? Explain your reasoning.

Guided Practice

Name the operation that should be performed first. Then find the value of each expression.

4. $3 \cdot 6 - 4$

5. $32 - 24 \div 2$

6. $5(8) + 7$

7. $6(15 - 4)$

8. $\frac{10 - 4}{1 + 2}$

9. $11 + 56 \div (2 \cdot 7)$

Write a numerical expression for each verbal phrase.

10. the quotient of fifteen and five

11. the difference of twelve and nine

Application

12. **MUSIC** Hector purchased 3 CDs for \$13 each and 2 cassette tapes for \$9 each. Write and then evaluate an expression for the total cost of the merchandise.

Practice and Apply

Homework Help

For Exercises See Examples

13–28 1
31–38 2
39–42, 47, 48 3

Extra Practice
See page 724.

Find the value of each expression.

13. $2 \cdot 6 - 8$

14. $12 - 3 \times 3$

15. $12 \div 3 + 21$

16. $9 + 18 \div 3$

17. $8 + 5(6)$

18. $4(7) - 11$

19. $\frac{15 + 9}{32 - 20}$

20. $\frac{45 - 18}{9 \div 3}$

21. $11(6 - 1)$

22. $(9 - 7) \cdot 13$

23. $56 \div (7 \cdot 2) \times 6$

24. $75 \div (7 + 8) - 3$

25. $2[5(11 - 3)] - 16$

26. $5[4 + (12 - 4) \div 2]$

27. $9[(22 - 17) + 5(1 + 2)]$

28. $10[9(2 + 4) - 6 \cdot 2]$

29. Find the value of *six added to the product of four and eleven*.
 30. What is the value of *sixty divided by the sum of two and ten*?

Write a numerical expression for each verbal phrase.

31. six minus three
 32. seven increased by two
 33. nine multiplied by five
 34. eleven more than fifteen
 35. twenty-four divided by six
 36. four less than eighteen
 37. the cost of 3 notebooks at \$6 each
 38. the total amount of CDs if Erika has 4 and Roberto has 5

GARDENING For Exercises 39 and 40, use the following information. A bag of potting soil sells for \$2, and a bag of fertilizer sells for \$13.

39. Write an expression for the total cost of 4 bags of soil and 2 bags of fertilizer.
 40. What is the total cost of the gardening supplies?

TRAVEL For Exercises 41 and 42, use the following information.

Miko is packing for a trip. The total weight of her luggage cannot exceed 200 pounds. She has 3 suitcases that weigh 57 pounds each and 2 sport bags that weigh 12 pounds each.

41. Write an expression for the total weight of the luggage.
 42. Is Miko's luggage within the 200-pound limit? Explain.

Copy each sentence. Then insert parentheses to make each sentence true.

43. $61 - 15 + 3 = 43$
 44. $12 \times 3 \div 1 + 2 = 12$
 45. $56 \div 2 + 6 - 4 = 3$
 46. $5 + 2 \cdot 9 - 3 = 42$

FOOTBALL For Exercises 47 and 48, use the table and the following information.

A national poll ranks college football teams using votes from sports reporters. Each vote is worth a certain number of points. Suppose The Ohio State University receives 50 first-place votes, 7 second-place votes, 4 fourth-place votes, and 3 tenth-place votes.

47. Write an expression for the number of points that Ohio State receives.
 48. Find the total number of points.

Number of Points for Each Vote	
Vote	Points
1 st place	25
2 nd place	24
3 rd place	23
4 th place	22
5 th place	21
⋮	⋮
25 th place	1

PUBLISHING For Exercises 49 and 50, use the following information.

An ISBN number is used to identify a published book. To determine if an ISBN number is correct, multiply each of the numbers in order by 10, 9, 8, 7, and so on. If the sum of the products can be divided by 11, with no remainder, the number is correct.

49. Find the ISBN number on the back cover of this book.
 50. Is the number correct? Explain why or why not.



More About . . .

Football

The Ohio State University Buckeyes ended the 2002 season ranked No. 1 in NCAA Division I-A college football with a 14–0 record.

Source: www.espn.com

51. **CRITICAL THINKING** Suppose only the 1, $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, $\boxed{(}$, $\boxed{)}$, and $\boxed{\text{ENTER}}$ keys on a calculator are working. How can you get a result of 75 if you are only allowed to push these keys fewer than 20 times?
52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

Why do we need to agree on an order of operations?

Include the following in your answer:

- an explanation of how the order operations are performed, and
- an explanation of what will happen to the value of an expression if the order of operations are not followed.



53. Which expression has a value of 18?
- (A) $2[2(6 - 3)] + 5$ (B) $27 \div 3 + (12 - 4)$
- (C) $(9 \times 3) - 63 \div 7$ (D) $6(3 + 2) \div (9 - 7)$
54. Identify the expression that represents *the quotient of ten and two*.
- (A) $2 \div 10$ (B) $\frac{10}{2}$ (C) 10×2 (D) $10 - 2$

Maintain Your Skills

Mixed Review Find the next term in each list. (Lesson 1-1)

55. 2, 4, 8, 16, 32, ... 56. 45, 42, 39, 36, 33, ...
57. 1, 3, 6, 10, 15, 21, ... 58. 15, 18, 22, 25, 29, ...

Solve each problem. (Lesson 1-1)

59. **BUSINESS** Mrs. Lewis is a sales associate for a computer company. She receives a salary, plus a bonus for any computer package she sells. Find Mrs. Lewis' bonus if she sells 16 computer packages.

Packages	Bonus
2	\$100
4	\$125
6	\$150
8	\$175

60. **TRAVEL** The graph shows the projected number of travelers for 2020. How many more people will travel to the United States than to Spain?

61. **SPACE SHUTTLE** The space shuttle can carry a payload of about 65,000 pounds. If a compact car weighs about 2450 pounds, about how many compact cars can the space shuttle carry?



Getting Ready for the Next Lesson **BASIC SKILL** Find each sum.

62. $18 + 34$ 63. $85 + 41$
64. $342 + 50$ 65. $535 + 28$



1-3

Variables and Expressions

What You'll Learn

- Evaluate expressions containing variables.
- Translate verbal phrases into algebraic expressions.

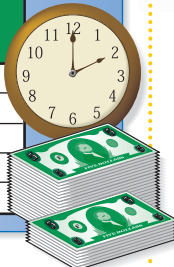
Vocabulary

- variable
- algebraic expression
- defining a variable

How are variables used to show relationships?

A baby-sitter earns \$5 per hour. The table shows several possibilities for number of hours and earnings.

Number of Hours	Money Earned
2	$5 \cdot 2$ or 10
5	$5 \cdot 5$ or 25
8	$5 \cdot 8$ or 40
11	$5 \cdot 11$ or 55
h	?



- Suppose the baby-sitter worked 10 hours. How much would he or she earn?
- What is the relationship between the number of hours and the money earned?
- If h represents *any number of hours*, what expression could you write to represent the amount of money earned?

Reading Math**Variable**

Root Word: Vary

The word *variable* means *likely to change or vary*.

EVALUATE EXPRESSIONS Algebra is a language of symbols. One symbol that is frequently used is a variable. A **variable** is a placeholder for any value. As shown above, h represents some *unknown number of hours*.

Any letter can be used as a variable. Notice the special notation for multiplication and division with variables.

The letter x is most often used as a variable.

$$x + 2$$

$4h$ means $4 \times h$.
 mn means $m \times n$.

$$4h - 5$$

$$mn$$

$\frac{y}{3}$ means $y \div 3$.

$$\frac{y}{3}$$

An expression like $x + 2$ is an **algebraic expression** because it contains sums and/or products of variables and numbers.

✓ Concept Check True or false: $2x$ is an example of an algebraic expression. Explain your reasoning.

To evaluate an algebraic expression, replace the variable or variables with known values and then use the order of operations.

Example 1 Evaluate Expressions

Evaluate $x + y - 9$ if $x = 15$ and $y = 26$.

$$\begin{aligned} x + y - 9 &= 15 + 26 - 9 && \text{Replace } x \text{ with } 15 \text{ and } y \text{ with } 26. \\ &= 41 - 9 && \text{Add } 15 \text{ and } 26. \\ &= 32 && \text{Subtract } 9 \text{ from } 41. \end{aligned}$$

Replacing a variable with a number demonstrates the **Substitution Property of Equality**.

Key Concept

Substitution Property of Equality

- **Words** If two quantities are equal, then one quantity can be replaced by the other.
- **Symbols** For all numbers a and b , if $a = b$, then a may be replaced by b .

Example 2 Evaluate Expressions

Evaluate each expression if $k = 2$, $m = 7$, and $n = 4$.

a. $6m - 3k$

$$\begin{aligned} 6m - 3k &= 6(7) - 3(2) && \text{Replace } m \text{ with } 7 \text{ and } k \text{ with } 2. \\ &= 42 - 6 && \text{Multiply.} \\ &= 36 && \text{Subtract.} \end{aligned}$$

b. $\frac{mn}{2}$

$$\begin{aligned} \frac{mn}{2} &= mn \div 2 && \text{Rewrite as a division expression.} \\ &= (7 \cdot 4) \div 2 && \text{Replace } m \text{ with } 7 \text{ and } n \text{ with } 4. \\ &= 28 \div 2 && \text{Multiply.} \\ &= 14 && \text{Divide.} \end{aligned}$$

c. $n + (k + 5m)$

$$\begin{aligned} n + (k + 5m) &= 4 + (2 + 5 \cdot 7) && \text{Replace } n \text{ with } 4, k \text{ with } 2, \text{ and } m \text{ with } 7. \\ &= 4 + (2 + 35) && \text{Multiply } 5 \text{ and } 7. \\ &= 4 + 37 && \text{Add } 2 \text{ and } 35. \\ &= 41 && \text{Add } 4 \text{ and } 37. \end{aligned}$$

TRANSLATE VERBAL PHRASES The first step in translating verbal phrases into algebraic expressions is to choose a variable and a quantity for the variable to represent. This is called **defining a variable**.

Example 3 Translate Verbal Phrases into Expressions

Translate each phrase into an algebraic expression.

a. twelve points more than the Dolphins scored

Words twelve points more than the Dolphins scored

Variable Let p represent the points the Dolphins scored.

Expression $\underbrace{\text{twelve points}}_{12} \underbrace{\text{more than}}_{+} \underbrace{\text{the Dolphins scored}}_p$ The expression is $p + 12$.

b. four times a number decreased by 6

Words four times a number decreased by 6

Variable Let n represent the number.

Expression $\underbrace{\text{four times a number}}_{4n} \underbrace{\text{decreased by}}_{-} \underbrace{\text{six}}_6$ The expression is $4n - 6$.

Study Tip

Look Back

To review **key words and phrases**, see p. 11.

More About . . .



Soccer

Soccer is the fastest growing and most popular sport in the world. It is estimated that more than 100,000,000 people in more than 150 countries play soccer.

Source: *The World Almanac for Kids*

Algebraic expressions can be used to represent real-world situations.

Example 4 Use an Expression to Solve a Problem

SOCCKER The Johnstown Soccer League ranks each team in their league using points. A team gets three points for a win and one point for a tie.

- a. Write an expression that can be used to find the total number of points a team receives.

Words three points for a win and one point for a tie

Variables Let w = number of wins and t = number of ties.

Expression $\underbrace{\text{three points for a win}}_{3w} \quad \text{and} \quad \underbrace{\text{one point for a tie}}_{1t}$
 $3w + 1t$

The expression $3w + 1t$ can be used to find the total number of points a team will receive.

- b. Suppose in one season, the North Rockets had 17 wins and 4 ties. How many points did they receive?

$$\begin{aligned} 3w + 1t &= 3(17) + 1(4) && \text{Replace } w \text{ with } 17 \text{ and } t \text{ with } 4. \\ &= 51 + 4 && \text{Multiply.} \\ &= 55 && \text{Add.} \end{aligned}$$

The North Rockets received 55 points.

Check for Understanding

Concept Check

- OPEN ENDED** Give two examples of an algebraic expression and two examples of expressions that are *not* algebraic.
- Define *variable*.
- Write an expression that is the same as $4cd$.

Guided Practice

ALGEBRA Evaluate each expression if $a = 5$, $b = 12$, and $c = 4$.

- $b + 6$
- $18 - 3c$
- $\frac{2b}{8}$
- $5a - (b - c)$

ALGEBRA Translate each phrase into an algebraic expression.

- eight more than the amount Kira saved
- five goals less than the Pirates scored
- the quotient of a number and four, minus five
- seven increased by the quotient of a number and eight

Application

- SPACE** Due to gravity, objects weigh three times as much on Earth as they do on Mercury.
 - Suppose the weight of an object on Mercury is w . Write an expression for the object's weight on Earth.
 - How much would an object weigh on Earth if it weighs 25 pounds on Mercury?



Practice and Apply

Homework Help

For Exercises	See Examples
13–32, 43, 44	1, 2
33–42	3
48–50	4

Extra Practice
See page 724.

ALGEBRA Evaluate each expression if $x = 7$, $y = 3$, and $z = 9$.

- | | | |
|--------------------------|--------------------------|-------------------------|
| 13. $z + 2$ | 14. $5 + x$ | 15. $2 + 4z$ |
| 16. $15 - 2x$ | 17. $\frac{6y}{z}$ | 18. $\frac{9x}{y}$ |
| 19. $\frac{xy}{3} + 2$ | 20. $10 - \frac{xz}{9}$ | 21. $4z - 3y$ |
| 22. $3x - 2y$ | 23. $2x + 3z + 5y$ | 24. $5z - 3x - 2y$ |
| 25. $7z - (y + x)$ | 26. $(8y + 5) - 2z$ | 27. $3y + (7z - 4x)$ |
| 28. $6x - (z - 2y) + 15$ | 29. $2x + (4z - 13) - 5$ | 30. $(9 - 3y) + 4z - 5$ |

SCIENCE For Exercises 31 and 32, use the following information.

The number of times a cricket chirps can be used to estimate the temperature in degrees Fahrenheit. Use $c \div 4 + 37$ where c is the number of chirps in one minute.

- Find the approximate temperature if a cricket chirps 136 times in a minute.
- What is the temperature if a cricket chirps 100 times in a minute?

ALGEBRA Translate each phrase into an algebraic expression.

- Mark's salary plus a \$200 bonus
 - three more than the number of cakes baked
 - six feet shorter than the mountain's height
 - two seconds faster than Sarah's time
 - five times a number, minus four
 - seven less than a number times eight
 - nine more than a number divided by six
 - the quotient of eight and twice a number
 - the difference of seventeen and four times a number
 - three times the product of twenty-five and a number
43. Evaluate $\frac{10mn}{3p-3}$ if $m = 6$, $n = 3$, and $p = 7$.
44. What is the value of $\frac{3(4a-3b)}{b-4}$ if $a = 6$ and $b = 7$?

ALGEBRA Write an algebraic expression that represents the relationship in each table.

45.

Age Now	Age in Three Years
10	13
12	15
15	18
20	23
x	■

46.

Number of Items	Total Cost
5	25
6	30
8	40
10	50
n	■

47.

Regular Price	Sale Price
\$12	8
\$15	11
\$18	14
\$24	20
$\$p$	■



48. **BUSINESS** Cornet Cable charges \$32.50 a month for basic cable television. Each premium channel selected costs an additional \$4.95 per month. Write an expression to find the cost of a month of cable service.

SALES For Exercises 49 and 50, use the following information.

The selling price of a sweater is the cost plus the markup minus the discount.

49. Write an expression to show the selling price s of a sweater. Use c for cost, m for markup, and d for discount.
50. Suppose the cost of a sweater is \$25, the markup is \$20, and the discount is \$6. What is the selling price of the sweater?
51. **CRITICAL THINKING** What value of t makes the expressions $6t$, $t + 5$, and $2t + 4$ equal?
52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are variables used to show relationships?

Include the following in your answer:

- an explanation of variables and what they represent, and
- an example showing how variables are used to show relationships.



53. If the value of $c + 5$ is 18, what is the value of c ?
 Ⓐ 3 Ⓑ 8 Ⓒ 7 Ⓓ 13
54. Which expression represents *four less than twice a number*?
 Ⓐ $4n - 2$ Ⓑ $2n - 4$ Ⓒ $4(2 + n)$ Ⓓ $2n + 4$

Maintain Your Skills

Mixed Review Find the value of each expression. (Lesson 1-2)

55. $3 + (6 \times 2) - 8$ 56. $5(16 - 5 \times 3)$ 57. $36 \div (9 \cdot 2) + 7$

58. **FOOD** The table shows the amount in pounds of certain types of pasta sold in a recent year. About how many million pounds of these types of pasta were sold? (Lesson 1-1)

Pasta	Amount (millions)
Spaghetti	308
Elbow	121
Noodles	70
Twirl	52
Penne	51
Lasagna	35
Fettuccine	24

Source: National Pasta Association

Getting Ready for the Next Lesson **BASIC SKILL** Find each difference.

59. $53 - 17$ 60. $97 - 28$ 61. $104 - 82$ 62. $152 - 123$

Practice Quiz 1

Lessons 1-1 through 1-3

1. What is the next term in the list 4, 5, 7, 10, ...? (Lesson 1-1)

Find the value of each expression. (Lesson 1-2)

2. $28 \div 4 \times 2$ 3. $7(3 + 10) - 2 \cdot 6$ 4. $3[6(12 - 3)] - 17$
5. Evaluate $7x - 3y$ if $x = 4$ and $y = 2$. (Lesson 1-3)



Spreadsheet Investigation

A Follow-Up of Lesson 1-3

Expressions and Spreadsheets

One of the most common computer applications is a spreadsheet program. A **spreadsheet** is a table that performs calculations. It is organized into boxes called **cells**, which are named by a letter and a number. In the spreadsheet below, cell B1 is highlighted.

An advantage of using a spreadsheet is that values in the spreadsheet are recalculated when a number is changed. You can use a spreadsheet to investigate patterns in data.

Example

Here's a mind-reading trick! Think of a number. Then double it, add six, divide by two, and subtract the original number. What is the result?

You can use a spreadsheet to test different numbers. Suppose we start with the number 10.

	A	B	C
1	Think of a number.	10	10
2	Double it.	$2*B1$	20
3	Add 6.	$B2+6$	26
4	Divide by 2.	$B3/2$	13
5	Subtract the original number.	$B4-B1$	3
6			

The spreadsheet takes the value in B1, doubles it, and enters the value in B2. Note the * is the symbol for multiplication.

The spreadsheet takes the value in B3, divides by 2, and enters the value in B4. Note that / is the symbol for division.

The result is 3.

Exercises

To change information in a spreadsheet, move the cursor to the cell you want to access and click the mouse. Then type in the information and press Enter. Find the result when each value is entered in B1.

- 6
- 8
- 25
- 100
- 1500

Make a Conjecture

- What is the result if a decimal is entered in B1? a negative number?
- Explain why the result is always 3.
- Make up your own mind-reading trick. Enter it into a spreadsheet to show that it works.

1-4 Properties

What You'll Learn

- Identify and use properties of addition and multiplication.
- Use properties of addition and multiplication to simplify algebraic expressions.

Vocabulary

- properties
- counterexample
- simplify
- deductive reasoning

How are real-life situations commutative?

Abraham Lincoln delivered the Gettysburg Address more than 130 years ago. The table lists the number of words in certain historic documents.

Historical Document	Words
Preamble to The U.S. Constitution	52
Mayflower Compact	196
Atlantic Charter	375
Gettysburg Address (Nicolay Version)	238

Source: U.S. Historical Documents Archive

- Suppose you read the Preamble to The U.S. Constitution first and then the Gettysburg Address. Write an expression for the total number of words read.
- Suppose you read the Gettysburg Address first and then the Preamble to the U.S. Constitution. Write an expression for the total number of words read.
- Find the value of each expression. What do you observe?
- Does it matter in which order you add any two numbers? Why or why not?

PROPERTIES OF ADDITION AND MULTIPLICATION In algebra, **properties** are statements that are true for any numbers. For example, the expressions $3 + 8$ and $8 + 3$ have the same value, 11. This illustrates the **Commutative Property of Addition**. Likewise, $3 \cdot 8$ and $8 \cdot 3$ have the same value, 24. This illustrates the **Commutative Property of Multiplication**.

Reading Math

Commutative

Root Word: Commute

The everyday meaning of the word *commute* means to change or exchange.


Key Concept

Commutative Property of Addition

- **Words** The order in which numbers are added does not change the sum.
- **Symbols** For any numbers a and b , $a + b = b + a$.
- **Example** $2 + 3 = 3 + 2$
 $5 = 5$

Commutative Property of Multiplication

- **Words** The order in which numbers are multiplied does not change the product.
- **Symbols** For any numbers a and b , $a \cdot b = b \cdot a$.
- **Example** $2 \cdot 3 = 3 \cdot 2$
 $6 = 6$

 **Concept Check** Write an example that shows the Commutative Property of Multiplication.

Associative

Root Word: Associate

The word *associate* means to join together, connect, or combine.

When evaluating expressions, it is often helpful to group or *associate* the numbers. The **Associative Property** says that the way in which numbers are grouped when added or multiplied does not change the sum or the product.

The Associative Property also holds true when multiplying numbers.


Key Concept

Associative Property of Addition

- **Words** The way in which numbers are grouped when added does not change the sum.
- **Symbols** For any numbers a , b , and c ,
 $(a + b) + c = a + (b + c)$.
- **Example** $(5 + 8) + 2 = 5 + (8 + 2)$
 $13 + 2 = 5 + 10$
 $15 = 15$

Associative Property of Multiplication

- **Words** The way in which numbers are grouped when multiplied does not change the product.
- **Symbols** For any numbers a , b , and c ,
 $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
- **Example** $(4 \cdot 6) \cdot 3 = 4 \cdot (6 \cdot 3)$
 $24 \cdot 3 = 4 \cdot 18$
 $72 = 72$

 **Concept Check** Write an example showing the Associative Property of Addition.

The following properties are also true.

Key Concept

Properties of Numbers

Property	Words	Symbols	Examples
Additive Identity	When 0 is added to any number, the sum is the number.	For any number a , $a + 0 = 0 + a = a$.	$5 + 0 = 5$ $0 + 9 = 9$
Multiplicative Identity	When any number is multiplied by 1, the product is the number.	For any number a , $a \cdot 1 = 1 \cdot a = a$.	$7 \cdot 1 = 7$ $1 \cdot 6 = 6$
Multiplicative Property of Zero	When any number is multiplied by 0, the product is 0.	For any number a , $a \cdot 0 = 0 \cdot a = 0$.	$4 \cdot 0 = 0$ $0 \cdot 2 = 0$

Example 1 Identify Properties

Name the property shown by each statement.

a. $3 + 7 + 9 = 7 + 3 + 9$

The order of the numbers changed. This is the Commutative Property of Addition.

b. $(a \cdot 6) \cdot 5 = a \cdot (6 \cdot 5)$

The grouping of the numbers and variables changed. This is the Associative Property of Multiplication.

c. $0 \cdot 12 = 0$

The number was multiplied by zero. This is the Multiplicative Property of Zero.

You can use the properties of numbers to find sums and products mentally. Look for sums or products that end in zero.

Example 2 Mental Math

Find $4 \cdot (25 \cdot 11)$ mentally.

Group 4 and 25 together because $4 \cdot 25 = 100$. It is easy to multiply by 100 mentally.

$$\begin{aligned} 4 \cdot (25 \cdot 11) &= (4 \cdot 25) \cdot 11 && \text{Associative Property of Addition} \\ &= 100 \cdot 11 && \text{Multiply 4 and 25 mentally.} \\ &= 1100 && \text{Multiply 100 and 11 mentally.} \end{aligned}$$

Study Tip

Counterexample

You can disprove a statement by finding only one counterexample.

You may wonder whether these properties apply to subtraction. One way to find out is to look for a counterexample. A **counterexample** is an example that shows a conjecture is not true.

Example 3 Find a Counterexample

State whether the following conjecture is *true* or *false*. If false, provide a counterexample.

Subtraction of whole numbers is associative.

Write two subtraction expressions using the Associative Property, and then check to see whether they are equal.

$$\begin{aligned} 9 - (5 - 3) &\stackrel{?}{=} (9 - 5) - 3 && \text{State the conjecture.} \\ 9 - 2 &\stackrel{?}{=} 4 - 3 && \text{Simplify within the parentheses.} \\ 7 &\neq 1 && \text{Subtract.} \end{aligned}$$

We found a counterexample. That is, $9 - (5 - 3) \neq (9 - 5) - 3$. So, subtraction is *not* associative. The conjecture is false.

ALGEBRA CONNECTION

SIMPLIFY ALGEBRAIC EXPRESSIONS To **simplify** algebraic expressions means to write them in a simpler form. You can use the Associative or Commutative Properties to simplify expressions.

Example 4 Simplify Algebraic Expressions

Simplify each expression.

a. $(k + 2) + 7$

$$\begin{aligned} (k + 2) + 7 &= k + (2 + 7) && \text{Associative Property of Addition} \\ &= k + 9 && \text{Substitution Property of Equality; } 2 + 7 = 9 \end{aligned}$$

b. $5 \cdot (d \cdot 9)$

$$\begin{aligned} 5 \cdot (d \cdot 9) &= 5 \cdot (9 \cdot d) && \text{Commutative Property of Multiplication} \\ &= (5 \cdot 9)d && \text{Associative Property of Multiplication} \\ &= 45d && \text{Substitution Property of Equality; } 5 \cdot 9 = 45 \end{aligned}$$

Study Tip

Inductive Reasoning

In inductive reasoning, conclusions are made based on past events or patterns.

Notice that each step in Example 4 was justified by a property. The process of using facts, properties, or rules to justify reasoning or reach valid conclusions is called **deductive reasoning**.

Check for Understanding

Concept Check

- OPEN ENDED** Write a numerical sentence that illustrates the Commutative Property of Multiplication.
- Tell the difference between the Commutative and Associative Properties.
- FIND THE ERROR** Kimberly and Carlos are using the Associative Properties of Addition and Multiplication to rewrite expressions.

Kimberly

$$(4 + 3) + 6 = 4 + (3 + 6)$$

Carlos

$$(2 + 7) \cdot 5 = 2 + (7 \cdot 5)$$

Who is correct? Explain your reasoning.

Guided Practice

Name the property shown by each statement.

4. $7 + 5 = 5 + 7$

5. $8 + 0 = 8$

6. $8 \cdot 4 \cdot 13 = 4 \cdot 8 \cdot 13$

Find each sum or product mentally.

7. $13 + 8 + 7$

8. $6 \cdot 9 \cdot 5$

9. $8 + 11 + 22 + 4$

10. State whether the conjecture *division of whole numbers is commutative* is true or false. If false, provide a counterexample.

ALGEBRA Simplify each expression.

11. $6 + (n + 7)$

12. $(3 \cdot w) \cdot 9$

Application

13. **SHOPPING** Denyce purchased a pair of jeans for \$26, a T-shirt for \$12, and a pair of socks for \$4. What is the total cost of the items? Explain how the Commutative Property of Addition can be used to find the total.

Practice and Apply

Homework Help

For Exercises

See Examples

14–25	1
26–34	2
35–37	3
39–47	4

Extra Practice
See page 725.

Name the property shown by each statement.

14. $5 \cdot 3 = 3 \cdot 5$

15. $1 \cdot 4 = 4$

16. $6 \cdot 2 \cdot 0 = 0$

17. $12 \cdot 8 = 8 \cdot 12$

18. $0 + 13 = 13 + 0$

19. $(4 + 5) + 15 = 4 + (5 + 15)$

20. $1h = h$

21. $7k + 0 = 7k$

22. $(5 + x) + 6 = 5 + (x + 6)$

23. $4(mn) = (4m)(n)$

24. $9(gh) = (9g)h$

25. $(3a + b) + 2c = 2c + (3a + b)$

Find each sum or product mentally.

26. $11 + 8 + 19$

27. $17 + 5 + 33$

28. $15 \cdot 0 \cdot 2$

29. $5 + 18 + 15 + 2$

30. $2 \cdot 7 \cdot 30$

31. $11 \cdot 9 \cdot 10$

32. $23 + 3 + 17 + 7$

33. $125 \cdot 4 \cdot 0$

34. $16 + 57 + 94 + 33$

State whether each conjecture is *true* or *false*. If false, provide a counterexample.

35. Division of whole numbers is associative.

36. The sum of two whole numbers is always greater than either addend.

37. Subtraction of whole numbers is commutative.

More About . . .



Science

A water molecule is composed of two atoms of hydrogen and one atom of oxygen. Thus, in chemistry, the formula for water is H_2O .

Source: Merrill Chemistry

- 38. **SCIENCE** In chemistry, water is used to dilute acid. Since pouring water into acid could cause spattering and burns, it's important to pour the acid into the water. Is combining acid and water commutative? Explain.

ALGEBRA Simplify each expression.

39. $(m + 8) + 4$ 40. $(17 + p) + 9$ 41. $15 + (12 + a)$
42. $21 + (k + 16)$ 43. $6 \cdot (y \cdot 2)$ 44. $7 \cdot (d \cdot 4)$
45. $(6 \cdot c) \cdot 8$ 46. $(3 \cdot w) \cdot 5$ 47. $25s(3)$

48. **CRITICAL THINKING** The **Closure Property** states that because the sum or product of two whole numbers (0, 1, 2, 3, ...) is also a whole number, the set of whole numbers is *closed* under addition and multiplication. Tell whether the set of whole numbers is closed under subtraction and division. If not, give counterexamples.

49. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are real-life situations commutative?

Include the following in your answer:

- an example of a real-life situation that is commutative,
- an example of a real-life situation that is not commutative, and
- an explanation of why each situation is or is not commutative.

Standardized Test Practice

A B C D

50. The statement $e + (f + g) = (f + g) + e$ is an example of which property of addition?
- (A) Commutative (B) Associative
(C) Identity (D) Substitution
51. Rewrite the expression $(7 \cdot m) \cdot 8$ using the Associative Property.
- (A) $(8 \cdot 7) \cdot m$ (B) $7 \cdot (m \cdot 8)$
(C) $8 \cdot (7 \cdot m)$ (D) $7 \cdot m \cdot 8$

Maintain Your Skills

Mixed Review ALGEBRA Evaluate each expression if $a = 6$, $b = 4$, and $c = 5$. (Lesson 1-3)

52. $a + c - b$ 53. $8a - 3b$ 54. $4a - (b + c)$

55. Translate the phrase *the difference of w and 12* into an algebraic expression. (Lesson 1-3)

Find the value of each expression. (Lesson 1-2)

56. $7 - 2 \times 3$ 57. $21 \div 3 \times 5$ 58. $4 \cdot (8 + 9) + 6$

59. Find the next two terms in the list 0, 1, 3, 6, 10, ... (Lesson 1-1)

Getting Ready for the Next Lesson

BASIC SKILL Find each product.

60. 48×5 61. 8×37 62. 16×12
63. 25×42 64. 106×13 65. 59×127

What You'll Learn

- Identify and solve open sentences.
- Translate verbal sentences into equations.

Vocabulary

- equation
- open sentence
- solution
- solving the equation

How is solving an open sentence similar to evaluating an expression?

Emilio is seven years older than his sister Rebecca.

- a. If Rebecca is x years old, what expression represents Emilio's age?

Suppose Emilio is 19 years old. You can write a mathematical sentence that shows two expressions are equal.

Words Emilio's age is 19.

Symbols $x + 7 = 19$

- b. What two expressions are equal?
c. If Emilio is 19, how old is Rebecca?

EQUATIONS AND OPEN SENTENCES A mathematical sentence that contains an equals sign (=) is called an **equation**. A few examples are shown.

$$5 + 9 = 14$$

$$2(6) - 3 = 9$$

$$x + 7 = 19$$

$$2m - 1 = 13$$

An equation that contains a variable is an **open sentence**. An open sentence is neither true nor false. When the variable in an open sentence is replaced with a number, you can determine whether the sentence is true or false.

$$\begin{array}{l} x + 7 = 19 \\ 11 + 7 \neq 19 \\ 18 \neq 19 \end{array}$$

Replace x with 11.

false

When $x = 11$, this sentence is false.

$$\begin{array}{l} x + 7 = 19 \\ 12 + 7 = 19 \\ 19 = 19 \end{array}$$

Replace x with 12.

true

When $x = 12$, this sentence is true.

A value for the variable that makes an equation true is called a **solution**. For $x + 7 = 19$, the solution is 12. The process of finding a solution is called **solving the equation**.

Example 1 Solve an Equation

Find the solution of $12 - m = 8$. Is it 2, 4, or 7?

Replace m with each value.

Value for m	$12 - m = 8$	True or False?
2	$12 - 2 \neq 8$	false
4	$12 - 4 = 8$	true ✓
7	$12 - 7 \neq 8$	false

Therefore, the solution of $12 - m = 8$ is 4.

Study Tip**Symbols**

The symbol \neq means is not equal to.

Most standardized tests include questions that ask you to solve equations.



Example 2 Solve an Equation

Multiple-Choice Test Item

Which value is the solution of $2x + 1 = 7$?

(A) 6

(B) 5

(C) 4

(D) 3

Read the Test Item

The *solution* is the value that makes the equation true.

Solve the Test Item Test each value.

$$2x + 1 = 7$$

$$2(6) + 1 = 7 \quad \text{Replace } x \text{ with } 6.$$

$$13 \neq 7$$

$$2x + 1 = 7$$

$$2(5) + 1 = 7 \quad \text{Replace } x \text{ with } 5.$$

$$11 \neq 7$$

$$2x + 1 = 7$$

$$2(4) + 1 = 7 \quad \text{Replace } x \text{ with } 4.$$

$$9 \neq 7$$

$$2x + 1 = 7$$

$$2(3) + 1 = 7 \quad \text{Replace } x \text{ with } 3.$$

$$7 = 7 \quad \checkmark$$

Since 3 makes the equation true, the answer is D.

Test-Taking Tip

The strategy of testing each value is called *backsolving*. You can also use this strategy with complex equations.

Example 3 Solve Simple Equations Mentally

Solve each equation mentally.

a. $5x = 30$

$$5 \cdot 6 = 30 \quad \text{Think: What number times 5 is 30?}$$

$$x = 6 \quad \text{The solution is 6.}$$

b. $\frac{72}{d} = 8$

$$\frac{72}{9} = 8 \quad \text{Think: 72 divided by what number is 8?}$$

$$d = 9 \quad \text{The solution is 9.}$$

In Lesson 1-4, you learned that certain properties are true for any number. Two properties of equality are shown below.

Key Concept		Properties of Equality	
Property	Words	Symbols	Example
Symmetric	If one quantity equals a second quantity, then the second quantity also equals the first.	For any numbers a and b , if $a = b$, then $b = a$.	If $10 = 4 + 6$, then $4 + 6 = 10$.
Transitive	If one quantity equals a second quantity and the second quantity equals a third quantity, then the first equals the third.	For any numbers a , b , and c , if $a = b$ and $b = c$, then $a = c$.	If $3 + 5 = 8$ and $8 = 2(4)$, then $3 + 5 = 2(4)$.

Example 4 Identify Properties of Equality

Name the property of equality shown by each statement.

a. If $5 = x + 2$, then $x + 2 = 5$.

If $a = b$, then $b = a$. This is the Symmetric Property of Equality.

b. If $y + 8 = 15$ and $15 = 7 + 8$, then $y + 8 = 7 + 8$.

If $a = b$ and $b = c$, then $a = c$. This is the Transitive Property of Equality.

TRANSLATE VERBAL SENTENCES INTO EQUATIONS Just as verbal phrases can be translated into algebraic expressions, verbal sentences can be translated into equations and then solved.

Example 5 Translate Sentences Into Equations

The difference of a number and ten is seventeen. Find the number.

Words The difference of a number and ten is seventeen.

Variables Let $n =$ the number. Define the variable.

The difference of a number and ten is seventeen.
 $n - 10 = 17$

Equation

$n - 10 = 17$ Write the equation.

$27 - 10 = 17$ Think: What number minus 10 is 17?

$n = 27$ The solution is 27.

Check for Understanding

Concept Check

- OPEN ENDED** Write two different equations whose solutions are 5.
- Tell what it means to *solve an equation*.

Guided Practice

ALGEBRA Find the solution of each equation from the list given.

3. $h + 15 = 21$; 5, 6, 7

4. $13 - m = 4$; 7, 8, 9

ALGEBRA Solve each equation mentally.

5. $a + 8 = 13$

6. $12 - d = 9$

7. $3x = 18$

8. $4 = \frac{36}{t}$

Name the property of equality shown by each statement.

9. If $x + 4 = 9$, then $9 = x + 4$.

10. If $5 + 7 = 12$ and $12 = 3 \cdot 4$, then $5 + 7 = 3 \cdot 4$.

ALGEBRA Define a variable. Then write an equation and solve.

11. A number increased by 8 is 23.

12. Twenty-five is 10 less than a number.

13. Find the value that makes $6 = \frac{48}{k}$ true.

(A) 6

(B) 7

(C) 8

(D) 12

Standardized Test Practice

Practice and Apply

Homework Help

For Exercises	See Examples
14–23	1
26–41	3
42–49, 54, 55	5
50–53	4

Extra Practice
See page 725.

ALGEBRA Find the solution of each equation from the list given.

- | | |
|-------------------------------------|-------------------------------------|
| 14. $c + 12 = 30$; 8, 16, 18 | 15. $g + 17 = 28$; 9, 11, 13 |
| 16. $23 - m = 14$; 7, 9, 11 | 17. $18 - k = 6$; 8, 10, 12 |
| 18. $14k = 42$; 2, 3, 4 | 19. $75 = 15n$; 3, 4, 5 |
| 20. $\frac{51}{z} = 3$; 15, 16, 17 | 21. $\frac{60}{p} = 4$; 15, 16, 17 |
22. What is the solution of $3n + 13 = 25$; 2, 3, 4?
23. Find the solution of $7 = 4w - 29$. Is it 8, 9, or 10?

Tell whether each sentence is *sometimes*, *always*, or *never* true.

24. An equation is an open sentence.
25. An open sentence contains a variable.

ALGEBRA Solve each equation mentally.

- | | | | |
|------------------------|------------------------|-------------------------|-------------------------|
| 26. $d + 7 = 12$ | 27. $19 = 4 + y$ | 28. $8 + j = 27$ | 29. $22 + b = 22$ |
| 30. $20 - p = 11$ | 31. $15 - m = 0$ | 32. $16 = x - 7$ | 33. $12 = y - 5$ |
| 34. $7s = 49$ | 35. $8c = 88$ | 36. $63 = 9h$ | 37. $72 = 8w$ |
| 38. $\frac{30}{r} = 3$ | 39. $\frac{24}{y} = 8$ | 40. $12 = \frac{36}{p}$ | 41. $14 = \frac{56}{d}$ |

ALGEBRA Define a variable. Then write an equation and solve.

42. The sum of 7 and a number is 23.
43. A number minus 10 is 27.
44. Twenty-four is the product of 8 and a number.
45. The sum of 9 and a number is 36.
46. The difference of a number and 12 is 54.
47. A number times 3 is 45.

MOVIE INDUSTRY For Exercises 48 and 49, use the following information. Megan purchased movie tickets for herself and two friends. The cost was \$24.

48. Define a variable. Then write an equation that can be used to find how much Megan paid for each ticket.
49. What was the cost of each ticket?

Name the property of equality shown by each statement.

50. If $2 + 3 = 5$ and $5 = 1 + 4$, then $2 + 3 = 1 + 4$.
51. If $3 + 4 = 7$ then $7 = 3 + 4$.
52. If $(1 + 2) + 6 = 9$, then $9 = (1 + 2) + 6$.
53. If $m + n = p$, then $p = m + n$.

HEIGHT For Exercises 54 and 55, use the following information. Sean grew from a height of 65 inches to a height of 68 inches.

54. Define a variable. Then write an equation that can be used to find the increase in height.
55. How many inches did Sean grow?

More About . . .



Movie Industry

In 1990, the total number of indoor movie screens was about 22,000. Today, there are over 37,000 indoor movie screens and the number keeps rising.

Source: National Association of Theatre Owners

1-6

Ordered Pairs and Relations

What You'll Learn

- Use ordered pairs to locate points.
- Use tables and graphs to represent relations.

Vocabulary

- coordinate system
- y-axis
- coordinate plane
- origin
- x-axis
- ordered pair
- x-coordinate
- y-coordinate
- graph
- relation
- domain
- range

How are ordered pairs used to graph real-life data?

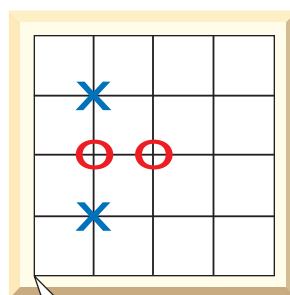
Maria and Hiroshi are playing a game. The player who gets four Xs or Os in a row wins.

1st move Maria places an X at 1 over and 3 up.

2nd move Hiroshi places an O at 2 over and 2 up.

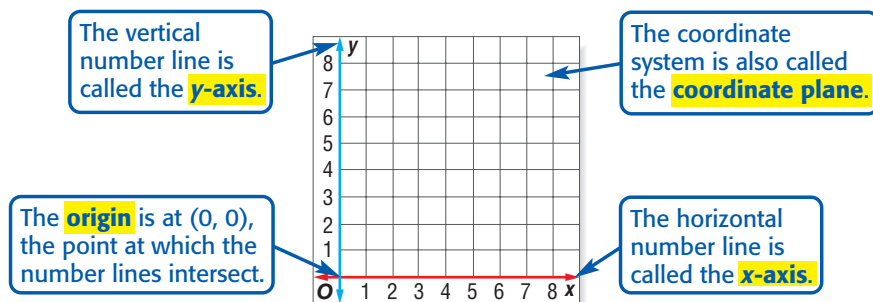
3rd move Maria places an X at 1 over and 1 up.

4th move Hiroshi places an O at 1 over and 2 up.

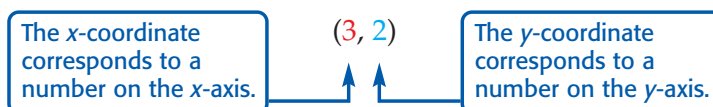


- Where should Maria place an X now? Explain your reasoning.
- Suppose (1, 2) represents 1 over and 2 up. How could you represent 3 over and 2 up?
- How are (5, 1) and (1, 5) different?
- Where is a good place to put the next O?
- Work with a partner to finish the game.

ORDERED PAIRS In mathematics, a **coordinate system** is used to locate points. The coordinate system is formed by the intersection of two number lines that meet at right angles at their zero points.



An **ordered pair** of numbers is used to locate any point on a coordinate plane. The first number is called the **x-coordinate**. The second number is called the **y-coordinate**.



To **graph** an ordered pair, draw a dot at the point that corresponds to the ordered pair. The coordinates are your directions to locate the point.

Study Tip

Coordinate System

You can assume that each unit on the x - and y -axis represents 1 unit. *Axes* is the plural of *axis*.

Example 1 Graph Ordered Pairs

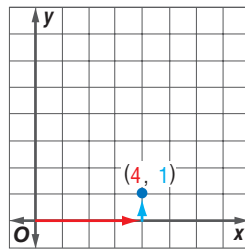
Graph each ordered pair on a coordinate system.

a. $(4, 1)$

Step 1 Start at the origin.

Step 2 Since the x -coordinate is 4, move 4 units to the right.

Step 3 Since the y -coordinate is 1, move 1 unit up. Draw a dot.

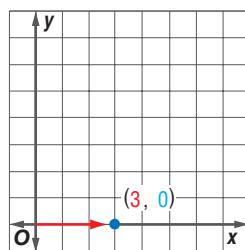


b. $(3, 0)$

Step 1 Start at the origin.

Step 2 The x -coordinate is 3. So, move 3 units to the right.

Step 3 Since the y -coordinate is 0, you will not need to move up. Place the dot on the axis.



Concept Check Where is the graph of $(0, 4)$ located?

Sometimes a point on a graph is named by using a letter. To identify its location, you can write the ordered pair that represents the point.

Example 2 Identify Ordered Pairs

Write the ordered pair that names each point.

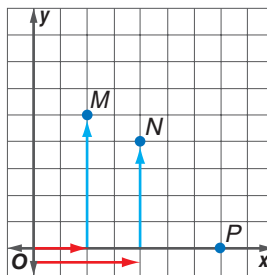
a. M

Step 1 Start at the origin.

Step 2 Move right on the x -axis to find the x -coordinate of point M , which is 2.

Step 3 Move up the y -axis to find the y -coordinate, which is 5.

The ordered pair for point M is $(2, 5)$.



b. N

The x -coordinate of N is 4, and the y -coordinate is 4.

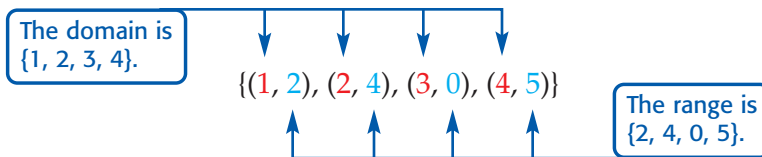
The ordered pair for point N is $(4, 4)$.

c. P

The x -coordinate of P is 7, and the y -coordinate is 0.

The ordered pair for point P is $(7, 0)$.

RELATIONS A set of ordered pairs such as $\{(1, 2), (2, 4), (3, 0), (4, 5)\}$ is a **relation**. The **domain** of the relation is the set of x -coordinates. The **range** of the relation is the set of y -coordinates.



A relation can be shown in several ways.

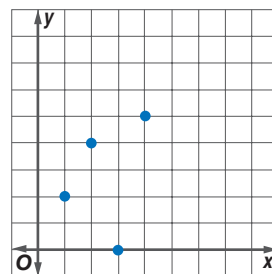
Ordered Pairs

- (1, 2)
- (2, 4)
- (3, 0)
- (4, 5)

Table

x	y
1	2
2	4
3	0
4	5

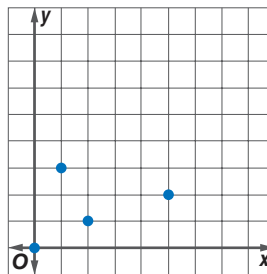
Graph



Example 3 *Relations as Tables and Graphs*

Express the relation $\{(0, 0), (2, 1), (1, 3), (5, 2)\}$ as a table and as a graph. Then determine the domain and range.

x	y
0	0
2	1
1	3
5	2



The domain is $\{0, 2, 1, 5\}$, and the range is $\{0, 1, 3, 2\}$.

More About...



Plants

Bamboo is a type of grass. It can vary in height from one-foot dwarf plants to 100-foot giant timber plants.

Source: American Bamboo Society

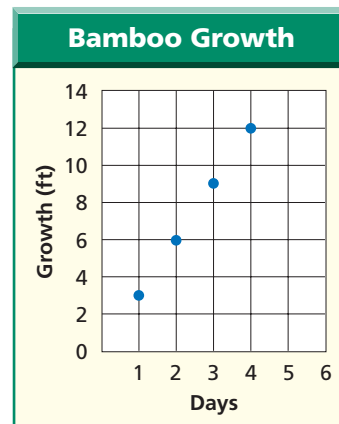
Example 4 *Apply Relations*

PLANTS Some species of bamboo grow 3 feet in one day.

- Make a table of ordered pairs in which the x -coordinate represents the number of days and the y -coordinate represents the amount of growth for 1, 2, 3, and 4 days.
- Graph the ordered pairs.

x	y	(x, y)
1	3	(1, 3)
2	6	(2, 6)
3	9	(3, 9)
4	12	(4, 12)

- Describe the graph.
The points appear to fall in a line.



Check for Understanding

Concept Check

- OPEN ENDED** Give an example of an ordered pair, and identify the x - and y -coordinate.
- Name three ways to represent a relation.
- Define *domain* and *range*.

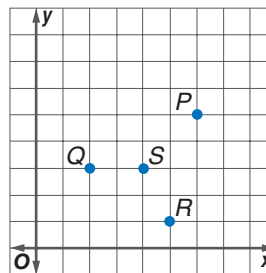
Guided Practice

Graph each point on a coordinate system.

- $H(5, 3)$
- $D(6, 0)$

Refer to the coordinate system shown at the right. Write the ordered pair that names each point.

- Q
- P



Express each relation as a table and as a graph. Then determine the domain and range.

- $\{(2, 5), (0, 2), (5, 5)\}$
- $\{(1, 6), (6, 4), (0, 2), (3, 1)\}$

Application

ENTERTAINMENT For Exercises 10 and 11, use the following information. It costs \$4 to buy a student ticket to the movies.

- Make a table of ordered pairs in which the x -coordinate represents the number of student tickets and the y -coordinate represents the cost for 2, 4, and 5 tickets.
- Graph the ordered pairs (number of tickets, cost).

Practice and Apply

Homework Help

For Exercises	See Examples
12–17	1
18–23	2
26–30, 37–43	4
31–36	3

Extra Practice

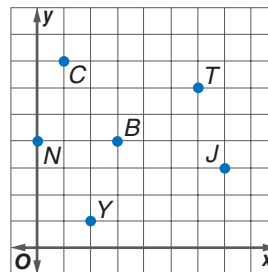
See page 725.

Graph each point on a coordinate system.

- $A(3, 3)$
- $D(1, 8)$
- $G(2, 7)$
- $X(7, 2)$
- $P(0, 6)$
- $N(4, 0)$

Refer to the coordinate system shown at the right. Write the ordered pair that names each point.

- C
- J
- N
- T
- Y
- B



- What point lies on both the x -axis and y -axis?
- Where are all of the possible locations for the graph of (x, y) if $y = 0$, if $x = 0$?

SCIENCE For Exercises 26 and 27, use the following information. The average speed of a house mouse is 12 feet per second.

Source: *Natural History Magazine*

- Find the distance traveled in 3, 5, and 7 seconds.
- Graph the ordered pairs (time, distance).

SCIENCE For Exercises 28–30, use the following information.

Keyson is conducting a physics experiment. He drops a tennis ball from a height of 100 centimeters and then records the height after each bounce. The results are shown in the table.

Bounce	0	1	2	3	4
Height (cm)	100	50	25	13	6

28. Write a set of ordered pairs for the data.
29. Graph the data.
30. How high do you think the ball will bounce on the fifth bounce? Explain.

Express each relation as a table and as a graph. Then determine the domain and range.

31. $\{(4, 5), (5, 2), (1, 6)\}$
32. $\{(6, 8), (2, 9), (0, 1)\}$
33. $\{(7, 0), (3, 2), (4, 4), (5, 1)\}$
34. $\{(2, 4), (1, 3), (5, 6), (1, 1)\}$
35. $\{(0, 1), (0, 3), (0, 5), (2, 0)\}$
36. $\{(4, 3), (3, 4), (1, 2), (2, 1)\}$

AIR PRESSURE For Exercises 37–39, use the table and the following information.

The air pressure decreases as the distance from Earth increases. The table shows the air pressure for certain distances.

Height (mi)	Pressure (lb/in ²)
sea level	14.7
1	10.2
2	6.4
3	4.3
4	2.7
5	1.6

37. Write a set of ordered pairs for the data.
38. Graph the data.
39. State the domain and the range of the relation.

SCIENCE For Exercises 40–43, use the following information and the information at the left.

Water boils at sea level at 100°C. The boiling point of water decreases about 5°C for every mile above sea level.

40. Make a table that shows the boiling point at sea level and at 1, 2, 3, 4, and 5 miles above sea level.
41. Show the data as a set of ordered pairs.
42. Graph the ordered pairs.
43. At about what temperature does water boil in Albuquerque, New Mexico? in Alpine, Texas? (*Hint*: 1 mile = 5280 feet)
44. **CRITICAL THINKING** Where are all of the possible locations for the graph of (x, y) if $x = 4$?
45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are ordered pairs used to graph real-life data?

Include the following in your answer:

- an explanation of how an ordered pair identifies a specific point on a graph, and
- an example of a situation where ordered pairs are used to graph data.

More About . . .



Science

Albuquerque, New Mexico, is at 7200 feet above sea level. Alpine, Texas, is at 4490 feet above sea level.

Source: *The World Almanac*



46. Graph each relation on a coordinate system. Then find the coordinates of another point that follows the pattern in the graph.

a.

x	1	3	5	7
y	2	4	6	8

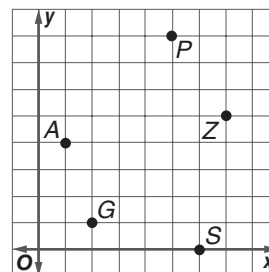
b.

x	0	2	4	6
y	10	8	6	4



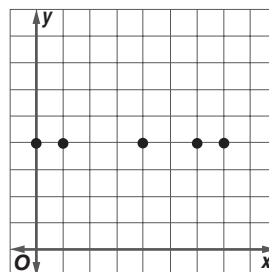
47. State the domain of the relation shown in the graph.

- (A) {0, 1, 4, 5, 8}
- (B) {A, G, P, S, Z}
- (C) {0, 1, 2, 4, 5}
- (D) {1, 2, 5, 6, 7}



48. What relationship exists between the x - and y -coordinates of each of the data points shown on the graph?

- (A) The y -coordinate varies, and the x -coordinate is always 4.
- (B) The y -coordinate is 4 more than the x -coordinate.
- (C) The sum of the x - and y -coordinate is always 4.
- (D) The x -coordinate varies, and the y -coordinate is always 4.



Extending the Lesson

49. Draw a coordinate grid.

- a. Graph (2, 1), (2, 4), and (5, 1).
- b. Connect the points with line segments. Describe the figure formed.
- c. Multiply each coordinate in the set of ordered pairs by 2.
- d. Graph the new ordered pairs. Connect the points with line segments. What figure is formed?
- e. **MAKE A CONJECTURE** How do the figures compare? Write a sentence explaining the similarities and differences of the figures.

Maintain Your Skills

Mixed Review

ALGEBRA Solve each equation mentally. (Lesson 1-5)

50. $a + 6 = 17$

51. $7t = 42$

52. $\frac{54}{n} = 6$

53. Name the property shown by $4 \cdot 1 = 4$. (Lesson 1-4)

ALGEBRA Evaluate each expression if $a = 5$, $b = 1$, and $c = 3$. (Lesson 1-3)

54. $ca - cb$

55. $5a - 6c$

Write a numerical expression for each verbal phrase. (Lesson 1-2)

56. fifteen less than twenty-one

57. the product of ten and thirty

Getting Ready for the Next Lesson

BASIC SKILL Find each quotient.

58. $74 \div 2$

59. $96 \div 8$

60. $102 \div 3$

61. $112 \div 4$

62. $80 \div 16$

63. $91 \div 13$

64. $132 \div 22$

65. $153 \div 17$





Algebra Activity

A Preview of Lesson 1-7

Scatter Plots

Sometimes, it is difficult to determine whether a relationship exists between two sets of data by simply looking at them. To determine whether a relationship exists, we can write the data as a set of ordered pairs and then graph the ordered pairs on a coordinate system.

Collect the Data

Let's investigate whether a relationship exists between height and arm span.

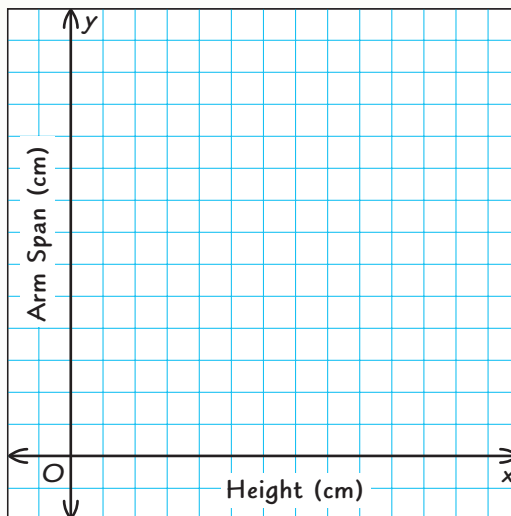
Step 1 Work with a partner. Use a centimeter ruler to measure the length of your partner's height and arm span to the nearest centimeter. Record the data in a table like the one shown.

Name	Height (cm)	Arm Span (cm)

Step 2 Extend the table. Combine your data with that of your classmates.

Step 3 Make a list of ordered pairs in which the x -coordinate represents height and the y -coordinate represents arm span.

Step 4 Draw a coordinate grid like the one shown and graph the ordered pairs (height, arm span).



Analyze the Data

1. Does there appear to be a trend in the data? If so, describe the trend.

Make a Conjecture

- Estimate the arm span of a person whose height is 60 inches. 72 inches.
- How does a person's arm span compare to his or her height?
- Suppose the variable x represents height, and the variable y represents arm span. Write an expression for arm span.

Extend the Activity

5. Collect and graph data to determine whether a relationship exists between height and shoe length. Explain your results.



1-7

Scatter Plots

What You'll Learn

- Construct scatter plots.
- Interpret scatter plots.

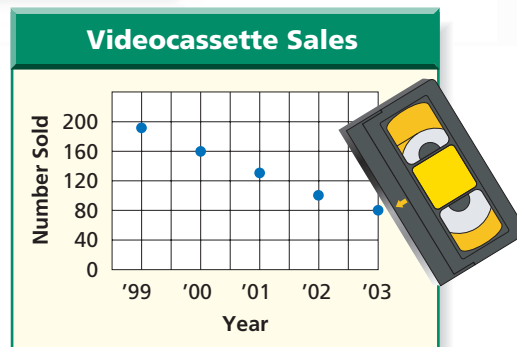
Vocabulary

- scatter plot

How can scatter plots help spot trends?

Suppose you work in the video department of a home entertainment store. The number of movies on videocassettes you have sold in a five-year period is shown in the graph.

- What appears to be the trend in sales of movies on videocassette?
- Estimate the number of movies on videocassette sold for 2005.



CONSTRUCT SCATTER PLOTS A **scatter plot** is a graph that shows the relationship between two sets of data. In a scatter plot, two sets of data are graphed as ordered pairs on a coordinate system.

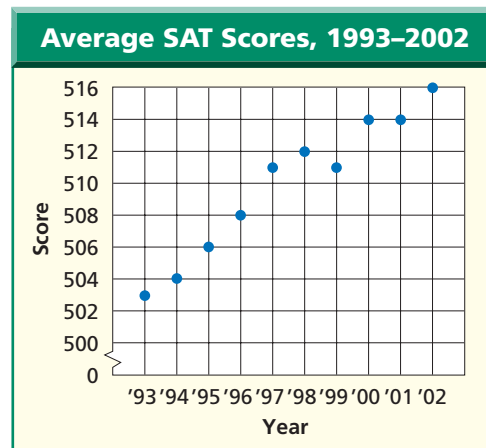
Example 1 Construct a Scatter Plot

TEST SCORES The table shows the average SAT math scores from 1993–2002. Make a scatter plot of the data.

Year	'93	'94	'95	'96	'97	'98	'99	'00	'01	'02
Score	503	504	506	508	511	512	511	514	514	516

Source: *The College Board*

Let the horizontal axis, or x -axis, represent the year. Let the vertical axis, or y -axis, represent the score. Then graph ordered pairs (year, score).



Concept Check True or false: A scatter plot represents one set of data. Explain.

INTERPRET SCATTER PLOTS

The following scatter plots show the types of relationships or patterns of two sets of data.

Study Tip

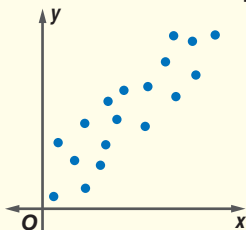
Scatter Plots

Data that appear to go uphill from left to right show a positive relationship. Data that appear to go downhill from left to right show a negative relationship. Data that appear to go downhill from left to right show a negative relationship.

Concept Summary

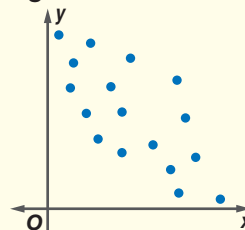
Types of Relationships

Positive Relationship



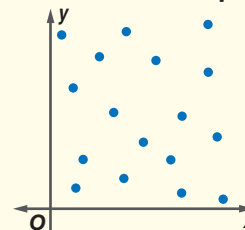
As x increases, y increases.

Negative Relationship



As x increases, y decreases.

No Relationship



No obvious pattern.

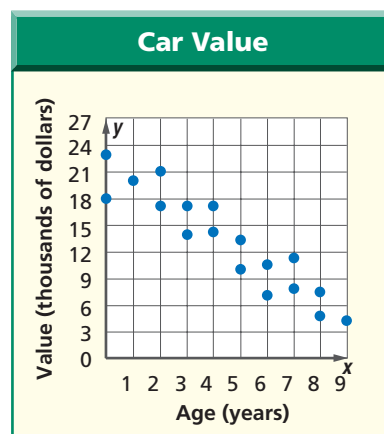
✓ Concept Check What type of relationship is shown on a graph that shows as the values of x increase, the values of y decrease?

Example 2 Interpret Scatter Plots

Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship. Explain your answer.

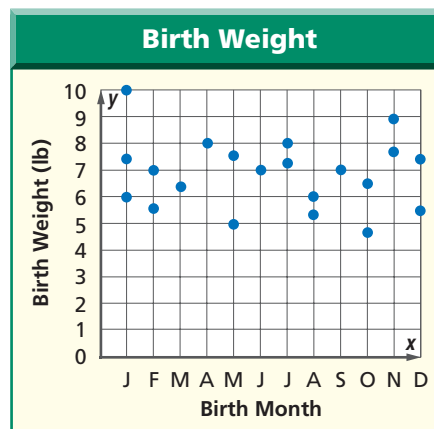
a. age of car and value of car

As the age of a car increases, the value of the car decreases. So, a scatter plot of the data would show a negative relationship.



b. birth month and birth weight

A person's birth weight is not affected by their birth month. Therefore, a scatter plot of the data would show no relationship.



You can also use scatter plots to spot trends, draw conclusions, and make predictions about the data.





Biologist

Wildlife biologists work in the field of fish and wildlife conservation. Duties may include studying animal populations and monitoring trends of migrating animals.

Online Research
For information about a career as a wildlife biologist, visit:
www.pre-alg.com/careers

Example 3 Use Scatter Plots to Make Predictions

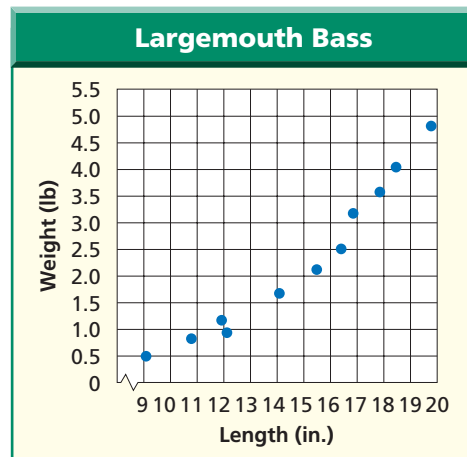
BIOLOGY A biologist recorded the lengths and weights of some largemouth bass. The table shows the results.

Length (in.)	9.2	10.9	12.3	12.0	14.1	15.5	16.4	16.9	17.7	18.4	19.8
Weight (lb)	0.5	0.8	0.9	1.3	1.7	2.2	2.5	3.2	3.6	4.1	4.8

a. **Make a scatter plot of the data.**
Let the horizontal axis represent length, and let the vertical axis represent weight. Graph the data.

b. **Does the scatter plot show a relationship between the length and weight of a largemouth bass? Explain.**

As the length of the bass increases, so does its weight. So, the scatter plot shows a positive relationship.



c. **Predict the weight of a bass that measures 22 inches.**

By looking at the pattern in the graph, we can predict that the weight of a bass measuring 22 inches would be between 5 and 6 pounds.

Check for Understanding

- Concept Check**
- List three ways a scatter plot can be used.
 - OPEN ENDED** Draw a scatter plot with ten ordered pairs that show a negative relationship.
 - Name the three types of relationships shown by scatter plots.

- Guided Practice** Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship. Explain your answer.
- hours worked and earnings
 - hair color and height

Application SCHOOL For Exercises 6 and 7, use the table that shows the heights and grade point averages of the students in Mrs. Stanley's class.

- Make a scatter plot of the data.
- Does there appear to be a relationship between the scores? Explain.

Name	Height (in.)	GPA
Jenna	66	3.6
Michael	61	3.2
Laura	59	3.9
Simon	64	2.8
Marcus	61	3.8
Timothy	65	3.1
Brandon	70	2.6
Emily	64	2.2
Eduardo	65	4.0

Practice and Apply

Homework Help

For Exercises	See Examples
8–13	2
14–20	1, 3

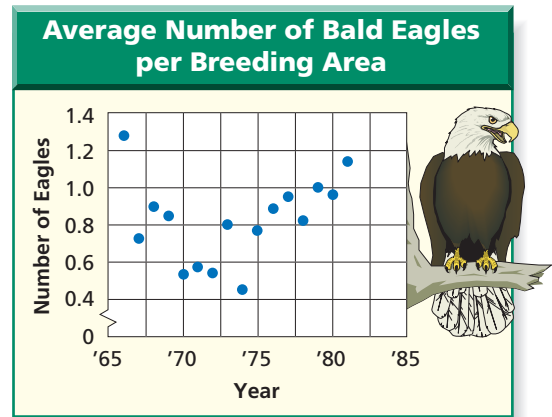
Extra Practice
See page 726.

Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship. Explain your answer.

- size of household and amount of water bill
- number of songs on a CD and cost of a CD
- size of a car's engine and miles per gallon
- speed and distance traveled
- outside temperature and amount of heating bill
- size of a television screen and the number of channels it receives

ANIMALS For Exercises 14–16, use the scatter plot shown.

- Do the data show a *positive*, *negative*, or *no* relationship between the year and the number of bald eagle hatchlings?
- What appears to be the trend in the number of hatchlings between 1965 and 1972?
- What appears to be the trend between 1972 and 1985?



Source: CHANCE

Online Research Data Update How has the total number of bald eagle pairs in the United States changed since 1980? Visit www.pre-alg.com/data_update to learn more.

WebQuest

The high and low temperatures for your vacation destinations can be shown in a scatter plot. Visit www.pre-alg.com/webquest to continue work on your WebQuest project.

BASKETBALL For Exercises 17–19, use the following information.

The number of minutes played and the number of field goal attempts for certain players of the Indiana Pacers for the 1999–2000 season is shown below.

Player	Minutes Played	Field Goal Attempts	Player	Minutes Played	Field Goal Attempts
Rose	2978	1196	Best	1691	561
Miller	2987	1041	Jackson	2190	570
Smits	1852	890	Perkins	1620	441
Croshere	1885	653	Mullin	582	187
Davis	2127	602	McKey	634	108

- Make a scatter plot of the data.
- Does the scatter plot show any relationship? If so, is it positive or negative? Explain your reasoning.
- Suppose a player played 2500 minutes. Predict the number of field goal attempts for that player.
- RESEARCH** Use the Internet or another source to find two sets of sports statistics that can be shown in a scatter plot. Identify any trends in the data.

21. **CRITICAL THINKING** Refer to Example 1 on page 40. Do you think the trend in the test scores would continue in the years to come? Explain your reasoning.
22. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

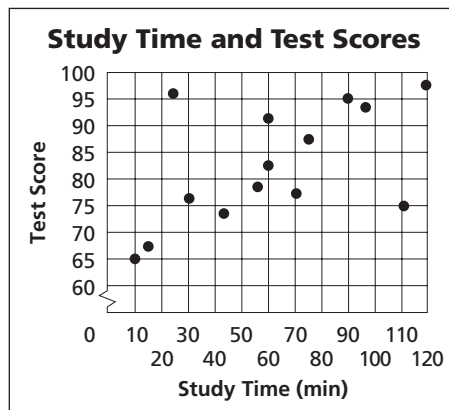
How can scatter plots help us spot trends?

Include the following in your answer:

- definitions of positive relationship, negative relationship, and no relationship, and
- examples of real-life situations that would represent each type of relationship.



The scatter plot shows the study time and test scores for the students in Mr. Mock's history class.



23. Based on the results, which of the following is an appropriate score for a student who studies for 1 hour?
- (A) 68 (B) 98
(C) 87 (D) 72
24. Which of the following is an appropriate score for a student who studies for 1.5 hours?
- (A) 78 (B) 92
(C) 81 (D) 74

Maintain Your Skills

Mixed Review

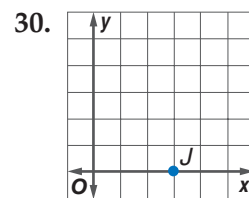
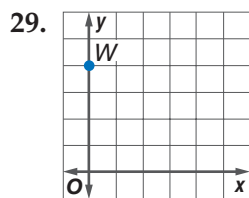
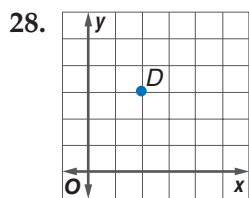
Graph each ordered pair on a coordinate system. (Lesson 1-6)

25. $M(3, 2)$

26. $X(5, 0)$

27. $K(0, 2)$

Write the ordered pair that names each point. (Lesson 1-6)



31. Determine the domain and range of the relation $\{(0, 9), (4, 8), (2, 3), (6, 1)\}$. (Lesson 1-6)

ALGEBRA Solve each equation mentally. (Lesson 1-5)

32. $3c = 81$

33. $15 - x = 8$

34. $8 = \frac{32}{m}$

35. **ALGEBRA** Simplify $15 + (b + 3)$. (Lesson 1-4)

ALGEBRA Evaluate each expression if $m = 8$ and $y = 6$. (Lesson 1-3)

36. $(2m + 3y) - m$

37. $3m + (y - 2) + 3$





Graphing Calculator Investigation

A Follow-Up of Lesson 1-7

Scatter Plots

You have learned that graphing ordered pairs as a scatter plot on a coordinate plane is one way to make it easier to “see” if there is a relationship. You can use a TI-83 Plus graphing calculator to create scatter plots.

SCIENCE A zoologist studied extinction times (in years) of island birds. The zoologist wanted to see if there was a relationship between the average number of nests and the time needed for each bird to become extinct on the islands. Use the table of data below to make a scatter plot.

Bird Name	Bird Size	Average Number of Nests	Extinction Time
Buzzard	Large	2.0	5.5
Quail	Large	1.0	1.5
Curlew	Large	2.8	3.1
Cuckoo	Large	1.4	2.5
Magpie	Large	4.5	10.0
Swallow	Small	3.8	2.6
Robin	Small	3.3	4.0
Stonechat	Small	3.6	2.4
Blackbird	Small	4.7	3.3
Tree-sparrow	Small	2.2	1.9

Step 1 Enter the data.

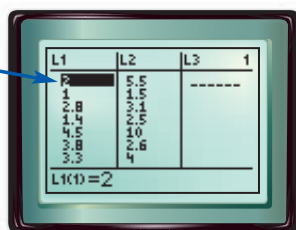
- Clear any existing lists.

KEYSTROKES: **STAT** **ENTER** **▲** **CLEAR**
ENTER

- Enter the average number of nests as L1 and extinction times as L2.

KEYSTROKES: **STAT** **ENTER** 2 **ENTER** 1
ENTER ... 2.2 **ENTER** **▶** 5.5
ENTER 1.5 **ENTER** ... 1.9
ENTER

The first data pair is (2, 5.5).



Step 2 Format the graph.

- Turn on the statistical plot.

KEYSTROKES: **2nd** **[STAT PLOT]** **ENTER**
ENTER

- Select the scatter plot, L1 as the Xlist and L2 as the Ylist.

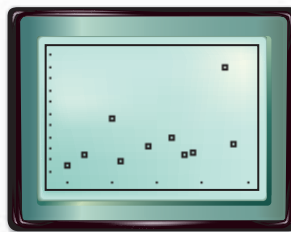
KEYSTROKES: **▼** **ENTER** **▼** **2nd** **[L1]** **ENTER**
2nd **[L2]** **ENTER**



Graphing Calculator Investigation

Step 3 *Graph the data.*

- Display the scatter plot.
KEYSTROKES: **ZOOM** 9
- Use the **TRACE** feature and the left and right arrow keys to move from one point to another.



Exercises

1. Press **TRACE**. Use the left and right arrow keys to move from one point to another. What do the coordinates of each data point represent?
2. Describe the scatter plot.
3. Is there a relationship between the average number of nests and extinction times? If so, write a sentence or two that describes the relationship.
4. Are there any differences between the extinction times of large birds versus small birds?
5. Separate the data by bird size. Enter average number of nests and extinction times for large birds as lists L1 and L2 and for small birds as lists L3 and L4. Use the graphing calculator to make two scatter plots with different marks for large and small birds. Does your scatter plot agree with your answer in Exercise 4? Explain.

For Exercises 6–8, make a scatter plot for each set of data and describe the relationship, if any, between the x - and y -values.

6.

x	y
70	323
80	342
40	244
50	221
30	121
80	399
60	230
60	200
50	215
40	170

7.

x	y
8	89
5	32
9	30
10	18
3	26
4	72
10	51
7	34
6	82
7	60

8.

x	5.2	5.8	6.3	6.7	7.4	7.6	8.4	8.5	9.1
y	12.1	11.9	11.5	9.8	10.2	9.6	8.8	9.1	8.5

9. **RESEARCH** Find two sets of data on your own. Then determine whether a relationship exists between the data.

Vocabulary and Concept Check

algebraic expression (p. 17)
 conjecture (p. 7)
 coordinate plane (p. 33)
 coordinate system (p. 33)
 counterexample (p. 25)
 deductive reasoning (p. 25)
 defining a variable (p. 18)
 domain (p. 35)
 equation (p. 28)
 evaluate (p. 12)

graph (p. 34)
 inductive reasoning (p. 7)
 numerical expression (p. 12)
 open sentence (p. 28)
 ordered pair (p. 33)
 order of operations (p. 12)
 origin (p. 33)
 properties (p. 23)
 range (p. 35)
 relation (p. 35)

scatter plot (p. 40)
 simplify (p. 25)
 solution (p. 28)
 solving the equation (p. 28)
 variable (p. 17)
 x -axis (p. 33)
 x -coordinate (p. 33)
 y -axis (p. 33)
 y -coordinate (p. 33)

Choose the letter of the term that best matches each statement or phrase.
 Use each letter once.

- $m + 3n - 4$
- to find the value of a numerical expression
- the set of all y -coordinates of a relation
- $20 + 12 \div 4 - 1 \times 2$
- the set of all x -coordinates of a relation

- numerical expression
- evaluate
- domain
- algebraic expression
- range

Lesson-by-Lesson Review

1-1 Using a Problem-Solving Plan

See pages
6–10.

Concept Summary

- The four steps of the four-step problem-solving plan are *explore*, *plan*, *solve*, and *examine*.
- Some problems can be solved using inductive reasoning.

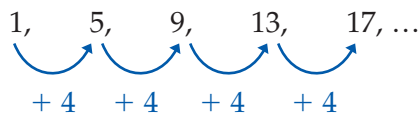
Example

What is the next term in the list 1, 5, 9, 13, 17, ...?

Explore We know the first five terms. We need to find the next term.

Plan Use inductive reasoning to determine the next term.

Solve Each term is 4 more than the previous term.



By continuing the pattern, the next term is $17 + 4$ or 21.

Examine Subtract 4 from each term. $21 - 4 = 17$, $17 - 4 = 13$, $13 - 4 = 9$, $9 - 4 = 5$, and $5 - 4 = 1$. So, the answer is correct.



Exercises Find the next term in each list. *See Example 2 on page 7.*

6. 2, 4, 6, 8, 10, ... 7. 5, 8, 11, 14, 17, ...
 8. 2, 6, 18, 54, 162, ... 9. 1, 2, 4, 7, 11, 16, ...

10. **FOOD** The table below shows the cost of various-sized hams. How much will it cost to buy a ham that weighs 7 pounds? *See Examples 1 and 3 on pages 7 and 8.*

Weight (lb)	1	2	3	4	5
Cost	\$4.38	\$8.76	\$13.14	\$17.52	\$21.90

1-2 Numbers and Expressions

See pages 12–16.

Concept Summary

- When evaluating an expression, follow the order of operations.
 - Step 1** Simplify the expressions inside grouping symbols.
 - Step 2** Do all multiplications and/or divisions from left to right.
 - Step 3** Do all additions and/or subtractions from left to right.

Example

Find the value of $3[(10 - 7) + 2]$.

$$\begin{aligned} 3[(10 - 7) + 2] &= 3[3 + 2] && \text{Evaluate } (10 - 7). \\ &= 3[5] && \text{Add 3 and 2.} \\ &= 15 && \text{Multiply 3 and 5.} \end{aligned}$$

Exercises Find the value of each expression. *See Example 1 on page 13.*

11. $7 + 3 \cdot 5$ 12. $36 \div 9 - 3$ 13. $5 \cdot (7 - 2) - 9$
 14. $\frac{2(17 + 4)}{3}$ 15. $18 \div (7 - 4) + 6$ 16. $4[9 + (1 \cdot 16) - 8]$

1-3 Variables and Expressions

See pages 17–21.

Concept Summary

- To evaluate an algebraic expression, replace each variable with its known value, and then use the order of operations.

Example

Evaluate $5a + 2$ if $a = 7$.

$$\begin{aligned} 5a + 2 &= 5(7) + 2 && \text{Replace } a \text{ with 7.} \\ &= 35 + 2 && \text{Multiply 5 and 7.} \\ &= 37 && \text{Add 35 and 2.} \end{aligned}$$

Exercises ALGEBRA Evaluate each expression if $x = 3$, $y = 8$, and $z = 5$. *See Examples 1 and 2 on pages 17 and 18.*

17. $y + 6$ 18. $17 - 2x$ 19. $z - 3 + y$
 20. $6x - 2z + 7$ 21. $\frac{6y}{x} + 9$ 22. $9x - (y + z)$

- Extra Practice, see pages 724–726.
- Mixed Problem Solving, see page 758.

1-6 Ordered Pairs and Relations

See pages
33–38.

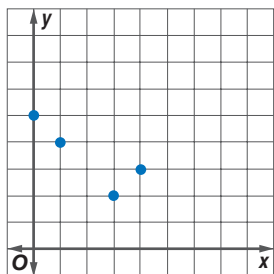
Concept Summary

- Ordered pairs are used to graph a point on a coordinate system.
- A relation is a set of ordered pairs. The set of x -coordinates is the domain, and the set of y -coordinates is the range.

Example

Express the relation $\{(1, 4), (3, 2), (4, 3), (0, 5)\}$ as a table and as a graph. Then determine the domain and range.

x	y
1	4
3	2
4	3
0	5



The domain is $\{1, 3, 4, 0\}$,
and the range is $\{4, 2, 3, 5\}$.

Exercises Express each relation as a table and as a graph. Then determine the domain and range. See Example 3 on page 35.

33. $\{(2, 3), (6, 1), (7, 5)\}$

34. $\{(0, 2), (1, 7), (5, 2), (6, 5)\}$

1-7 Scatter Plots

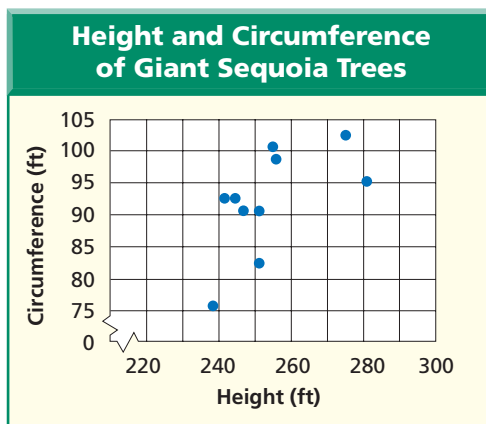
See pages
40–44.

Concept Summary

- A scatter plot is a graph that shows the relationship between two sets of data.

Example

The scatter plot shows the approximate heights and circumferences of various giant sequoia trees.



Exercises Refer to the scatter plot. See Example 3 on page 42.

- Does the scatter plot show a *positive*, *negative*, or *no* relationship? Explain.
- Predict the circumference of a 245-foot sequoia. Explain your reasoning.

Practice Test

Vocabulary and Concepts

- Write the steps of the four-step problem-solving plan.
- List the order of operations used to find the value of a numerical expression.

Skills and Applications

Find the value of each expression.

- $24 - 8 \div 2 \cdot 3$
- $16 \div 4 + 3(9 - 7)$
- $3[18 - 5(7 - 5 + 1)]$

Write a numerical expression for each verbal phrase.

- three less than fifteen
- twelve increased by seven
- the quotient of twelve and six

ALGEBRA Evaluate each expression if $a = 7$, $b = 3$, and $c = 5$.

- $4a - 3c$
- $42 \div [a(c - b)]$
- $5c + (a + 2b) - 8$
- What property is shown by $(5 \cdot 6) \cdot 8 = 5 \cdot (6 \cdot 8)$?

ALGEBRA Simplify each expression.

- $9 + (p + 3)$
- $6 \cdot (7 \cdot k)$

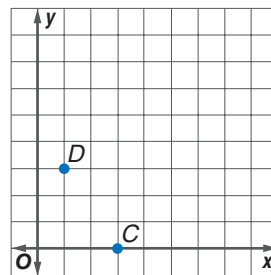
ALGEBRA Solve each equation mentally.

- $4m = 20$
- $16 - a = 9$
- Graph $A(2, 5)$ on a coordinate system.

Refer to the coordinate system shown at the right. Write the ordered pair that names each point.

- C
- D

- Express $\{(8, 5), (4, 3), (2, 2), (6, 1)\}$ as a table and as a graph. Then determine the domain and range.



Determine whether a scatter plot of the data for the following might show a *positive*, *negative*, or *no* relationship. Explain your answer.

- outside temperature and air conditioning bill
- number of siblings and height
- Find the next three terms in the list 3, 5, 9, 15,
- MONEY** Mrs. Adams rents a car for a week and pays \$79 for the first day and \$49 for each additional day. Mr. Lowe rents a car for \$350 a week. Which was the better deal? Explain.
- STANDARDIZED TEST PRACTICE** Katie purchased 6 loaves of bread at the grocery store and paid a total of \$12. Which equation can be used to find how much Katie paid for each loaf of bread?

(A) $x + 6 = 12$

(B) $6x = 12$

(C) $x - 6 = 12$

(D) $x \div 6 = 12$



Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

- Find the next two terms in the pattern 4, 12, 36, 108, ... (Lesson 1-1)
 - (A) 116 and 124
 - (B) 116 and 140
 - (C) 324 and 648
 - (D) 324 and 972
- Evaluate $2(15 - 3 \cdot 4)$. (Lesson 1-2)
 - (A) 6
 - (B) 16
 - (C) 18
 - (D) 96
- Of the six books in a mystery series, four have 200 pages and two have 300 pages. Which expression represents the total number of pages in the series? (Lesson 1-3)
 - (A) $200 + 300$
 - (B) $6(200 + 300)$
 - (C) $4(200) + 2(300)$
 - (D) $6(200) + 6(300)$
- The postage for a first-class letter is \$0.34 for the first ounce and \$0.21 for each additional ounce. Which expression best represents the cost of postage for a letter that weighs 5 ounces? (Lesson 1-3)
 - (A) $0.34 + 0.21(5)$
 - (B) $0.21 + 0.34(4)$
 - (C) $0.34(5)$
 - (D) $0.34 + 0.21(4)$
- Which property is represented by the equation below? (Lesson 1-4)

$$8 \cdot (5 \cdot 3) = (8 \cdot 5) \cdot 3$$
 - (A) Commutative Property of Addition
 - (B) Commutative Property of Multiplication
 - (C) Associative Property of Multiplication
 - (D) Identity Property of Multiplication
- Which number is the solution of the equation $17 - 2x = 9$? (Lesson 1-5)
 - (A) 2
 - (B) 4
 - (C) 6
 - (D) 8

Test-Taking Tip



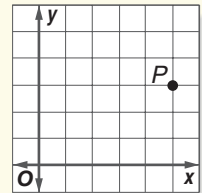
Question 6

To solve an equation, you can replace the variable in the equation with the values given in each answer choice. The answer choice that results in a true statement is the correct answer.

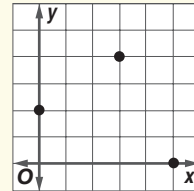
- Which sentence does the equation $n + 9 = 15$ represent? (Lesson 1-5)
 - (A) A number is the sum of 9 and 15.
 - (B) A number decreased by 9 is 15.
 - (C) The product of a number and 9 is 15.
 - (D) Nine more than a number is 15.

- What are the coordinates of point P? (Lesson 1-6)

- (A) (3, 5)
- (B) (5, 3)
- (C) (3, 3)
- (D) (5, 5)



- Which table shows the set of ordered pairs that represents the points graphed on the grid below? (Lesson 1-6)



- (A)

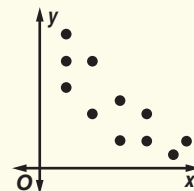
x	y
2	0
3	4
0	5
- (B)

x	y
0	2
4	3
5	0
- (C)

x	y
0	2
3	4
5	0
- (D)

x	y
2	0
4	3
0	5

- What type of relationship does the scatter plot below show? (Lesson 1-7)

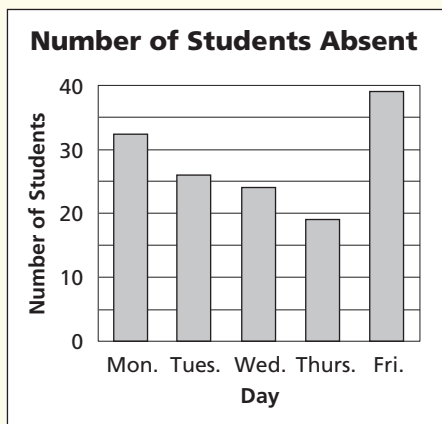


- (A) positive
- (B) negative
- (C) associative
- (D) none

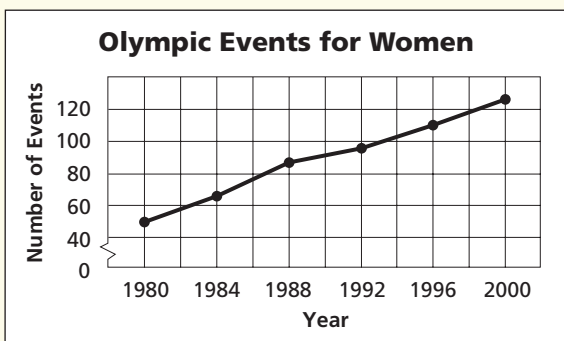
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

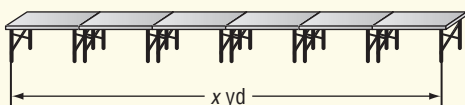
11. The graph below shows the number of students absent from school each day of one week. On what day were the fewest students absent? (Prerequisite Skill, p. 722)



12. The number of Olympic events for women is shown. About how many more events for women were held in 2000 than in 1980? (Prerequisite Skill, p. 722)



13. Six tables positioned in a row will be used to display science projects. Each table is 8 feet long. How many yards of fabric are needed to make a banner that will extend from one end of the row of tables to the other? (Lesson 1-1)



14. What is the value of the expression $5 + 4 \times 6 \div 3$? (Lesson 1-2)
15. Evaluate $x(xy + 3)$ if $x = 5$ and $y = 2$. (Lesson 1-3)
16. Write *14 is 12 less than twice the value of x* as an equation. (Lesson 1-5)

Part 3 Extended Response

Record your answers on a sheet of paper. Show your work.

17. Kenneth is recording the time it takes him to run various distances. The results are shown. (Lesson 1-6)

Distance (mi)	2	3	5	7	9
Time (min)	13	20	35	53	72

- a. Write a set of ordered pairs for the data.
- b. Graph the data.
- c. How many minutes do you think it will take Kenneth to run 4 miles? Explain.
- d. Predict how far Kenneth will run if he runs for 1 hour.
18. The table below shows the results of a survey about the average time that individual students spend studying on weekday evenings. (Lesson 1-7)

Grade	Time (min)	Grade	Time (min)
2	20	6	60
2	15	6	45
2	20	6	55
4	30	6	60
4	20	8	70
4	25	8	80
4	40	8	75
4	30	8	60

- a. Make a scatter plot of the data.
- b. What are the coordinates of the point that represents the longest time spent on homework?
- c. Does a relationship exist between grade level and time spent studying? If so, write a sentence to describe the relationship. If not, explain why not.